

ON THE STRUCTURE OF CERTAIN $K3$ SURFACES

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$(I_4^{*2}) := \{X \rightarrow P_t^1 \mid \text{an elliptic } K3 \text{ surface } X \text{ with a section and two singular fibers both of type } I_4^* \text{ at } t = 0, \infty \in P_t^1 \text{ over } C\}$

E_1, E_2 : elliptic curves

$S = \text{Km}(E_1 \times E_2)$ has an elliptic fibration having a section and 3 singular fibers of type $I_4^*, I_{\mu_1}^*, I_{\mu_2}^*$ at $\infty, 2, -2 \in P^1$ ($0 \leq \mu_1, \mu_2 \leq 1$).

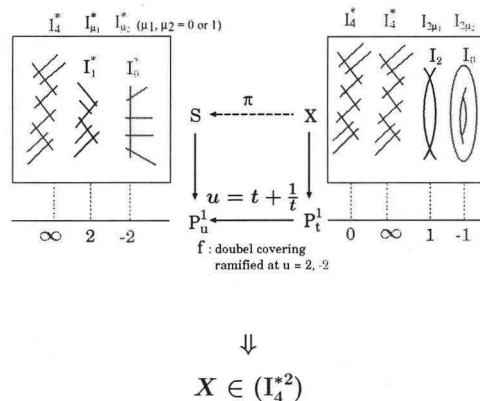
X : the double covering surface of S induced by

$$f: P_t^1 \rightarrow P_u^1; t \mapsto u = t + \frac{1}{t}$$

$$\downarrow$$

$$X \in (I_4^{*2})$$

(♣) Conversely is it possible to recover such a Kummer surface S from given $X \in (I_4^{*2})$?



Every $X \in (I_4^{*2})$ has the following Weierstrass equation

$$y^2 = x^3 - 3t^2(t^4 + a_3t^3 + a_2t^2 + a_1t + 1)x + 2t^3(t^6 + b_5t^5 + b_4t^4 + b_3t^3 + b_2t^2 + b_1t + 1),$$

where

$$b_1 = \frac{3}{2}a_1$$

$$b_2 = \frac{3}{8}a_1^2 + \frac{3}{2}a_2$$

$$b_3 = -\frac{1}{16}a_1^3 + \frac{3}{4}a_1a_2 + \frac{3}{2}a_3 = -\frac{1}{16}a_3^3 + \frac{3}{4}a_2a_3 + \frac{3}{2}a_1$$

$$b_4 = \frac{3}{8}a_3^2 + \frac{3}{2}a_2$$

$$b_5 = \frac{3}{2}a_3$$

$$a_1^4 - 8a_1^2a_2 + 32a_1a_3 + 16a_2^2 - 64a_2 - 16a_3^2 + 64 \neq 0$$

$$a_3^4 - 8a_2a_3^2 + 32a_1a_3 + 16a_2^2 - 64a_2 - 16a_1^2 + 64 \neq 0.$$

$$(I_4^{*2})_2 := \{X \in (I_4^{*2}) \mid a_1 = a_3\}$$

$$(I_4^{*2})_* := \{X \in (I_4^{*2}) \mid a_1 \neq a_3\}$$

The answer of the question (♣)

$$X \in (I_4^{*2})_2 \Rightarrow \text{YES.}$$

$$X \in (I_4^{*2})_* \text{ and } \rho(X) = 18 \Rightarrow \text{NO.}$$

Definition. (Shioda-Inose structure)

A $K3$ surface X admits a Shioda-Inose structure if there exists a symplectic involution ι on X with the rational quotient map $\pi: X \dashrightarrow Y$ of ι such that Y is a Kummer surface and $\pi_*: H^2(X, \mathbb{Z}) \rightarrow H^2(Y, \mathbb{Z})$ induces a Hodge isometry $T_X(2) \simeq T_Y$.

If X admits a Shioda-Inose structure, the complex torus such that $Y = \text{Km}(Z)$ gives a diagram



of rational maps of degree 2. This diagram induces a Hodge isometry

$$T_X \simeq T_Z.$$

Theorem 1.

If $X \in (I_4^{*2})_2$, then X admits a Shioda-Inose structure.

Theorem 2.

If $X \in (I_4^{*2})_*$ and Picard number of X is 18, then X does not admit a Shioda-Inose structure. However X is isomorphic to a Kummer surface of product type.