

FLUID MODELING FOR THE KNUDSEN COMPRESSOR: CASE OF POLYATOMIC GASES

SHIGERU TAKATA, HITOSHI FUNAGANE, AND KAZUO AOKI

Department of Mechanical Engineering and Science and
Advanced Research Institute of Fluid Science and Engineering
Graduate School of Engineering, Kyoto University
Kyoto 606-8501, Japan

(Communicated by Pierre Degond)

ABSTRACT. A fluid-dynamic system describing the behavior of a polyatomic gas in a Knudsen compressor, based on a periodic arrangement of narrower and wider two-dimensional channels and on a periodic saw-tooth temperature distribution, is derived, using the polyatomic version of the ellipsoidal statistical (ES) model of the Boltzmann equation, under the assumption that the channel width is much smaller than the length of a unit of the compressor (narrow channel approximation). The difference from the corresponding fluid-dynamic system for a monatomic gas is shown to be confined in the transport coefficients occurring in the fluid-dynamic equation. It is also shown that these coefficients in the present polyatomic-gas case are readily obtained by a simple conversion from the corresponding coefficients for the BGK model for a monatomic gas. Some numerical simulations based on the fluid-dynamic model are carried out, the results of which show that the properties of the Knudsen pump are little affected by the internal structure of a molecule.

1. Introduction. The Knudsen pump (or compressor) is a non-mechanical thermal pump for gases, the prototype of which is the device made by Knudsen in 1910 [24, 25]. The driving mechanism is the thermal transpiration, a gas flow caused by a temperature gradient along the wall of a pipe, which is peculiar to rarefied gases (gases in low-density circumstances or in micro scales). Since the pump does not contain any moving parts, it has a potential applicability for specific purposes in vacuum and micro technologies. Therefore, the properties of the pump have been investigated in detail numerically and experimentally [30, 41, 43, 23, 39, 4, 37, 40, 20, 19].

A typical configuration of the Knudsen pump is a long tube, consisting of alternately arranged thin and thick pipes, with an imposed periodic temperature distribution along the pipe wall (say, saw-tooth distribution increasing in the thin pipes and decreasing in the thick pipes). Because of the opposite temperature gradients, the flows induced in the thick pipes are in the opposite direction to the flows induced in the thin pipes. However, these flows in the opposite directions do not cancel out completely because of the different thickness of the pipes. As a result, the flows in the thin pipes become dominant and cause a global one way flow, which

2000 *Mathematics Subject Classification.* Primary: 58F15, 58F17; Secondary: 53C35.

Key words and phrases. kinetic theory of gases, Boltzmann equation, diffusion model, Knudsen pump, thermal transpiration, Poiseuille flow.

This work was supported by the Grant-in-Aid for Scientific Research No. 20360046 from JSPS.

has a pumping effect. As is seen from this mechanism, the performance of the pump can be improved by a cascade system using many thick and thin pipes (i.e., many units).

Since the thermal transpiration is an effect of a rarefied gas, one has to use kinetic theory to analyze the behavior of the gas in the Knudsen pump. The most widely used tool for simulations of rarefied gas flows is the direct simulation Monte Carlo (DSMC) method [8]. However, for the pump consisting of large number of units, the computational cost required for the DSMC method makes it impossible to perform practical computations. The same is true for other numerical methods, such as finite-difference methods based on the model Boltzmann equations. In contrast, a simple and convenient macroscopic system, composed of an equation of convection-diffusion type and a connection condition at the junctions of the thick and thin pipes, was proposed recently for the purpose of overcoming the computational difficulty [2, 42, 3]. The system, which is valid for any Knudsen number (the mean free path of the gas molecules divided by the characteristic length of the system), was derived systematically from the Boltzmann equation under the assumption that the pipes are sufficiently thin compared with their length. The convection-diffusion-type equation contains variable coefficients (say, transport coefficients), which correspond to the mass-flow rate of the thermal transpiration and that of the Poiseuille flow (the flow caused by the pressure gradient along the pipe axis) in an infinitely long single pipe as the functions of the Knudsen number. Applying the macroscopic system with the transport coefficients based on the model Boltzmann equations, some quantitative results for two-dimensional Knudsen pumps composed of wider and narrower channels between two plates have been obtained [42, 3]. For instance, the fact that the Knudsen pump is applicable to gas separation was pointed out in [42].

The previous results are formally based on the Boltzmann equation for a single gas or a mixture of gases of monatomic molecules, and the quantitative results are based on the transport coefficients obtained by using the model Boltzmann equations for monatomic molecules [the Bhatnagar–Gross–Krook (BGK) model [7, 44] for a single gas in [3], the ellipsoidal statistical (ES) model [21, 22] for a single gas in [5], and the McCormack model [26] for a gaseous mixture in [42]]. The aim of the present paper is to extend the macroscopic system to a single polyatomic gas and clarify the effect of the polyatomic molecules on the properties of the Knudsen pump. We employ the ES model for a polyatomic gas [1] as the basic equation and derive the macroscopic system following the procedure in [2, 42, 3]. As in the examples in [42, 3], we restrict ourselves to the case of a two-dimensional Knudsen pump consisting of wider and narrower channels between two plates.

The paper is organized as follows. In Sec. 2, we first formulate the flow of a rarefied polyatomic gas between two parallel infinite plates using the ES model and then derive a convection-diffusion-type equation by a formal asymptotic analysis, under the assumption that the channel width is much smaller than the length scale of variation of the wall temperature along the channel. In Sec. 3, we derive the connection condition at the junctions of wider and narrower channels for the convection-diffusion-type equation when it is applied to each channels. In Sec. 4, we show that the transport coefficients in the convection-diffusion-type equation can be obtained readily from the corresponding quantities for the BGK model for

a monatomic gas by a simple conversion. Finally in Sec. 5, the behavior of a polyatomic gas in the Knudsen pump is investigated with the help of the macroscopic system.

2. Fluid-dynamic model for a single channel. In this section, we consider a rarefied gas in a single channel between two parallel plates, and following the analysis in [42], [3], we derive a macroscopic equation of convection-diffusion type in the case where the channel width is much smaller than the length scale of variation along the channel.

2.1. Problem and assumptions. Let us consider a rarefied gas in a channel between two infinite parallel plates. We take the X_1 axis (of a rectangular coordinate system X_i) perpendicular to the plates and X_2 axis along them and let the plates be located at $X_1 = \pm D/2$. The temperature of the plates, which is common to both plates and depends only on X_2 , is denoted by $T_w(X_2)$.

We investigate the motion of the gas in the channel under the following assumptions:

- (i) The behavior of the gas is described by the ES model [1] of the Boltzmann equation for a polyatomic gas.
- (ii) The gas molecules undergo diffuse reflection on the plates.
- (iii) The problem is two dimensional, i.e., the physical quantities do not depend on X_3 .
- (iv) The scale of variation L_* of the plate temperature is much longer than the width of the channel D , i.e., $L_* \gg D$.

2.2. Basic equation and boundary conditions. Let us consider a polyatomic gas consisting of molecules with internal degree of freedom δ . The number of the molecules with position in $d\mathbf{X}$ around $\mathbf{X}$, velocity in $d\boldsymbol{\xi}$ around $\boldsymbol{\xi}$, and energy related to the internal degree of freedom in $d\mathcal{E}$ around $\mathcal{E}$ at time t is written as

$$\frac{1}{m} f(t, \mathbf{X}, \boldsymbol{\xi}, \mathcal{E}) d\mathbf{X} d\boldsymbol{\xi} d\mathcal{E}, \quad (1)$$

where f is the velocity and energy distribution function of the gas molecules, and m is the mass of a molecule. The equation for f , the ES model for a polyatomic gas, can be written in the following form [1] (see the last paragraph of this subsection for the difference in notations between [1] and the present paper):

$$\frac{\partial f}{\partial t} + \xi_1 \frac{\partial f}{\partial X_1} + \xi_2 \frac{\partial f}{\partial X_2} = A_c(T) \rho (\mathcal{G} - f), \quad (2a)$$

$$\mathcal{G} = \frac{\rho \Lambda_\delta \mathcal{E}^{\delta/2-1}}{(2\pi)^{3/2} |\underline{\mathcal{T}}|^{1/2} (RT_{\text{rel}})^{\delta/2}} \exp\left(-\frac{1}{2} {}^t(\boldsymbol{\xi} - \mathbf{v}) \cdot \underline{\mathcal{T}}^{-1} \cdot (\boldsymbol{\xi} - \mathbf{v}) - \frac{\mathcal{E}}{RT_{\text{rel}}}\right), \quad (2b)$$

$$\underline{\mathcal{T}} = (1 - \eta) \left\{ (1 - \nu) RT_{\text{tr}} \underline{Id} + \nu \underline{\Theta} \right\} + \eta RT \underline{Id}, \quad (2c)$$

$$\underline{\Theta} = \frac{1}{\rho} \int \int_0^\infty (\boldsymbol{\xi} - \mathbf{v}) \cdot {}^t(\boldsymbol{\xi} - \mathbf{v}) f d\mathcal{E} d\boldsymbol{\xi}, \quad (2d)$$

$$\rho = \int \int_0^\infty f d\mathcal{E} d\boldsymbol{\xi}, \quad (2e)$$

$$v_i = \frac{1}{\rho} \int \int_0^\infty \xi_i f d\mathcal{E} d\boldsymbol{\xi}, \quad (2f)$$

$$T_{\text{tr}} = \frac{1}{3\rho R} \int \int_0^\infty (\xi_i - v_i)^2 f d\mathcal{E} d\boldsymbol{\xi}, \quad (2g)$$

$$T_{\text{int}} = \frac{2}{\delta\rho R} \int \int_0^\infty \mathcal{E} f d\mathcal{E} d\boldsymbol{\xi}, \quad (2h)$$

$$T = \frac{3T_{\text{tr}} + \delta T_{\text{int}}}{3 + \delta}, \quad (2i)$$

$$T_{\text{rel}} = \eta T + (1 - \eta)T_{\text{int}}. \quad (2j)$$

Here, ρ is the mass density of the gas, v_i is the flow velocity, T is the temperature, T_{tr} is the temperature related to the translational energy, T_{int} is the temperature related to the energy of the internal degree of freedom, and R is the specific gas constant (the Boltzmann constant divided by the mass of a molecule); $\nu \in [-1/2, 1)$ and $\eta \in (0, 1]$ are the parameters to adjust the Prandtl number and the bulk viscosity to the gas under consideration [see Eqs. (6c) and (6d)]; $A_c(T)$ is a function of T such that $A_c(T)\rho$ is the collision frequency of the gas molecules, and Λ_δ is a dimensionless constant defined by

$$\Lambda_\delta^{-1} = \int_0^\infty s^{\delta/2-1} e^{-s} ds. \quad (3)$$

In addition, $\underline{\mathcal{T}}$ and $\underline{\Theta}$ are 3×3 symmetric matrices, $\underline{Id}$ is the 3×3 identity matrix, $|\underline{\mathcal{T}}|$ and $\underline{\mathcal{T}}^{-1}$ are the determinant and the inverse matrix of $\underline{\mathcal{T}}$, respectively, and the symbol t indicates the transpose operation. In what follows, for an arbitrary matrix $\underline{A}$, its (i, j) component, determinant, inverse matrix, and transposed matrix are denoted by A_{ij} , $|\underline{A}|$, $\underline{A}^{-1}$, and $^t\underline{A}$, respectively. In the above equations, $d\boldsymbol{\xi} = d\xi_1 d\xi_2 d\xi_3$, and the domain of integration with respect to ξ_i is the whole space of ξ_i . It should be noted that the pressure p and the stress tensor p_{ij} are given by

$$p = \rho RT, \quad (4a)$$

$$p_{ij} = \rho \Theta_{ij} = \int \int_0^\infty (\xi_i - v_i)(\xi_j - v_j) f d\mathcal{E} d\boldsymbol{\xi}. \quad (4b)$$

The vanishing right-hand side of Eq. (2a) is equivalent to the fact that f is a local equilibrium distribution f_{eq} [1]:

$$f_{\text{eq}} = \frac{\rho \Lambda_\delta}{(2\pi RT)^{3/2} (RT)^{\delta/2}} \mathcal{E}^{\delta/2-1} \exp\left(-\frac{(\xi_i - v_i)^2}{2RT} - \frac{\mathcal{E}}{RT}\right). \quad (5)$$

It is also known [1] that Eq. (2) leads to the viscosity μ , the thermal conductivity κ , the Prandtl number Pr , and the bulk viscosity μ_b in the following form:

$$\mu = \frac{1}{1 - \nu + \eta\nu} \frac{RT}{A_c(T)}, \quad (6a)$$

$$\kappa = \frac{\gamma}{\gamma - 1} R \frac{RT}{A_c(T)}, \quad (6b)$$

$$\text{Pr} = \frac{\gamma}{\gamma - 1} \frac{R\mu}{\kappa} = \frac{1}{1 - \nu + \eta\nu}, \quad (6c)$$

$$\mu_b = \frac{1}{\eta} \left(\frac{5}{3} - \gamma\right) \frac{\mu}{\text{Pr}}, \quad (6d)$$

with γ the ratio of the specific heats given by

$$\gamma = \frac{\delta + 5}{\delta + 3}. \quad (7)$$

Equation (2a) contains the set of parameters (ν, η, δ) characterizing the gas under consideration. In place of this set, we may use another set $(\text{Pr}, \mu_b/\mu, \delta)$ because of relations (6c), (6d), and (7) (cf. Secs. 2.4 and 4 and Appendix A).

The diffuse-reflection condition [10, 11, 35, 36], adapted to the present polyatomic case, on the channel walls are expressed as

$$f = \frac{\rho_w \Lambda_\delta}{(2\pi RT_w)^{3/2} (RT_w)^{\delta/2}} \mathcal{E}^{\delta/2-1} \exp\left(-\frac{\xi_i^2}{2RT_w} - \frac{\mathcal{E}}{RT_w}\right), \quad (X_1 = \pm D/2, \quad \xi_1 \leq 0), \quad (8a)$$

$$\rho_w = \pm \left(\frac{2\pi}{RT_w}\right)^{1/2} \int_{\xi_1 \geq 0} \int_0^\infty \xi_1 f d\mathcal{E} d\xi, \quad (X_1 = \pm D/2), \quad (8b)$$

where the upper (or lower) signs go together. The initial condition is given by

$$f(0, X_1, X_2, \xi, \mathcal{E}) = f^0(X_1, X_2, \xi, \mathcal{E}). \quad (9)$$

The mass-flow rate M in the X_2 direction (per unit time and per unit length in X_3) is expressed as

$$M = \int_{-D/2}^{D/2} \rho v_2 dX_1. \quad (10)$$

In [1], the energy $\mathcal{E}$, which is denoted by ε there, is assumed to be expressed as $\mathcal{E} = I^{2/\delta}$ in terms of a variable I , and I is used as an independent variable. More specifically, the distribution function in [1], which we denote by $\bar{f}(t, \mathbf{X}, \xi, I)$ here, is defined in such a way that

$$\frac{1}{m} \bar{f}(t, \mathbf{X}, \xi, I) d\mathbf{X} d\xi dI, \quad (11)$$

indicates the number of the molecules with position in $d\mathbf{X}$ around $\mathbf{X}$, velocity in $d\xi$ around ξ , and the variable I in dI around I at time t . Therefore, the relation between $\bar{f}$ and the present f is as follows:

$$f(t, \mathbf{X}, \xi, \mathcal{E}) = (\delta/2) \mathcal{E}^{\delta/2-1} \bar{f}(t, \mathbf{X}, \xi, \mathcal{E}^{\delta/2}). \quad (12)$$

In addition, Λ_δ in [1], which we denote $\bar{\Lambda}_\delta$ here, is related to Λ_δ in Eq. (3) as

$$\Lambda_\delta^{-1} = (2/\delta) (\bar{\Lambda}_\delta)^{-1}. \quad (13)$$

2.3. Dimensionless form. Let us introduce the following dimensionless quantities:

$$\begin{aligned} \hat{t} &= t/t_*, & x_1 &= X_1/D_*, & x_2 &= X_2/L_*, \\ \zeta_i &= \xi_i/(2RT_*)^{1/2}, & \hat{\mathcal{E}} &= \mathcal{E}/RT_*, \\ (\hat{f}, \hat{\mathcal{G}}) &= (f, \mathcal{G})/2\rho_*(2RT_*)^{-5/2}, \\ \hat{\rho} &= \rho/\rho_*, & \hat{v}_i &= v_i/(2RT_*)^{1/2}, \\ (\hat{T}_{\text{tr}}, \hat{T}_{\text{int}}, \hat{T}, \hat{T}_{\text{rel}}) &= (T_{\text{tr}}, T_{\text{int}}, T, T_{\text{rel}})/T_*, \\ \hat{p} &= p/p_*, & \hat{p}_{ij} &= p_{ij}/p_*, & \hat{M} &= M/\rho_*(2RT_*)^{1/2}D_*, \\ \hat{\rho}_w &= \rho_w/\rho_*, & \hat{T}_w &= T_w/T_*, & a &= D/D_*, \\ \hat{A}_c(\hat{T}) &= A_c(T)/A_c(T_*), & (\hat{\mathcal{T}}, \hat{\mathcal{Q}}) &= (\mathcal{T}, \mathcal{Q})/RT_*, \end{aligned} \quad (14)$$

where t_* is the reference time, D_* ($\ll L_*$) is the reference width of the channel, ρ_* is the reference density, T_* is the reference temperature, $p_* = R\rho_*T_*$ is the reference

pressure. The mean free path l_* of the gas molecules in the equilibrium state at rest is given as $l_* = (2/\sqrt{\pi})(2RT_*)^{1/2}/A_c(T_*)\rho_*$.

Then, Eq. (2) is transformed to the following dimensionless form:

$$\text{Sh} \frac{\partial \hat{f}}{\partial \hat{t}} + \zeta_1 \frac{\partial \hat{f}}{\partial x_1} + \epsilon \zeta_2 \frac{\partial \hat{f}}{\partial x_2} = \frac{2}{\sqrt{\pi}} \frac{1}{K_*} \hat{A}_c \hat{\rho} (\hat{\mathcal{G}} - \hat{f}), \quad (15a)$$

$$\hat{\mathcal{G}} = \frac{\hat{\rho} \Lambda_\delta}{\pi^{3/2} |\hat{\underline{T}}|^{1/2} \hat{T}_{\text{rel}}^{\delta/2}} \hat{\mathcal{E}}^{\delta/2-1} \exp\left(-{}^t(\underline{\zeta} - \underline{\hat{v}}) \cdot \hat{\underline{T}}^{-1} \cdot (\underline{\zeta} - \underline{\hat{v}}) - \frac{\hat{\mathcal{E}}}{\hat{T}_{\text{rel}}}\right), \quad (15b)$$

$$\hat{\underline{T}} = (1 - \eta) \{(1 - \nu) \hat{T}_{\text{tr}} \underline{Id} + \nu \hat{\underline{\Theta}}\} + \eta \hat{T} \underline{Id}, \quad (15c)$$

$$\hat{\underline{\Theta}} = \frac{2}{\hat{\rho}} \int \int_0^\infty (\underline{\zeta} - \underline{\hat{v}}) \cdot {}^t(\underline{\zeta} - \underline{\hat{v}}) \hat{f} d\hat{\mathcal{E}} d\underline{\zeta}, \quad (15d)$$

$$\hat{\rho} = \int \int_0^\infty \hat{f} d\hat{\mathcal{E}} d\underline{\zeta}, \quad (15e)$$

$$\hat{v}_i = \frac{1}{\hat{\rho}} \int \int_0^\infty \zeta_i \hat{f} d\hat{\mathcal{E}} d\underline{\zeta}, \quad (15f)$$

$$\hat{T}_{\text{tr}} = \frac{2}{3} \frac{1}{\hat{\rho}} \int \int_0^\infty (\zeta_i - \hat{v}_i)^2 \hat{f} d\hat{\mathcal{E}} d\underline{\zeta}, \quad (15g)$$

$$\hat{T}_{\text{int}} = \frac{2}{\delta} \frac{1}{\hat{\rho}} \int \int_0^\infty \hat{\mathcal{E}} \hat{f} d\hat{\mathcal{E}} d\underline{\zeta}, \quad (15h)$$

$$\hat{T} = \frac{3\hat{T}_{\text{tr}} + \delta \hat{T}_{\text{int}}}{3 + \delta}, \quad (15i)$$

$$\hat{T}_{\text{rel}} = \eta \hat{T} + (1 - \eta) \hat{T}_{\text{int}}, \quad (15j)$$

where $d\underline{\zeta} = d\zeta_1 d\zeta_2 d\zeta_3$, and the domain of integration with respect to ζ_i is the whole space of ζ_i . The Sh, ϵ , and K_* in Eq. (15) are the dimensionless parameters defined by

$$\text{Sh} = \frac{D_*}{t_*(2RT_*)^{1/2}}, \quad \epsilon = \frac{D_*}{L_*}, \quad K_* = \frac{l_*}{D_*}, \quad (16)$$

where Sh is the reference Strouhal number, and K_* is the reference Knudsen number. Corresponding to Eqs. (4a) and (4b), we have

$$\hat{p} = \hat{\rho} \hat{T}, \quad (17a)$$

$$\hat{p}_{ij} = \hat{\rho} \hat{\Theta}_{ij} = \int \int_0^\infty 2(\zeta_i - \hat{v}_i)(\zeta_j - \hat{v}_j) \hat{f} d\hat{\mathcal{E}} d\underline{\zeta}. \quad (17b)$$

The dimensionless form of the boundary condition (8) is given by

$$\hat{f} = \frac{\hat{\rho}_w \Lambda_\delta}{(\pi \hat{T}_w)^{3/2} \hat{T}_w^{\delta/2}} \hat{\mathcal{E}}^{\delta/2-1} \exp\left(-\frac{\zeta_i^2}{\hat{T}_w} - \frac{\hat{\mathcal{E}}}{\hat{T}_w}\right) \quad (x_1 = \pm a/2, \quad \zeta_1 \leq 0), \quad (18a)$$

$$\hat{\rho}_w = \pm 2 \sqrt{\frac{\pi}{\hat{T}_w}} \int_{\zeta_1 \geq 0} \int_0^\infty \zeta_1 \hat{f} d\hat{\mathcal{E}} d\underline{\zeta}, \quad (18b)$$

and that of the initial condition (9) is given by

$$\hat{f}(0, x_1, x_2, \underline{\zeta}, \hat{\mathcal{E}}) = \hat{f}^0(x_1, x_2, \underline{\zeta}, \hat{\mathcal{E}}). \quad (19)$$

From Eq. (10), we have, for the dimensionless mass-flow rate $\hat{M}$,

$$\hat{M} = \int_{-a/2}^{a/2} \hat{\rho} \hat{v}_2 dx_1. \quad (20)$$

Because of assumption (iv) in Sec. 2.1, ϵ in Eq. (16) is the small parameter in our problem, whereas the reference Knudsen number K_* is arbitrary. In this situation, the induced gas flow is expected to be slow. If we assume that $\hat{v}_i$ is of the order of ϵ , the reference time t_* and thus the Strouhal number Sh may be chosen as

$$t_* = L_*/(2RT_*)^{1/2}\epsilon, \quad \text{Sh} = \epsilon^2. \quad (21)$$

Therefore, Eq. (15a) becomes

$$\epsilon^2 \frac{\partial \hat{f}}{\partial \hat{t}} + \zeta_1 \frac{\partial \hat{f}}{\partial x_1} + \epsilon \zeta_2 \frac{\partial \hat{f}}{\partial x_2} = \frac{2}{\sqrt{\pi}} \frac{1}{K_*} \hat{A}_c \hat{\rho} (\hat{\mathcal{G}} - \hat{f}). \quad (22)$$

2.4. Macroscopic equation. We analyze the problem (22) [with Eqs. (15b)–(15j)], (18), and (19) by expanding $\hat{f}$ in terms of ϵ :

$$\hat{f} = \hat{f}_{(0)} + \hat{f}_{(1)}\epsilon + \hat{f}_{(2)}\epsilon^2 + \dots. \quad (23)$$

Correspondingly, the moments of $\hat{f}$ occurring in Eq. (22), i.e., $\hat{\rho}$, $\hat{v}_i$, $\hat{T}_{\text{tr}}$, $\hat{T}_{\text{int}}$, and $\hat{\underline{\theta}}$, and thus all the other (macroscopic) quantities that are defined in terms of these moments are expanded as

$$\hat{h} = \hat{h}_{(0)} + \hat{h}_{(1)}\epsilon + \hat{h}_{(2)}\epsilon^2 + \dots. \quad (24)$$

Here, $\hat{h}$ stands for $\hat{\rho}$, $\hat{v}_i$, $\hat{T}_{\text{tr}}$, $\hat{T}_{\text{int}}$, and $\hat{\underline{\theta}}$ and also $\hat{T}$, $\hat{T}_{\text{rel}}$, $\hat{A}_c$, $|\hat{\underline{T}}|$, $\hat{\underline{T}}^{-1}$, $\hat{p}$ and $\hat{p}_{ij}$. For instance, for $\hat{\rho}$, $\hat{\underline{\theta}}$, $\hat{A}_c$, we write

$$\hat{\rho} = \hat{\rho}_{(0)} + \hat{\rho}_{(1)}\epsilon + \hat{\rho}_{(2)}\epsilon^2 + \dots, \quad (25a)$$

$$\hat{\underline{\theta}} = \hat{\underline{\theta}}_{(0)} + \hat{\underline{\theta}}_{(1)}\epsilon + \hat{\underline{\theta}}_{(2)}\epsilon^2 + \dots, \quad (25b)$$

$$\hat{A}_c = \hat{A}_{c(0)} + \hat{A}_{c(1)}\epsilon + \hat{A}_{c(2)}\epsilon^2 + \dots. \quad (25c)$$

The expansion of the macroscopic quantities (24) leads to the corresponding expansion of the Gaussian $\hat{\mathcal{G}}$:

$$\hat{\mathcal{G}} = \hat{\mathcal{G}}_{(0)} + \hat{\mathcal{G}}_{(1)}\epsilon + \hat{\mathcal{G}}_{(2)}\epsilon^2 + \dots. \quad (26)$$

In addition, $\hat{\rho}_w$ in the boundary condition (18) and the dimensionless mass-flow rate $\hat{M}$ [Eq. (20)] are expanded as well:

$$\hat{\rho}_w = \hat{\rho}_{w(0)} + \hat{\rho}_{w(1)}\epsilon + \hat{\rho}_{w(2)}\epsilon^2 + \dots, \quad (27a)$$

$$\hat{M} = \hat{M}_{(0)} + \hat{M}_{(1)}\epsilon + \hat{M}_{(2)}\epsilon^2 + \dots. \quad (27b)$$

The explicit expressions of the coefficients in the expansions (24), (26), and (27), some of which will be given when necessary in the sequel, are omitted here. By the substitution of these expansions in Eqs. (22) [with Eqs. (15b)–(15j)], (18), and (19), we can solve the problem from the lowest order of ϵ .

2.4.1. *Zeroth order.* The equation at the zeroth order in ϵ is as follows:

$$\zeta_1 \frac{\partial \hat{f}_{(0)}}{\partial x_1} = \frac{2}{\sqrt{\pi}} \frac{1}{K_*} \hat{A}_{c(0)} \hat{\rho}_{(0)} (\hat{\mathcal{G}}_{(0)} - \hat{f}_{(0)}), \quad (28)$$

where $\hat{A}_{c(0)} = \hat{A}_c(\hat{T}_{(0)})$, and the explicit forms of $\hat{\rho}_{(0)}$ and $\hat{\mathcal{G}}_{(0)}$, which can be obtained from Eqs. (15b)–(15j) straightforwardly, are omitted here. The corresponding boundary condition is given by

$$\hat{f}_{(0)} = \frac{\hat{\rho}_{w(0)} \Lambda_\delta}{(\pi \hat{T}_w)^{3/2} \hat{T}_w^{\delta/2}} \hat{\mathcal{E}}^{\delta/2-1} \exp\left(-\frac{\zeta_i^2}{\hat{T}_w} - \frac{\hat{\mathcal{E}}}{\hat{T}_w}\right) \quad (x_1 = \pm a/2, \quad \zeta_1 \leq 0), \quad (29a)$$

$$\hat{\rho}_{w(0)} = \pm 2 \sqrt{\frac{\pi}{\hat{T}_w}} \int_{\zeta_1 \geq 0} \int_0^\infty \zeta_1 \hat{f}_{(0)} d\hat{\mathcal{E}} d\zeta. \quad (29b)$$

It should be noted that $\hat{t}$ and x_2 appear only as parameters in the problem (28) and (29). It is seen by the direct substitution that Eqs. (28) and (29) have a solution of the following form:

$$\hat{f}_{(0)} = \frac{\sigma_{(0)}(\hat{t}, x_2) \Lambda_\delta}{[\pi \hat{T}_w(x_2)]^{3/2} [\hat{T}_w(x_2)]^{\delta/2}} \hat{\mathcal{E}}^{\delta/2-1} \exp\left(-\frac{\zeta_i^2 + \hat{\mathcal{E}}}{\hat{T}_w(x_2)}\right), \quad (30)$$

where $\sigma_{(0)}$ is an arbitrary function of $\hat{t}$ and x_2 to be determined later. In fact, Eq. (30) gives the following macroscopic quantities:

$$\begin{aligned} \hat{\rho}_{(0)} &= \sigma_{(0)}(\hat{t}, x_2), & \hat{v}_{i(0)} &= 0, \\ \hat{T}_{\text{tr}(0)} &= \hat{T}_{\text{int}(0)} = \hat{T}_{(0)} = \hat{T}_{\text{rel}(0)} = \hat{T}_w(x_2), \\ \hat{p}_{(0)} &= \sigma_{(0)}(\hat{t}, x_2) \hat{T}_w(x_2), & \hat{p}_{ij(0)} &= \hat{p}_{(0)} \delta_{ij}, & \hat{M}_{(0)} &= 0, \end{aligned} \quad (31)$$

which reduce $\hat{\mathcal{G}}_{(0)}$ and $\hat{A}_{c(0)}$ to

$$\hat{\mathcal{G}}_{(0)} = \hat{f}_{(0)}, \quad \hat{A}_{c(0)} = \hat{A}_c(\hat{T}_w). \quad (32)$$

Moreover, we can show, by following [14], that Eq. (30) is the unique solution (up to the arbitrariness of $\sigma_{(0)}$).

2.4.2. *First order.* The equation in the first order of ϵ is obtained as follows:

$$\zeta_1 \frac{\partial \hat{f}_{(1)}}{\partial x_1} + \zeta_2 \frac{\partial \hat{f}_{(0)}}{\partial x_2} = \frac{2}{\sqrt{\pi}} \frac{1}{K_*} \hat{A}_{c(0)} \sigma_{(0)} (\hat{\mathcal{G}}_{(1)} - \hat{f}_{(1)}), \quad (33a)$$

$$\begin{aligned} \hat{\mathcal{G}}_{(1)} &= \hat{\mathcal{G}}_{(0)} \left[\frac{\hat{\rho}_{(1)}}{\sigma_{(0)}} - \frac{1}{2} \frac{|\hat{\underline{T}}|_{(1)}}{\hat{T}_w^3} - \frac{\delta}{2} \frac{\hat{T}_{\text{rel}(1)}}{\hat{T}_w} + \frac{\hat{\mathcal{E}} \hat{T}_{\text{rel}(1)}}{\hat{T}_w^2} \right. \\ &\quad \left. + 2 \frac{\zeta_i \hat{v}_{i(1)}}{\hat{T}_w} - {}^t \underline{\zeta} \cdot (\hat{\underline{T}}^{-1})_{(1)} \cdot \underline{\zeta} \right], \end{aligned} \quad (33b)$$

$$|\hat{\underline{T}}|_{(1)} = \hat{T}_w^2 \left[3(1 - \eta) \hat{T}_{\text{tr}(1)} + 3\eta \hat{T}_{(1)} \right], \quad (33c)$$

$$\begin{aligned} {}^t \underline{\zeta} \cdot (\hat{\underline{T}}^{-1})_{(1)} \cdot \underline{\zeta} &= -\frac{1}{\hat{T}_w^2} \left[\left((1 - \eta) \hat{T}_{\text{tr}(1)} + \eta \hat{T}_{(1)} \right) \zeta_i^2 \right. \\ &\quad \left. + (1 - \eta) \nu \frac{\hat{p}_{ij(1)}}{\sigma_{(0)}} \left(\zeta_i \zeta_j - \frac{1}{3} \zeta_k^2 \delta_{ij} \right) \right], \end{aligned} \quad (33d)$$

$$\hat{\rho}_{(1)} = \iint_0^\infty \hat{f}_{(1)} d\hat{\mathcal{E}} d\zeta, \quad (33e)$$

$$\sigma_{(0)}\hat{v}_{i(1)} = \iint_0^\infty \zeta_i \hat{f}_{(1)} d\hat{\mathcal{E}} d\zeta, \quad (33f)$$

$$\sigma_{(0)}\hat{T}_{\text{tr}(1)} = \frac{2}{3} \iint_0^\infty \zeta_i^2 \hat{f}_{(1)} d\hat{\mathcal{E}} d\zeta - \hat{\rho}_{(1)}\hat{T}_w, \quad (33g)$$

$$\sigma_{(0)}\hat{T}_{\text{int}(1)} = \frac{2}{\delta} \iint_0^\infty \hat{\mathcal{E}} \hat{f}_{(1)} d\hat{\mathcal{E}} d\zeta - \hat{\rho}_{(1)}\hat{T}_w, \quad (33h)$$

$$\hat{T}_{(1)} = \frac{3\hat{T}_{\text{tr}(1)} + \delta\hat{T}_{\text{int}(1)}}{3 + \delta}, \quad (33i)$$

$$\hat{T}_{\text{rel}(1)} = \eta\hat{T}_{(1)} + (1 - \eta)\hat{T}_{\text{int}(1)}. \quad (33j)$$

The corresponding boundary condition is given by

$$\hat{f}_{(1)} = \frac{\hat{\rho}_w(1)\Lambda\delta}{(\pi\hat{T}_w)^{3/2}\hat{T}_w^{\delta/2}} \hat{\mathcal{E}}^{\delta/2-1} \exp\left(-\frac{\zeta_i^2}{\hat{T}_w} - \frac{\hat{\mathcal{E}}}{\hat{T}_w}\right) \quad (x_1 = \pm a/2, \quad \zeta_1 \leq 0), \quad (34a)$$

$$\hat{\rho}_w(1) = \pm 2\sqrt{\frac{\pi}{\hat{T}_w}} \int_{\zeta_1 \geq 0} \int_0^\infty \zeta_1 \hat{f}_{(1)} d\hat{\mathcal{E}} d\zeta. \quad (34b)$$

In addition, the pressure, the stress tensor, and the mass-flow rate in the first order of ϵ are expressed as follows:

$$\hat{p}_{(1)} = \sigma_{(0)}\hat{T}_{(1)} + \hat{\rho}_{(1)}\hat{T}_w, \quad (35a)$$

$$\hat{p}_{ij(1)} = \iint_0^\infty 2\zeta_i \zeta_j \hat{f}_{(1)} d\hat{\mathcal{E}} d\zeta, \quad (35b)$$

$$\hat{M}_{(1)} = \int_{-a/2}^{a/2} \sigma_{(0)}\hat{v}_{2(1)} dx_1. \quad (35c)$$

Here again, the problem (33) and (34) contains $\hat{t}$ and x_2 only as parameters [see Eqs. (31) and (32)]. Now we can show, as in the same way as [42], that the solution to Eqs. (33) and (34) is obtained in the following form:

$$\begin{aligned} \hat{f}_{(1)} = & \hat{f}_{(0)} \left[\phi_g(\hat{t}, x_2) + \frac{\zeta_2}{\zeta_r} a \phi_P \left(\frac{x_1}{a}, \frac{\zeta_1}{\hat{T}_w^{1/2}}, \frac{\zeta_r}{\hat{T}_w^{1/2}}, \frac{\hat{\mathcal{E}}}{\hat{T}_w}; K(\hat{t}, x_2) \right) \frac{1}{\hat{p}_{(0)}} \frac{\partial \hat{p}_{(0)}}{\partial x_2} \right. \\ & \left. + \frac{\zeta_2}{\zeta_r} a \phi_T \left(\frac{x_1}{a}, \frac{\zeta_1}{\hat{T}_w^{1/2}}, \frac{\zeta_r}{\hat{T}_w^{1/2}}, \frac{\hat{\mathcal{E}}}{\hat{T}_w}; K(\hat{t}, x_2) \right) \frac{1}{\hat{T}_w} \frac{d\hat{T}_w}{dx_2} \right], \end{aligned} \quad (36)$$

where

$$\zeta_r = (\zeta_2^2 + \zeta_3^2)^{1/2}, \quad (37a)$$

$$K(\hat{t}, x_2) = \frac{K_* \hat{T}_w^{3/2}(x_2)}{\hat{p}_{(0)}(\hat{t}, x_2) \hat{A}_{c(0)}(x_2) a}, \quad (37b)$$

$\phi_g(\hat{t}, x_2)$ is an arbitrary function of $\hat{t}$ and x_2 , and ϕ_P and ϕ_T are, respectively, the solutions of the specific boundary-value problems for the linearized ES model. The equation and boundary condition for ϕ_P and those for ϕ_T are given in Appendix A, where the parameter set $(\text{Pr}, \mu_b/\mu, \delta)$ is used in place of (ν, η, δ) in Eq. (67) [see the comment following Eq. (7)]. The ϕ_P corresponds to the solution for the flow induced between two parallel plates by the uniform pressure gradient (Poiseuille flow) [36], whereas ϕ_T corresponds to the solution for the flow induced between two parallel plates by the uniform temperature gradient along the plates (thermal transpiration) [36]. They are fundamental and classical problems in rarefied gas

dynamics and have been investigated by many authors by various methods. Since an exhaustive list of references is beyond the scope of the present paper, we only mention some early works, [12],[28],[29], and more recent works, [34], [15] (see also [36], [32] and references therein). We will discuss these problems in Sec. 4.

The macroscopic quantities in the first order of ϵ , i.e., those corresponding to the solution (36), are obtained as follows:

$$\begin{aligned}
\hat{\rho}_{(1)} &= \sigma_{(0)}(\hat{t}, x_2) \phi_g(\hat{t}, x_2), & \hat{v}_{1(1)} &= \hat{v}_{3(1)} = 0, \\
\hat{v}_{2(1)} &= a \hat{T}_w^{1/2} \left(u_P \frac{1}{\hat{p}_{(0)}} \frac{\partial \hat{p}_{(0)}}{\partial x_2} + u_T \frac{1}{\hat{T}_w} \frac{d \hat{T}_w}{dx_2} \right), \\
\hat{T}_{tr(1)} &= \hat{T}_{int(1)} = \hat{T}_{(1)} = \hat{T}_{rel(1)} = 0, \\
\hat{p}_{(1)} &= \hat{p}_{(0)}(\hat{t}, x_2) \phi_g(\hat{t}, x_2), \\
\hat{p}_{ij(1)} &= \hat{p}_{(1)} \delta_{ij} + a \hat{p}_{(0)} \left(\Sigma_P \frac{1}{\hat{p}_{(0)}} \frac{\partial \hat{p}_{(0)}}{\partial x_2} + \Sigma_T \frac{1}{\hat{T}_w} \frac{d \hat{T}_w}{dx_2} \right) (\delta_{i1} \delta_{j2} + \delta_{i2} \delta_{j1}),
\end{aligned} \tag{38}$$

where, with $\alpha = P, T$,

$$\begin{aligned}
u_\alpha &= u_\alpha \left(\frac{x_1}{a}; K(\hat{t}, x_2) \right) \\
&= \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{\infty} c_r^2 Y^{\delta/2-1} \phi_\alpha \left(\frac{x_1}{a}, c_1, c_r, Y; K(\hat{t}, x_2) \right) \tilde{E}_Y dY dc_r dc_1,
\end{aligned} \tag{39a}$$

$$\begin{aligned}
\Sigma_\alpha &= \Sigma_\alpha \left(\frac{x_1}{a}; K(\hat{t}, x_2) \right) \\
&= 2 \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{\infty} c_1 c_r^2 Y^{\delta/2-1} \phi_\alpha \left(\frac{x_1}{a}, c_1, c_r, Y; K(\hat{t}, x_2) \right) \tilde{E}_Y dY dc_r dc_1,
\end{aligned} \tag{39b}$$

$$\tilde{E}_Y = \Lambda_\delta \pi^{-1/2} \exp(-c_1^2 - c_r^2 - Y). \tag{39c}$$

[It turns out that $\Sigma_P = -x_1/a$ and $\Sigma_T = 0$ (see Appendix A).] In addition, the corresponding mass-flow rate is given as

$$\hat{M}_{(1)} = a^2 \frac{\hat{p}_{(0)}}{\hat{T}_w^{1/2}} \left(M_P \frac{1}{\hat{p}_{(0)}} \frac{\partial \hat{p}_{(0)}}{\partial x_2} + M_T \frac{1}{\hat{T}_w} \frac{d \hat{T}_w}{dx_2} \right), \tag{40}$$

where

$$M_\alpha = M_\alpha(K(\hat{t}, x_2)) = \int_{-1/2}^{1/2} u_\alpha(y; K(\hat{t}, x_2)) dy, \quad (\alpha = P, T). \tag{41}$$

2.4.3. Second order and macroscopic equation. The equation in the second order of ϵ reads as follows:

$$\begin{aligned}
&\frac{\partial \hat{f}_{(0)}}{\partial \hat{t}} + \zeta_1 \frac{\partial \hat{f}_{(2)}}{\partial x_1} + \zeta_2 \frac{\partial \hat{f}_{(1)}}{\partial x_2} \\
&= \frac{2}{\sqrt{\pi}} \frac{1}{K_*} \left[\hat{A}_{c(0)} \sigma_{(0)} (\hat{\mathcal{G}}_{(2)} - \hat{f}_{(2)}) + (\hat{A}_{c(0)} \hat{\rho}_{(1)} + \hat{A}_{c(1)} \sigma_{(0)}) (\hat{\mathcal{G}}_{(1)} - \hat{f}_{(1)}) \right],
\end{aligned} \tag{42}$$

where the explicit form of $\hat{\mathcal{G}}_{(2)}$ is omitted. The boundary condition for Eq. (42) is given by

$$\hat{f}_{(2)} = \frac{\hat{\rho}_{w(2)}\Lambda_\delta}{(\pi\hat{T}_w)^{3/2}\hat{T}_w^{\delta/2}}\hat{\mathcal{E}}^{\delta/2-1}\exp\left(-\frac{\zeta_i^2}{\hat{T}_w}-\frac{\hat{\mathcal{E}}}{\hat{T}_w}\right) \quad (x_1 = \pm a/2, \quad \zeta_1 \leq 0), \quad (43a)$$

$$\hat{\rho}_{w(2)} = \pm 2\sqrt{\frac{\pi}{\hat{T}_w}}\int_{\zeta_1 \geq 0}\int_0^\infty \zeta_1\hat{f}_{(2)}d\hat{\mathcal{E}}d\zeta. \quad (43b)$$

We now integrate Eq. (42) with respect to ζ_i and $\hat{\mathcal{E}}$ over their whole ranges and then with respect to x_1 from $-a/2$ to $a/2$. If we take into account, in this process, the explicit forms of $\hat{f}_{(0)}$ [Eq. (30)] and $\hat{f}_{(1)}$ [Eq. (36)], as well as the relation

$$\iint_0^\infty \zeta_1\hat{f}_{(2)}|_{x_1=\pm a/2}d\hat{\mathcal{E}}d\zeta = 0, \quad (44)$$

obtained from the boundary condition (43), then we have the following equation:

$$\frac{\partial\hat{p}_{(0)}}{\partial\hat{t}} + \hat{T}_w\frac{\partial\mathcal{M}}{\partial x_2} = 0, \quad (45a)$$

$$\mathcal{M} = \frac{\hat{p}_{(0)}}{\hat{T}_w^{1/2}}a\left[M_P(K; \delta, \text{Pr})\frac{1}{\hat{p}_{(0)}}\frac{\partial\hat{p}_{(0)}}{\partial x_2} + M_T(K; \delta)\frac{1}{\hat{T}_w}\frac{d\hat{T}_w}{dx_2}\right], \quad (45b)$$

$$K = K_*\hat{T}_w^{3/2}/\hat{p}_{(0)}\hat{A}_{c(0)}a, \quad \hat{A}_{c(0)} = \hat{A}_c(\hat{T}_w), \quad (45c)$$

where, $\hat{p}_{(0)} = \sigma_{(0)}\hat{T}_w$, rather than $\sigma_{(0)}$, has been used as the unknown function. In Eq. (45), $\hat{p}_{(0)}$, $\mathcal{M}$, and K are the functions of $\hat{t}$ and x_2 , whereas $\hat{T}_w$ and $\hat{A}_{c(0)}$ are the functions of x_2 only. As shown in Appendix A, ϕ_P depends only on (Pr, δ) and ϕ_T only on δ among the set of parameters $(\text{Pr}, \mu_b/\mu, \delta)$. Thus, the same dependency holds for M_P and M_T . This fact is shown explicitly in Eq. (45). Further relevant properties of M_P and M_T will be shown in Sec. 4. The original (dimensional) mass-flow rate M [cf. Eq. (10)] is expressed as

$$M/\rho_*(2RT_*)^{1/2}D_* = a\mathcal{M}\epsilon + O(\epsilon^2). \quad (46)$$

Thus, $\mathcal{M}$ is the dimensionless first-order mass flux.

In summary, Eq. (45) is the equation of convection-diffusion type for the pressure $\hat{p}_{(0)}$ at the zeroth order in ϵ if we have the data of M_P and M_T as the functions of K . The construction of the database will be made in Sec. 4.

The steady version of Eq. (45) is essentially the same as the generalized Reynolds equation derived in [17] and often used in micro-fluid applications [33, 13]. Mathematical study of the derivation of diffusion-type equations from the Boltzmann equation is found, for instance, in Refs. [6, 18, 16]. In Ref. [3], the term corresponding to ϕ_g in Eq. (36) is set to be zero because this setting does not make any change in the resulting equation of convection-diffusion type.

3. Connected channels and connection condition. Let us consider a long channel composed of several (or many) channels with different width connected linearly one after another. If each channel is long enough compared with its width, the macroscopic equation (45) should be valid in each channel except the regions near the junctions. If we assume that the macroscopic equation is valid even in the junction regions, it should be complemented by an appropriate connection condition at the junctions. This condition can be derived by the analysis of the original kinetic

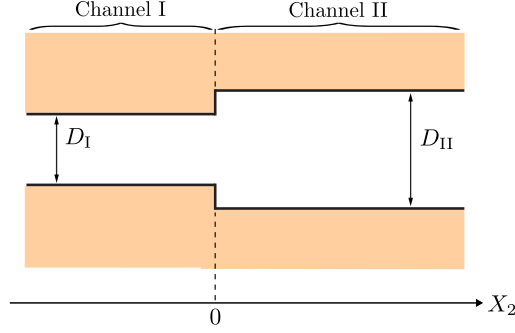


FIGURE 1. Junction of two channels with different width.

equation in the junction regions (see [2, 42, 3]). Here, we only give the result thus obtained.

Let us consider the junction of two channels shown in Fig. 1. That is, the channel I with width D_I ($X_2 < 0$) is connected with the channel II with width D_{II} ($X_2 > 0$) at $X_2 = 0$. We assume the diffuse reflection on the wall at $X_2 = 0$ and that the temperature of the channel wall is continuous at $X_2 = 0$ though its derivative may be discontinuous. Then, the connection condition for Eq. (45) applied to each channel is given by

$$\hat{p}_{(0)}^I(\hat{t}, x_2 = 0_-) = \hat{p}_{(0)}^{II}(\hat{t}, x_2 = 0_+), \quad (47a)$$

$$\mathcal{M}^I(\hat{t}, x_2 = 0_-)a_I = \mathcal{M}^{II}(\hat{t}, x_2 = 0_+)a_{II}, \quad (47b)$$

with

$$a_I = D_I/D_*, \quad a_{II} = D_{II}/D_*. \quad (48)$$

Here, $\hat{p}_{(0)}^J$ and $\mathcal{M}^J$ ($J = I, II$) are the zeroth-order pressure and the first-order mass flux in the channel J , respectively. The condition (47) is valid irrespective of the positions of the center lines of the two channels (i.e., the center lines do not need to be common).

In the case of a channel composed of many channels with different width, the condition (47) is to be applied at each junction.

4. Data for M_P and M_T : reduction to BGK model. The convection-diffusion-type equation (45) is of the same form as in the case of a monatomic gas [42, 3]. In other words, the effect of the polyatomic gas is confined in the transport coefficients M_P and M_T . As we have seen in Sec. 2.4.2, M_P and M_T are obtained by Eqs. (39a) and (41) from the solutions ϕ_P and ϕ_T to Eq. (67) with boundary condition (66). In this section, we investigate this problem in detail.

We first note that we can eliminate the variable Y from our problem by introducing the following marginal velocity distribution functions:

$$\Phi_\alpha(y, c_1, c_r; K(\hat{t}, x_2)) = \Lambda_\delta \int_0^\infty Y^{\delta/2-1} \phi_\alpha(y, c_1, c_r, Y; K(\hat{t}, x_2)) \exp(-Y) dY, \quad (\alpha = P, T). \quad (49)$$

More specifically, if we multiply Eq. (67a) by $\Lambda_\delta Y^{\delta/2-1} \exp(-Y)$ and integrate with respect to Y from 0 to ∞ , we obtain the boundary-value problems for Φ_P and Φ_T .

That is, the problem for Φ_P is

$$c_1 \frac{\partial \Phi_P}{\partial y} = \frac{2}{\sqrt{\pi}} \frac{1}{K} \left[-\Phi_P + 2c_r u_P - 2 \left(1 - \frac{1}{\text{Pr}} \right) c_1 c_r y \right] - c_r, \quad (50a)$$

$$u_P = \int_{-\infty}^{\infty} \int_0^{\infty} c_r^2 \Phi_P E_c dc_r dc_1, \quad (50b)$$

$$\Phi_P = 0, \quad (y = \pm 1/2, \quad c_1 \leq 0), \quad (50c)$$

where

$$E_c = \pi^{-1/2} \exp(-c_1^2 - c_r^2), \quad (51)$$

and the problem for Φ_T is

$$c_1 \frac{\partial \Phi_T}{\partial y} = \frac{2}{\sqrt{\pi}} \frac{1}{K} (-\Phi_T + 2c_r u_T) - c_r \left(c_1^2 + c_r^2 - \frac{5}{2} \right), \quad (52a)$$

$$u_T = \int_{-\infty}^{\infty} \int_0^{\infty} c_r^2 \Phi_T E_c dc_r dc_1, \quad (52b)$$

$$\Phi_T = 0, \quad (y = \pm 1/2, \quad c_1 \leq 0). \quad (52c)$$

In addition, we have

$$M_P = \int_{-1/2}^{1/2} u_P dy, \quad M_T = \int_{-1/2}^{1/2} u_T dy. \quad (53)$$

It should be remarked that the parameter δ of the internal degree of freedom of a molecule has been eliminated from the problems (50) and (52). As seen from Eqs. (50) and (52), Φ_P depends only on Pr and Φ_T on none of the parameters $(\text{Pr}, \mu_b/\mu, \delta)$. Thus, we may write M_P and M_T as $M_P(K, \text{Pr})$ and $M_T(K)$.

In fact, Eqs. (50) and (52) are of the same form as the equations and boundary conditions corresponding to the plane Poiseuille flow and thermal transpiration based on the ES model for a monatomic gas. The only difference lies in the different values of Pr in the former. Furthermore, Eq. (52) is identical to the equation and boundary condition corresponding to the thermal transpiration based on the BGK model for a monatomic gas. Therefore, we can just exploit the existing data of M_T based on the BGK model [38]. On the other hand, the problem (50) differs from that based on the BGK model. However, if we let

$$\Phi_P = \Psi_P - \frac{2}{\sqrt{\pi}} \frac{1}{K} \left(1 - \frac{1}{\text{Pr}} \right) c_r \left(y^2 - \frac{1}{4} \right), \quad (54)$$

then we have the following equation and boundary condition for Ψ_P :

$$c_1 \frac{\partial \Psi_P}{\partial y} = \frac{2}{\sqrt{\pi}} \frac{1}{K} (-\Psi_P + 2c_r u_P[\Psi_P]) - c_r, \quad (55a)$$

$$u_P[\Psi_P] = \int_{-\infty}^{\infty} \int_0^{\infty} c_r^2 \Psi_P E_c dc_r dc_1, \quad (55b)$$

$$\Psi_P = 0, \quad (y = \pm 1/2, \quad c_1 \leq 0), \quad (55c)$$

which does not contain Pr and is identical to the equation and boundary condition for the plane Poiseuille flow based on the BGK model for a single-component gas. Let $u_P[\Phi_P]$ and $M_P[\Phi_P]$ denote u_P and M_P based on Φ_P , and $u_P[\Psi_P]$ and $M_P[\Psi_P]$

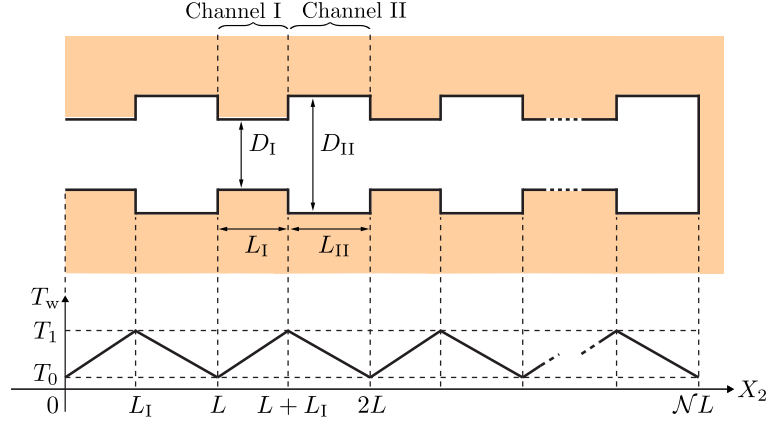


FIGURE 2. Knudsen pump composed of Channel I and Channel II.

those based on Ψ_P . Then, they are related as

$$u_P[\Phi_P] = u_P[\Psi_P] - \frac{1}{\sqrt{\pi}} \frac{1}{K} \left(1 - \frac{1}{\text{Pr}}\right) \left(y^2 - \frac{1}{4}\right), \quad (56)$$

$$M_P[\Phi_P] = M_P[\Psi_P] + \frac{1}{6} \frac{1}{\sqrt{\pi}} \frac{1}{K} \left(1 - \frac{1}{\text{Pr}}\right). \quad (57)$$

Since $M_P[\Psi_P]$ is independent of Pr , the functional form of $M_P(K, \text{Pr})$, as the function of Pr , is given by the last term on the right-hand side of Eq. (57). In this way, the data of $M_P(K, \text{Pr})$ can be obtained readily from the existing data of $M_P[\Psi_P]$ for the BGK model for a monatomic gas [38]. The same conversions as (54), (56), and (57) can be used even when the boundary condition other than the diffuse reflection (e.g., Cercignani–Lampis model [10]) is used in the formulation of the problem [or in place of (50c)]. It should be noted that, for a monatomic gas, the conversion between the ES and BGK models is mentioned in [9, 10], where a conversion formula corresponding to Eq. (57) is given for the cylindrical Poiseuille flow, assuming the diffuse reflection boundary condition.

In summary, we can obtain the database for $M_P(K, \text{Pr})$ and $M_T(K)$ without any new computation.

5. Knudsen pump and its performance. In this section, we apply the macroscopic equation (45) and connection condition (47) to a Knudsen pump. We take the nitrogen gas (N_2), for which the experimental values of Pr and μ_b/μ are available ($\text{Pr} = 0.718$ [27] and $\mu_b/\mu = 0.731$ [31]). We set the values of ν and η in such a way that the resulting Pr and μ_b/μ are close to the above experimental values, that is, $\nu = -0.50$ and $\eta = 0.46$, which lead to $\text{Pr} = 0.787$ and $\mu_b/\mu = 0.722$.

Now, let us consider a Knudsen pump (or compressor) shown in Fig. 2, that is, a long channel consisting of alternately arranged $\mathcal{N}$ narrower channels (Channel I: length L_I and width D_I) and $\mathcal{N}$ wider channels [Channel II: length L_{II} and width $D_{II}(> D_I)$] along the X_2 axis. We let $L = L_I + L_{II}$ and assume that $D_I \ll L_I$ and $D_{II} \ll L_{II}$. In addition, we assume that the temperature $T_w(X_2)$ of the channel walls is a piecewise linear and continuous periodic function (period L) of X_2 , varying between T_0 and $T_1(> T_0)$. With this setting, we can apply Eq. (45) to each channel

and Eq. (47) to each junction. To be more specific, if we denote the dimensionless pressure $\hat{p}_{(0)}$ in Channel J ($J = \text{I, II}$) by $\hat{p}_{(0)}^J$, we have

$$\frac{\partial \hat{p}_{(0)}^J}{\partial \hat{t}} + \hat{T}_w \frac{\partial \mathcal{M}^J}{\partial x_2} = 0, \quad (58a)$$

$$\mathcal{M}^J = \frac{\hat{p}_{(0)}^J}{\hat{T}_w^{1/2}} a_J \left[M_P(K_J, \text{Pr}) \frac{1}{\hat{p}_{(0)}^J} \frac{\partial \hat{p}_{(0)}^J}{\partial x_2} + M_T(K_J) \frac{1}{\hat{T}_w} \frac{d\hat{T}_w}{dx_2} \right], \quad (58b)$$

$$a_J = \frac{D_J}{D_*}, \quad K_J = \frac{K_* \hat{T}_w^{3/2}}{\hat{p}_{(0)}^J a_J}, \quad (58c)$$

where

$$J = \begin{cases} \text{I} & \text{for } mL/L_* < x_2 < (L_I + mL)/L_*, \\ \text{II} & \text{for } (L_I + mL)/L_* \leq x_2 \leq (1+m)L/L_*, \end{cases} \quad (m = 0, \dots, \mathcal{N} - 1). \quad (59)$$

Here, we have assumed that $A_c(T)$ does not depend on T , so that $\hat{A}_c = 1$ (or $\hat{A}_{c(0)} = 1$). We also have to impose the following conditions at the junctions:

$$\hat{p}_{(0)}^{\text{I}} = \hat{p}_{(0)}^{\text{II}}, \quad \mathcal{M}^{\text{I}} a_{\text{I}} = \mathcal{M}^{\text{II}} a_{\text{II}}, \quad \text{at } x_2 = L_I/L_*, (L_I + mL)/L_* \text{ and } mL/L_*, \quad (m = 1, \dots, \mathcal{N} - 1). \quad (60)$$

If we assign the appropriate initial condition and conditions at both ends, we can describe the time evolution of the pressure distribution along the Knudsen pump. Here, we consider the following two types of end condition that are simple and natural:

- (i) Open end: the pressure of the gas is specified there.
- (ii) Closed end: the mass flow vanishes there.

Let us consider the following two problems for the Knudsen pump:

- (A) The left end ($X_2 = 0$) is open, and the right end ($X_2 = \mathcal{N}L$) is closed. The pressure at the left end is kept at p_0 . The initial condition is that the density is uniform at $t = 0$.
- (B) Both ends are open. The pressure at the left end is kept at p_0 and that at the right end is kept at $p_1 (> p_0)$. The initial condition is that the density is uniform at $t = 0$.

We take p_0 , T_0 , D_I , and L as the reference pressure p_* , temperature T_* , channel width D_* , and channel length L_* , respectively. Therefore, the reference mean free path becomes $l_* = (2/\sqrt{\pi})(2RT_0)^{1/2}/A_c \rho_0$ (with $\rho_0 = p_0/RT_0$), and thus the reference Knudsen number $K_* = l_*/D_I$. In what follows, we show the steady pressure distribution for Problem (A) and the steady mass-flow rate for Problem (B), obtained as the long-time limits of the time-dependent solutions. The results for the N_2 gas ($\text{Pr} = 0.787$) are compared with those for a monatomic gas with $\text{Pr} = 0.666$ (Note that the values of Pr obtained experimentally as well as theoretically are very close to $2/3$ for any monatomic gas).

The steady pressure distribution $p/p_0 (= \hat{p})$ along the pump at $K_* = 0.1, 1$, and 10 in Problem (A) is shown in Fig. 3 for $D_{\text{II}}/D_I = 2$, $L_I/L = 0.5$, $T_1/T_0 = 1.5$, and $\mathcal{N} = 100$. Figure 3(a) shows the result for the N_2 gas. Figure 3(b) shows the distribution of the pressure averaged over each unit. The result for the monatomic gas is also shown in Fig. 3(b) for comparison. In this problem, we observe the property of the Knudsen pump as a compressor. If the Knudsen number K_* , which

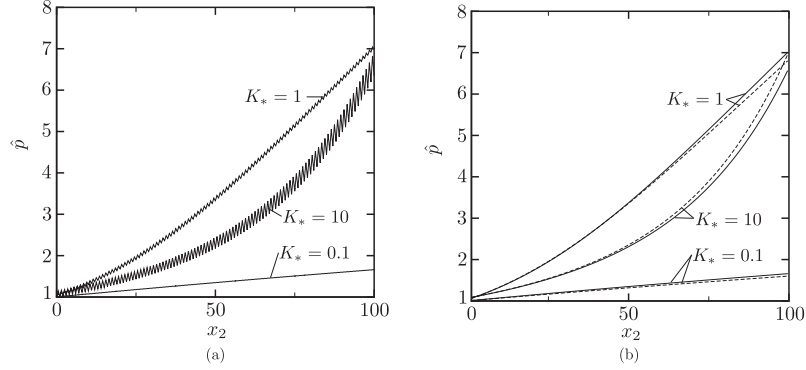


FIGURE 3. Steady pressure distribution along the Knudsen pump in Problem (A). (a) Pressure distribution for the N_2 gas, (b) Averaged pressure distribution for the N_2 and monatomic gases. In (b), the solid line indicates the result for the N_2 gas, and the dashed line that for the monatomic gas.

TABLE 1. Steady mass-flow rate $\mathcal{M}^I a_I$ versus the number of the units $\mathcal{N}$ in Problem (B).

$\mathcal{N}$	$\mathcal{M}^I(0.666)a_I$			$\mathcal{M}^I(0.787)a_I$		
	$K_* = 10$	$K_* = 1$	$K_* = 0.1$	$K_* = 10$	$K_* = 1$	$K_* = 0.1$
10	-0.1107	-0.7529(-1) ¹	-0.3029	-0.1112	-0.7321(-1)	-0.2664
50	0.3414(-2)	0.1970(-1)	-0.4618(-1)	0.2783(-2)	0.1875(-1)	-0.3905(-1)
100	0.1595(-1)	0.2910(-1)	-0.1410(-1)	0.1539(-1)	0.2788(-1)	-0.1064(-1)
150	0.1917(-1)	0.3097(-1)	-0.3410(-2)	0.1867(-1)	0.2971(-1)	-0.1182(-2)
200	0.2031(-1)	0.3142(-1)	0.1928(-2)	0.1984(-1)	0.3017(-1)	0.3542(-2)

¹ Read as -0.7529×10^{-1} .

is the Knudsen number at the left end, is not small, the pressure ratio of $6.5 \sim 7$ is obtained at the right end. The difference between the pressure at the right end for the N_2 gas and that for the monatomic gas is relatively small: the difference is around 6% at $K_* = 10$, 3% at $K_* = 1$, and 4% at $K_* = 0.1$.

Table 1 shows the (dimensionless) steady mass-flow rate $\mathcal{M}^I a_I$ [cf. Eq. (46)] versus the number of the units $\mathcal{N}$ in Problem (B) at three different Knudsen numbers ($K_* = 10, 1, 0.1$) for $D_{II}/D_I = 2$, $L_I/L = 0.5$, $T_1/T_0 = 1.5$, and $p_1/p_0 = 2$: $\mathcal{M}^I(0.787)a_I$ indicates the result for the N_2 gas, whereas $\mathcal{M}^I(0.666)a_I$ that for the monatomic gas. In this problem, we can see the pumping effect against the pressure difference. As the number of units increases, the pumping effect finally overcomes the flow caused by the pressure difference, and the gas flows from the low-pressure circumstances to the high-pressure ones. The mass-flow rate of the N_2 gas does not differ much from that of the monatomic gas.

6. Concluding remarks. In the present study, we considered the Knudsen pump consisting of alternately arranged wider and narrower channels between two parallel plates and investigated its properties when the working gas is a polyatomic gas, using a polyatomic version of the ES model. We first derived, under the assumption

that the width of each channel is sufficiently small compared with its length, the macroscopic system composed of a differential equation of convection-diffusion type and its connection condition at the junctions of the wider and narrower channels. The resulting macroscopic system is the same form as that derived from the BGK model for a monatomic gas, and the effect of the polyatomic gas is confined only in the transport coefficients in the equation. We also showed that the transport coefficients can be obtained readily from those corresponding to the BGK model for a monatomic gas by a simple conversion. Finally, we applied the macroscopic system to investigate numerically the properties of the Knudsen pump for a diatomic N_2 gas. As the result, we confirmed that the polyatomic-gas effect on the performance of the Knudsen pump is quite limited, so that the existing macroscopic system derived for a monatomic gas can be applied safely also to a polyatomic gas in the level of practical applications.

Appendix A. Equations for ϕ_P and ϕ_T . Let us consider Eqs. (33) and (34). We note that, with the help of Eq. (30), the inhomogeneous term of Eq. (33a), i.e., the term $-\zeta_2 \partial \hat{f}_{(0)} / \partial x_2$, can be expressed as the sum of two terms, one in proportion to $(1/\hat{p}_{(0)})(\partial \hat{p}_{(0)} / \partial x_2)$ and the other to $(1/\hat{T}_w)(d\hat{T}_w/dx_2)$.

We first introduce the following new variables:

$$\phi = \frac{\hat{f}_{(1)}}{\hat{f}_{(0)}}, \quad \psi = \frac{\hat{G}_{(1)}}{\hat{f}_{(0)}}, \quad c_i = \frac{\zeta_i}{\hat{T}_w^{1/2}}, \quad Y = \frac{\hat{\mathcal{E}}}{\hat{T}_w}, \quad y = \frac{x_1}{a}, \quad (61)$$

so that $\phi = \phi(\hat{t}, y, x_2, c_i, Y)$. Then, taking into account the form of the inhomogeneous term, we let

$$\phi = \phi_g + \frac{1}{\hat{p}_{(0)}} \frac{\partial \hat{p}_{(0)}}{\partial x_2} \phi_s^P + \frac{1}{\hat{T}_w} \frac{d\hat{T}_w}{dx_2} \phi_s^T. \quad (62)$$

From the form of the equation and boundary condition for ϕ_g , it is easy to show that ϕ_g is independent of y , c_i , and Y (cf. [42]), i.e.,

$$\phi_g = \phi_g(\hat{t}, x_2). \quad (63)$$

On the other hand, it can be shown that ϕ_s^P and ϕ_s^T of the following form:

$$\phi_s^\alpha = a(c_2/c_r) \phi_\alpha(y, c_1, c_r, Y; K(\hat{t}, x_2)), \quad (\alpha = P, T), \quad (64a)$$

$$c_r = (c_2^2 + c_3^2)^{1/2}, \quad (64b)$$

are compatible with their equations and boundary conditions (similarity solutions). The resulting equation and boundary condition for $\phi_\alpha(y, c_1, c_r, Y; K(\hat{t}, x_2))$ are as follows:

$$c_1 \frac{\partial \phi_\alpha}{\partial y} = \frac{2}{\sqrt{\pi}} \frac{1}{K} \left[-\phi_\alpha + 2c_r u_\alpha + 2(1-\eta) \nu c_1 c_r \Sigma_\alpha \right] + I_\alpha \quad (\alpha = P, T), \quad (65a)$$

$$u_\alpha = \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{\infty} c_r^2 Y^{\delta/2-1} \phi_\alpha \tilde{E}_Y dY dc_r dc_1, \quad (65b)$$

$$\Sigma_\alpha = 2 \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{\infty} c_1 c_r^2 Y^{\delta/2-1} \phi_\alpha \tilde{E}_Y dY dc_r dc_1, \quad (65c)$$

$$I_P = -c_r, \quad I_T = -c_r \left(c_1^2 + c_r^2 + Y - \frac{5+\delta}{2} \right), \quad (65d)$$

$$\tilde{E}_Y = \Lambda_\delta \pi^{-1/2} \exp(-c_1^2 - c_r^2 - Y), \quad (65e)$$

and

$$\phi_\alpha = 0, \quad (y = \pm 1/2, \quad c_1 \leq 0). \quad (66)$$

But, by integrating Eq. (65a) multiplied by $2c_r^2 Y^{\delta/2-1} \tilde{E}_Y$ over the whole ranges of Y , c_r , and c_1 , we have $\Sigma_P = -y$ and $\Sigma_T = 0$ because of the symmetry $\phi_\alpha(y, c_1, c_r, Y) = \phi_\alpha(-y, -c_1, c_r, Y)$. Therefore, Eq. (65) is recast as

$$c_1 \frac{\partial \phi_\alpha}{\partial y} = \frac{2}{\sqrt{\pi}} \frac{1}{K} \left[-\phi_\alpha + 2c_r u_\alpha + 2 \left(1 - \frac{1}{\text{Pr}} \right) c_1 c_r \Sigma_\alpha \right] + I_\alpha \quad (\alpha = \text{P}, \text{T}), \quad (67a)$$

$$u_\alpha = \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{\infty} c_r^2 Y^{\delta/2-1} \phi_\alpha \tilde{E}_Y dY dc_r dc_1, \quad (67b)$$

$$\Sigma_P = -y, \quad \Sigma_T = 0, \quad (67c)$$

$$I_P = -c_r, \quad I_T = -c_r \left(c_1^2 + c_r^2 + Y - \frac{5 + \delta}{2} \right), \quad (67d)$$

where $(\text{Pr}, \mu_b/\mu, \delta)$, rather than (ν, η, δ) , has been used as the set of parameters characterizing the gas under consideration [see the comment following Eq. (7)]. It is seen from Eq. (67) that ϕ_P does not depend on μ_b/μ and ϕ_T depends on neither μ_b/μ nor Pr .

REFERENCES

- [1] P. Andries, P. Le Tallec, J.-P. Perlat, and B. Perthame, *The Gaussian-BGK model of Boltzmann equation with small Prandtl number*, Eur. J. Mech. B/Fluids, **19** (2000), 813–830.
- [2] K. Aoki and P. Degond, *Homogenization of a flow in a periodic channel of small section*, Multiscale Model. Simul., **1** (2003), 304–334.
- [3] K. Aoki, P. Degond, S. Takata, and H. Yoshida, *Diffusion models for Knudsen compressors*, Phys. Fluids, **19** (2007) 117103.
- [4] K. Aoki, Y. Sone, S. Takata, K. Takahashi, and G. A. Bird, *One-way flow of a rarefied gas induced in a circular pipe with a periodic temperature distribution*, in “Rarefied Gas Dynamics” (eds. T. J. Bartel and M. A. Gallis), AIP, (2001), 940–947.
- [5] K. Aoki, S. Takata, and K. Kugimoto, *Diffusion approximation for the Knudsen compressor composed of circular tubes*, in “Rarefied Gas Dynamics” (ed. T. Abe), AIP, (2009), 953–958.
- [6] H. Babovsky, C. Bardos, and T. Platkowski, *Diffusion approximation for a Knudsen gas in a thin domain with accommodation on the boundary*, Asymptotic Analysis, **3** (1991), 265–289.
- [7] P. L. Bhatnagar, E. P. Gross, and M. Krook, *A model for collision processes in gases. I. Small amplitude processes in charged and neutral one-component systems*, Phys. Rev., **94** (1954), 511–525.
- [8] G. A. Bird, “Molecular Gas Dynamics and the Direct Simulation of Gas Flows,” Oxford University Press, Oxford, 1994.
- [9] C. Cercignani, “Mathematical Methods in Kinetic Theory,” Plenum Press, New York, 1969, Chap. VIII, Sec. 5.
- [10] C. Cercignani, “The Boltzmann Equation and Its Applications,” Springer-Verlag, Berlin, 1988.
- [11] C. Cercignani, “Rarefied Gas Dynamics, From Basic Concepts to Actual Calculations,” Cambridge Univ. Press, Cambridge, 2000.
- [12] C. Cercignani and A. Daneri, *Flow of a rarefied gas between two parallel plates*, J. Appl. Phys., **34** (1963), 3509–3513.
- [13] C. Cercignani, M. Lampis, and S. Lorenzani, *Flow of a rarefied gas between parallel and almost parallel plates*, in “Rarefied Gas Dynamics” (ed. M. Capitelli), AIP, (2005), 719–724.
- [14] P. Charrier and B. Dubroca, *Asymptotic transport models for heat and mass transfer in reactive porous media*, Multiscale Model. Simul., **2** (2003), 124–157.
- [15] C.-C. Chen, I.-K. Chen, T.-P. Liu, and Y. Sone, *Thermal transpiration for the linearized Boltzmann equation*, Commn. Pure Appl. Math., **60** (2007), 147–163.
- [16] P. Degond, V. Latocha, S. Mancini, and A. Mellet, *Diffusion dynamics of an electron gas confined between two plates*, Methods and Applications of Analysis, **9** (2002), 127–150.

- [17] S. Fukui and R. Kaneko, *Analysis of ultra-thin gas film lubrication based on linearized Boltzmann equation including thermal creep flow*, J. Tribol., **110** (1988), 253–262.
- [18] F. Golse, *Anomalous diffusion limit for the Knudsen gas*, Asymptotic Analysis, **17** (1998), 1–12.
- [19] Y.-L. Han, *Thermal-creep-driven flows in Knudsen compressors and related nano/microscale gas transport channels*, J. Microelectromech. Systems, **17** (2008), 984–997.
- [20] Y.-L. Han, M. Young, E. P. Muntz, and G. Shiflett, *Knudsen compressor performance at low pressures*, in “Rarefied Gas Dynamics” (ed. M. Capitelli), AIP, (2005), 162–167.
- [21] L. H. Holway, Jr., *Approximation Procedures for Kinetic Theory*, Ph.D. Thesis, Harvard University, 1963.
- [22] L. H. Holway, Jr., *New statistical models for kinetic theory: Methods of construction*, Phys. Fluids, **9** (1966), 1658–1673.
- [23] M. L. Hudson and T. J. Bartel, *DSMC simulation of thermal transpiration and accommodation pumps*, in “Rarefied Gas Dynamics, vol. 1” (eds. R. Brun, R. Campargue, R. Gatignol, J.-C. Lengrand), Cépaduès-Éditions, (1999), 719–726.
- [24] M. Knudsen, *Eine Revision der Gleichgewichtsbedingung der Gase. Thermische Molekularströmung*, Ann. Phys., **31** (1910), 205–229.
- [25] M. Knudsen, *Thermischer Molekulardruck der Gase in Röhren*, Ann. Phys., **33** (1910), 1435–1448.
- [26] F. J. McCormack, *Construction of linearized kinetic models for gaseous mixtures and molecular gases*, Phys. Fluids, **16** (1973), 2095–2105.
- [27] National Institutes of Natural Sciences and National Astronomical Observatory of Japan (eds.), “Chronological Scientific Tables,” Marzen, Tokyo (1994) (in Japanese).
- [28] H. Niimi, *Thermal creep flow of rarefied gas between two parallel plates*, J. Phys. Soc. Jpn., **30** (1971), 572–574.
- [29] T. Ohwada, Y. Sone, and K. Aoki, *Numerical analysis of the Poiseuille and thermal transpiration flows between two parallel plates on the basis of the Boltzmann equation for hard-sphere molecules*, Phys. Fluids A, **1** (1989), 2042–2049.
- [30] G. Pham-Van-Diep, P. Keeley, E. P. Muntz, and D. P. Weaver, *A micromechanical Knudsen compressor*, in “Rarefied Gas Dynamics” (eds. J. Harvey and G. Lord), Oxford University Press, (1995), 715–721.
- [31] G. J. Prangma, A. H. Alberga, and J. J. M. Beenakker, *Ultrasonic determination of the volume viscosity of N₂, CO, CH₄ and CD₄ between 77 and 300 K*, Physica, **64** (1973), 278–288.
- [32] F. Sharipov and V. Seleznev, *Data on internal rarefied gas flows*, J. Phys. Chem. Ref. Data, **27** (1998), 657–706.
- [33] C. Shen, *Use of the degenerated Reynolds equation in solving the microchannel flow problem*, Phys. Fluids, **17** (2005), 046101.
- [34] C. E. Siewert, *The linearized Boltzmann equation: Concise and accurate solutions to basic flow problems*, Z. angew. Math. Phys., **54** (2003), 273–303.
- [35] Y. Sone, “Kinetic Theory and Fluid Dynamics,” Birkhäuser, Boston, 2002; Supplementary Notes and Errata: Kyoto University Research Information Repository (<http://hdl.handle.net/2433/66099>).
- [36] Y. Sone, “Molecular Gas Dynamics: Theory, Techniques, and Applications,” Birkhäuser, Boston, 2007; Supplementary Notes and Errata: Kyoto University Research Information Repository (<http://hdl.handle.net/2433/66098>).
- [37] Y. Sone, T. Fukuda, T. Hokazono, and H. Sugimoto, *Experiment on a one-way flow of a rarefied gas through a straight circular pipe without average temperature and pressure gradients*, in “Rarefied Gas Dynamics” (eds. T. J. Bartel and M. A. Gallis), AIP, (2001), 948–955.
- [38] Y. Sone and E. Itakura, *Analysis of Poiseuille and thermal transpiration flows for arbitrary Knudsen numbers by a modified Knudsen number expansion and their database*, J. Vac. Soc. Jpn., **33** (1990), 92–94 (in Japanese) (the software package, based on this reference, made by Y. Sone, E. Itakura, and M. Handa, can be downloaded from the webpage <http://www.users.kudpc.kyoto-u.ac.jp/~a51424/Sone/database-e.html>).
- [39] Y. Sone and K. Sato, *Demonstration of a one-way flow of a rarefied gas induced through a pipe without average pressure and temperature gradients*, Phys. Fluids, **12** (2000), 1864–1868.
- [40] Y. Sone and H. Sugimoto, *Vacuum pump without a moving part and its performance*, in “Rarefied Gas Dynamics” (eds. A. D. Ketsdever and E. P. Muntz), AIP, (2003), 1041–1048.

- [41] Y. Sone, Y. Waniguchi, and K. Aoki, *One-way flow of a rarefied gas induced in a channel with a periodic temperature distribution*, Phys. Fluids, **8** (1996), 2227–2235.
- [42] S. Takata, H. Sugimoto, and S. Kosuge, *Gas separation by means of the Knudsen compressor*, Eur. J. Mech., B/Fluids, **26** (2007), 155–181.
- [43] S. E. Vargo and E. P. Muntz, *An evaluation of a multiple-stage micromechanical Knudsen compressor and vacuum pump*, in “Rarefied Gas Dynamics” (ed. C. Shen), Peking University Press, (1997), 995–1000.
- [44] P. Welander, *On the temperature jump in a rarefied gas*, Ark. Fys., **7** (1954), 507–553.
E-mail address: `takata@aero.mbox.media.kyoto-u.ac.jp`
E-mail address: `f.hitoshi@at8.ecs.kyoto-u.ac.jp`
E-mail address: `aoki@aero.mbox.media.kyoto-u.ac.jp`