

On the Unit Groups of Burnside Rings.

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1. Introduction.

Let G be a finite group and let $\underline{\text{Set}}_f^G$ be the category of finite (right) G -sets and G -maps. The Grothendieck ring of this category (with respect to coproduct $+$ and product $\times$) is called the Burnside ring of G and is denoted by $A(G)$.

A super class function is a map of the set of subgroups of G to $\underline{\mathbb{Z}}$ which is constant on each conjugate class of subgroups. Let $\tilde{A}(G) = \underline{\mathbb{Z}}^{\text{Cl}(G)}$ be the ring of super class functions. For any subgroup S of G , the map $[X] \longrightarrow |X^S|$ extends to a ring homomorphism $\varphi_S : A(G) \longrightarrow \underline{\mathbb{Z}}$, and so we have a ring homomorphism

$$\varphi = \prod_{(S)} \varphi_S : A(G) \longrightarrow \tilde{A}(G) = \underline{\mathbb{Z}}^{\text{Cl}(G)}.$$

This map is injective. Thus we can identify any element x of $A(G)$ with the super class function $\varphi(x)$, and so we simply write $x(S) := \varphi(x)(S) = \varphi_S(x)$ for a subgroup S . See tom Dieck [Di79] Chapter 1.

Now, by geometric methods, tom Dieck proved that for an $\underline{RG}$ -module V , the function

$$u(V) : S \longmapsto \operatorname{sgn} \dim V^S$$

belongs to the Burnside ring $A(G)$, where $\operatorname{sgn} n := (-1)^n$ ([Di79] Proposition 5.5.9). The first purpose of this paper is to prove this fact by purely algebraic methods. In fact we shall prove the following theorem in Section 2.

Theorem A. Let G be a finite group and let V be a $\underline{CG}$ -module with real valued character. Then the function

$$u(V) : S \longmapsto \operatorname{sgn} \dim_{\underline{C}} V^S$$

is a member of the Burnside ring $A(G)$.

Since clearly $u(V)^2 = 1$ and $u(V \oplus W) = u(V) u(W)$, we have a group homomorphism into the unit group :

$$u = u_G : \bar{R}(G) \longrightarrow A(G)^*,$$

where $\bar{R}(G)$ is the ring of real valued virtual characters of G . We call this map u_G a tom Dieck homomorphism.

There are various maps between Burnside rings (and unit groups), and the assignment $A^* : H \longmapsto A(H)^*$ together with restrictions and multiplicative inductions forms a so called G -functor (= a Mackey functor from the

category of finite G -sets) and further that the tom Dieck homomorphism gives a morphism between G -functors. Since A^* is a G -functor, we have that $A(G)^*$ is a module over $A(G)$ (and also over $A(G)_{(2)}$, the localization at 2). In fact, the action $A(G)^* \times A(G) \longrightarrow A(G)^*$ is induced by the exponential map $(Y, X) \longrightarrow Y^X$ (the set of all maps of X to Y). From the theory of Burnside rings and G -functors, we can show some transfer theorems about $A(G)^*$. See Section 3. The proof will appear in another paper.

Notation and terminology. We always denote by G a finite group. The set of G -conjugate classes (H) of subgroups H of G is denoted by $Cl(G)$. For subsets A, B of G , we mean by $A =_G B$ (resp. $A \leq_G B$) that A and B are conjugate in G (resp. A is G -conjugate to a subgroup of B). We put $A^g := g^{-1}Ag$. When a group G acts on a set X , we denote by X^G the set of elements fixed by G . The ordinary character ring of G is denoted by $R(G)$. For a ring R , the unit group of R is denoted by R^* . The inner product of characters χ and ϑ is denoted by $\langle \chi, \vartheta \rangle$. Other notation and terminology are standard. See [Go68], [Di79].

2. Proof of Theorem A.

In this section, we prove Theorem A. As in the introduction, let G be a finite group and let $\text{Cl}(G)$ denote the set of conjugate classes of subgroups of G . We mean by (S) the class of a subgroup S . We set $WS := N_G(S)/S$ for a subgroup S of G .

Lemma 2.1. There is an exact sequence of abelian groups:

$$0 \longrightarrow A(G) \xrightarrow{\varphi} \underline{\mathbb{Z}}^{\text{Cl}(G)} \xrightarrow{\gamma} \prod_{(S)} (\underline{\mathbb{Z}}/|WS|\underline{\mathbb{Z}}) \longrightarrow 0,$$

where φ is the injective ring homomorphism given in the introduction, and for a super class function x , the S -component of $\gamma(x)$ is defined by

$$(x)_S := \sum_{gS \in WS} x(\langle g \rangle S) \pmod{|WS|}.$$

This lemma is well-known and its proof is found in, for example, tom Dieck [Di79] 1.3. We can now prove Theorem A.

Proof of Theorem A. Let χ be the real valued character afforded by the $\underline{\mathbb{C}}G$ -module V , and let $u(V)$ be the present super class function:

$$u(V) : (S) \longmapsto \text{sgn dim } V^S.$$

By Lemma 2.1, we must show that for each subgroup S of G ,

$$(1) \quad \sum_{gS \in WS} u(V)(\langle g \rangle S) \equiv 0 \pmod{|WS|}.$$

Let χ' be the character afforded by the $\mathbb{C}WS$ -module V^H , so that by an easy representation theory, we have that

$$\chi'(gS) = \frac{1}{|S|} \sum_{h \in S} \chi(gh), \quad gS \in WS,$$

and so χ' is also a real valued character. Thus in order to prove (1), we may assume that $S = 1$. Set $u_\chi(g) := u(V)(\langle g \rangle)$, then it has the value

$$u_\chi(g) = \text{sgn} \langle \chi|_{\langle g \rangle}, 1_{\langle g \rangle} \rangle,$$

where $\langle , \rangle$ stands for the inner product of characters.

Now when $S = 1$, (1) becomes

$$(2) \quad \sum_{g \in G} u_\chi(g) \equiv 0 \pmod{|G|}.$$

In order to prove (2), it will suffice to show that u is a virtual character of G . In fact, we can show that

$$(3) \quad u_\chi = (-1)^{\chi(1)} \det \chi,$$

where $\det \chi$ is the linear character of G defined by the

composition

$$\det \chi : G \longrightarrow GL(V) \xrightarrow{\det} \mathbb{C}^*.$$

See Yoshida [Yo78]. In order to prove (3), we may assume that G is cyclic. Since $u_{\chi+\varphi} = u_\chi u_\varphi$ and $\det(\chi+\varphi) = \det \chi \cdot \det \varphi$, we may further assume that either χ is a real valued linear character or $\chi = \lambda + \bar{\lambda}$ for some nonreal linear character λ . In the first case, we have that $\langle \chi|_{\langle g \rangle}, 1_{\langle g \rangle} \rangle = 1$ if g is in the kernel of χ and $= 0$ otherwise, and so $u_\chi = -1$ or $+1$, respectively. Since $\det \chi = \chi$, (3) holds in this case. Next assume that $\chi = \lambda + \bar{\lambda}$, so that $\langle \chi|_{\langle g \rangle}, 1_{\langle g \rangle} \rangle = 0$ or 2 and $\det \chi = 1_G$. Thus (3) holds also in this case. The theorem is proved.

3. Some transfer theorems for the unit groups.

Let p be a prime. We put

$$\begin{aligned} \mathbb{Z}_{(p)} &:= \{ a/b \in \mathbb{Q} \mid a \in \mathbb{Z}, b \in \mathbb{Z} - p\mathbb{Z} \}, \\ A(G)_{(p)} &:= \mathbb{Z}_{(p)} \otimes_{\mathbb{Z}} A(G). \end{aligned}$$

For a finite group H , the subgroup generated by all p' -elements of H is denoted by $O^p(H)$. When $O^p(Q) = Q$ (that is, Q has no normal subgroup of index p), the group

Q is called to be p-perfect. Let $Cl_p(G) \subseteq Cl(G)$ denote the classes of p-perfect subgroups.

There is a one-to-one correspondence between primitive idempotents of $A(G)_{(p)}$ and $Cl_p(G)$ (cf. [Di79] 1.4). An explicit formula of primitive idempotents was obtained by Gluck [Gl81] and Yoshida [Yo83]. Let μ be the Mobius function of the subgroup lattice of G and δ_G the function defined by

$$\delta_G(H, K) := \begin{cases} 1 & \text{if } H =_G K \\ 0 & \text{otherwise.} \end{cases}$$

Each primitive idempotent of $A(G)_{(p)}$ is then written in the form

$$e_{G,Q}^p = \sum_{(D) \in Cl(G)} \lambda_{G,Q}^{(D)} [D \setminus G],$$

where $(Q) \in Cl_p(G)$ and

$$\lambda_{G,Q}^{(D)} := \frac{1}{|N_G(D)|} \sum_{K \leq G} \mu(D, K) \delta_G(O^p(K), Q).$$

As a super class function, $e_{G,Q}^p$ has the value

$$e_{G,Q}^p(S) = \begin{cases} 1 & \text{if } O^p(S) =_G Q \\ 0 & \text{otherwise.} \end{cases}$$

For the finite group G , let $\underline{\underline{Set}}_f^G$ denote the

category of finite (right) G -sets and G -maps. For two G -sets X and Y , let Y^X be the G -set consisting of all mappings of X to Y with G -action defined by $\alpha^g(x) := \alpha(xg^{-1}) \cdot g$. For an element $a = [A] - [B]$ of $A(G)$, we furthermore define the exponential map

$$(-)^a : A(G)^* \longrightarrow A(G)^* ; u \longmapsto u^{A+B}.$$

We often write $u \uparrow a$ for u^a . By this action, $A(G)^*$ is an $A(G)_{(2)}$ -module whose annihilator contains $2A(G)_{(2)}$.

Theorem B. (i) There is a decomposition

$$A(G)^* = \bigsqcup_{(Q)} A(G)^* \uparrow e_{G,Q}^2,$$

where (Q) runs over $Cl_2(G)$, classes of 2-perfect subgroups.

(ii) Let Q be a 2-perfect subgroup of G and let P be a subgroup of $N := N_G(Q)$ such that P/Q is a Sylow 2-subgroup of N/Q . Then there are group isomorphisms:

$$A(G)^* \uparrow e_{G,Q}^2 \cong A(N)^* \uparrow e_{N,Q}^2 \cong (A(P)^*)^N \uparrow e_{P,Q}^2.$$

where the last group is the subgroup of $A(P)^* \uparrow e_{P,Q}^P$ consisting of all elements x such that

$$\text{res}_{P^n \cap P}^P \text{con}_P^n(x) = \text{res}_{P^n \cap P}^P(x)$$

for any element n of N .

Theorem C. Let N be a finite group with 2-perfect normal subgroup Q . Put $W := N/Q$. Let $\bar{P} = P/Q$ be a Sylow 2-subgroup of W . Then the following groups are isomorphic:

- (a) $A(N)^* \uparrow e_{N,Q}^2$,
- (b) $\{\bar{u} \in A(W)^* \uparrow e_{W,1}^2 \mid \bar{u}(S/Q) = 1 \text{ if } o^{2'}(S) \not\leq Q\}$,
- (c) $\{\bar{v} \in A(\bar{P})^{*W} \mid \bar{v}(S/Q) = 1 \text{ if } o^{2'}(S) \not\leq Q\}$,

where $o^{2'}(S)$ is the subgroup of S generated by all 2-elements and $A(\bar{P})^{*W}$ is the set of elements v of $A(\bar{P})^*$ such that

$$\text{res}_{\bar{P}^W \cap \bar{P}} \text{con}^G(x) = \text{res}_{\bar{P}^W \cap \bar{P}}(x) \quad \text{for all } w \text{ in } W.$$

Theorem D. Assume that the finite group G has an abelian Sylow 2-subgroup. Let Q be a 2-perfect subgroup of G . Put $N := N_G(Q)$, $W := N/Q$, and let P/Q be an (abelian) Sylow 2-subgroup of W . Put $L := N_G(P) (\leq N)$, $\bar{L} := L/Q'$, $\bar{Q} := Q/Q'$, and $\bar{P} := P/Q'$, where Q' is the intersection of subgroups of Q of odd prime index. Then the following hold :

$$(i) \quad A(G)^* \uparrow e_{GQ}^2 \cong A(\bar{L})^* \uparrow e_{\bar{L}\bar{Q}}^2.$$

(ii) If Q is perfect, then

$$A(\bar{L})^* \uparrow e_{\bar{L}Q}^2 \cong A(P/Q)^{*L/Q} \cong C_{(P/QP^2)}(L/Q),$$

where C stands for the centralizer group.

(iii) Assume that Q is not perfect. If $\bar{P}$ is generated by elements t with $C_{\bar{Q}}(t) \neq 1$, then $A(\bar{L})^* \uparrow e_{\bar{L}Q}^2 = 1$, and otherwise it is of order 2.

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Appendix

(References on Burnside rings)

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