

The Initial State of a Primordial Anisotropic Stage of Inflation

Jose J. Blanco-Pillado^{a,b} and Masato Minamitsuji^{c,d}

^aDepartment of Theoretical Physics, University of the Basque Country UPV/EHU, 48080 Bilbao, Spain

^bIKERBASQUE, Basque Foundation for Science, Maria Diaz de Haro 3, 48013 Bilbao, Spain

^cYukawa Institute for Theoretical Physics, Kyoto University, Kitashirakawa-Oiwakecho, Sakyo-Ku, Kyoto 606-8502, Japan

^dCENTRA, Instituto Superior Tecnico, Universidade de Lisboa, Avenida Rovisco Pais 1, 1049-001 Lisboa, Portugal

E-mail: jo세juan.blanco@ehu.es, masato.minamitsuji@ist.utl.pt

Received February 28, 2015

Accepted May 22, 2015

Published June 12, 2015

Abstract. We investigate the possibility that the inflationary period in the early universe was preceded by a primordial stage of strong anisotropy. In particular we focus on the simplest model of this kind, where the spacetime is described by a non-singular Kasner solution that quickly evolves into an isotropic de Sitter space, the so-called Kasner-de Sitter solution. The initial Big Bang singularity is replaced, in this case, by a horizon. We show that the extension of this metric to the region behind the horizon contains a timelike singularity which will be visible by cosmological observers. This makes it impossible to have a reliable prediction of the quantum state of the cosmological perturbations in the region of interest. In this paper we consider the possibility that this Kasner-de Sitter universe is obtained as a result of a quantum tunneling process effectively substituting the region behind the horizon by an anisotropic parent vacuum state, namely a $1+1$ dimensional spacetime compactified over an internal flat torus, T_2 , which we take it to be of the form $\text{de Sitter}_2 \times T_2$ or $\text{Minkowski}_2 \times T_2$. As a first approximation to understand the effects of this anisotropic initial state, we compute the power spectrum of a massless scalar field in these backgrounds. In both cases, the spectrum converges at small scales to the isotropic scale invariant form and only present important deviations from it at the largest possible scales. We find that the decompactification scenario from $M_2 \times T_2$ leads to a suppressed and slightly anisotropic power spectrum at large scales which could be related to some of the anomalies present in the current CMB data. On the other hand, the spectrum of the universe with a $dS_2 \times T_2$ parent vacuum presents an enhancement in power at large scales not consistent with observations.

Keywords: cosmology with extra dimensions, inflation

ArXiv ePrint: [1501.07427](https://arxiv.org/abs/1501.07427)



Contents

1	Introduction	1
2	The background anisotropic geometry	3
2.1	A cosmological background	3
2.2	Maximally extended spacetime	4
2.3	Quantum initial state	6
3	Anisotropic inflation as a result of quantum tunneling	8
4	Quantization of a scalar field	11
4.1	Choice of a parent vacuum	12
4.1.1	$M_2 \times T_2$	12
4.1.2	$dS_2 \times T_2$	13
5	The power spectrum	14
6	Conclusions	15
A	Kruskal extension of the Kasner-de Sitter universe	17
B	The Kasner-de Sitter metric in several gauges	18
C	Adding an electric field	19
D	The $M_2 \times T_2$ parent vacuum	20
D.1	Rindler and Milne spaces	20
D.2	Scalar field quantization in $M_2 \times T_2$	21
E	The $dS_2 \times T_2$ parent vacuum	23
E.1	dS_2 metrics	23
E.2	Scalar field quantization in $dS_2 \times T_2$	25
E.2.1	Outside of the horizon	25
E.2.2	Normalizing the mode functions	26
E.2.3	Constructing the vacuum state inside of the lightcone	27

1 Introduction

The latest results from the WMAP and *Planck* collaborations fit beautifully within a very simple model of inflation [1, 2]. On the other hand, there are a number of intriguing large scale anomalies in the cosmic microwave background (CMB) data that clearly deserve some attention. These anomalies include the low power of the quadrupole [3], the alignment of the quadrupole and octopole [4, 5], the oscillation in the power of low $\ell < 10$ multiples with $P_{\text{odd}} < P_{\text{even}}$ [6], as well as the so-called dipolar modulation [7, 8]. Although the statistical significance of some of these effects is still under debate, it is particularly interesting to think

that they might be related. Here we explore the possibility that they may be due to a particular state of the universe at the onset of inflation.

Several authors have investigated the idea that some of these anomalies could be due to a period of anisotropic inflation [9–14]. These models require the existence of some kind of matter during inflation that sustains an anisotropic energy momentum tensor and bypass the no-hair theorems for a spacetime with a positive cosmological constant [15]. It is interesting to see that some of these models lead to an attractor behavior for this anisotropic period making their predictions more robust (See, for example [16]).

There is however another way to explain these large scale anomalies without invoking the presence of new anisotropic energy sources. The idea is to assume that inflation only lasted for a relatively short number of e-foldings, in fact, just enough to solve the horizon, flatness and isotropy problems. In a situation like this one could be seeing the effects of the initial state of inflation at the largest possible scales of the CMB today. This obviously requires a degree of fine-tuning of the number of e-foldings, but taking into account the number of suspicious effects at those scales one is tempted to consider this possibility seriously. In particular one would like to explain the apparent violation of rotational symmetry at the largest scales by a initial period of anisotropic evolution. Considering a universe dominated by a pure cosmological constant, one would find (in agreement with the no-hair theorems) a rapid approach to isotropic expansion. In other words, there is only a *primordial anisotropic stage* of inflation. This is the kind of scenario we are contemplating in this paper.

Such period of primordial anisotropic inflation was first considered in [17] and [18]. One of the crucial points of this scenario is that, of course, one does not have a long period of inflation that would settle the quantum state to a Bunch-Davis vacuum, as one has for a regular inflationary model with a large number of e-folds. This makes the initial state of the vacuum before inflation potentially observable. Here we would like to explore this possibility in more detail by looking at some of the simple models that have been proposed for primordial anisotropic inflation.

The rest of the paper is organized as follows. We show in section II that the models studied in the literature can be extended past their apparent Big Bang hypersurface to a spacelike region that has a timelike singularity. Furthermore, the quantum state for perturbations in these models could not be specified without some knowledge about the conditions on the singularity. This makes it impossible to make precise predictions on these scenarios. We present in section III a different scenario where we replace the region with the singularity with a lower dimensional compactified spacetime. This gives a new interpretation to the background geometry for anisotropic inflation as an anisotropic bubble created by the decompactification of the lower dimensional state and allows us to obtain a consistent quantum initial state for the cosmological perturbations in this model. We discuss the way to find this quantum state in section IV. Finally in section V we study the observational consequences of such model and demonstrate that in some cases the new quantum state alleviates some of the pathologies found in the power spectrum of perturbations. We end in section VI with some discussion and conclusions.

2 The background anisotropic geometry

2.1 A cosmological background

In this paper, we consider the possibility that the geometry of our universe is described by the Bianchi I metric of the form,

$$ds^2 = -dt^2 + \sum_{i=1}^3 a_i(t)^2 dx_i^2, \quad (2.1)$$

where $a_i(t)$ ($i = 1, 2, 3$) are the scale factors in the three different spatial directions. If the existence of matter is ignored in the primordial stage of the universe, the initial metric can be approximated by the Kasner solution,

$$ds^2 = -dt^2 + \sum_{i=1}^3 t^{2p_i} dx_i^2, \quad (2.2)$$

the vacuum solution of Einstein's equations, where the three exponents satisfy the relations

$$\sum_i p_i = \sum_i p_i^2 = 1. \quad (2.3)$$

In the presence of a positive cosmological constant $\Lambda > 0$ (or the equivalent potential energy), one can find solutions of Einstein's equations whose geometries interpolate between the initial Kasner (2.2) solution and a late-time isotropic de Sitter phase. These solutions are given by the so-called Kasner-de Sitter solution [17–26],

$$ds^2 = -dt^2 + \sum_{i=1}^3 \sinh^{\frac{2}{3}}(3Ht) \left\{ \tanh\left(\frac{3Ht}{2}\right) \right\}^{2(p_i - \frac{1}{3})} dx_i^2, \quad (2.4)$$

where $H := \sqrt{\frac{\Lambda}{3}}$ is the Hubble rate of the de Sitter in the late-time limit and the p_i parameters satisfy the same conditions as before, namely, eq. (2.3).

These spacetimes are initially anisotropic but this phase is short lived and within a period of $t \sim (\text{a few}) \times H^{-1}$ the universe becomes isotropic in agreement with the expectation of the cosmic no-hair theorem [15]. The curvature invariant $R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}$ at $t = 0$ diverges for a generic Kasner spacetime (2.2), and thus the geometry is initially singular except for the particular branch where $p_1 = 1$ and $p_2 = p_3 = 0$ of the Kasner-de Sitter solution (2.4) that takes the form

$$ds^2 = -dt^2 + \left(\frac{2}{3} H^{-1} \sinh \frac{3Ht}{2} \left(\cosh \frac{3Ht}{2} \right)^{-\frac{1}{3}} \right)^2 dr^2 + \left(\cosh \frac{3Ht}{2} \right)^{\frac{4}{3}} dx_{\perp}^2, \quad (2.5)$$

where $0 < t < \infty$ and $x_{\perp}$ represents the coordinates of the 2d symmetric plane.¹ This is

¹This is a Bianchi I spacetime with an extra rotational symmetry in the $x_{\perp}$ plane. The authors of [17] found two different solutions of this form for a universe with a pure cosmological constant. By choosing $p_1 = -\frac{1}{3}$ and $p_2 = p_3 = \frac{2}{3}$ in (2.4), the other planar branch of the Kasner-de Sitter solution can be found in this gauge to be,

$$ds^2 = -dt^2 + \left(\frac{2}{3} H^{-1} \cosh \frac{3Ht}{2} \left(\sinh \frac{3Ht}{2} \right)^{-\frac{1}{3}} \right)^2 dr^2 + \left(\sinh \frac{3Ht}{2} \right)^{\frac{4}{3}} dx_{\perp}^2, \quad (2.6)$$

However, this spacetime is singular at $t = 0$.

indeed a very anisotropic spacetime near $t = 0$ where the r direction grows linearly with time while the expansion rate in the $x_\perp$ plane goes to zero, in other words, it becomes static.

The authors in [17] noted that $t = 0$ is not a real singularity in this case, but just a coordinate singularity and the universe near this point is represented by a non-singular Kasner solution of the form,

$$ds^2 \approx -dt^2 + t^2 dr^2 + dx_\perp^2. \quad (2.7)$$

This is in fact a piece of the 4d Minkowski space. Looking at the $t-r$ subspace one identifies a 2d Milne space, which covers the interior of the future lightcone of any point in the 2d Minkowski spacetime so one can perform a change of variables that brings this metric into the usual 4d Minkowski space metric form. This means that the hypersurface of $t = 0$ is not the real Big Bang singularity but there is a part of spacetime that lies behind it. One could be tempted to set the initial state of the vacuum at $t = 0$ [17, 18, 23–26], but it is clear that in this geometry one should go beyond this hypersurface since any disturbances can propagate freely through this null hypersurface all the way to us. We will show this in the following section.

2.2 Maximally extended spacetime

Taking into account the considerations made above regarding the $t = 0$ hypersurface, it is clear that one would like to know how to extend the spacetime beyond this point. As shown in the appendix B, there are several different gauges to describe the Kasner-de Sitter spacetime (2.5). In order to do this it will prove convenient to rewrite the metric given above in a different gauge, namely,

$$ds^2 = -\frac{dT^2}{f(T)} + f(T)dr^2 + T^2 dx_\perp^2 \quad (2.8)$$

where

$$f(T) := H^2 T^2 - \frac{R_0}{T}. \quad (2.9)$$

In order for T to be the timelike coordinate, and $f(T) > 0$, we take $L < T < \infty$, where

$$L := H^{-2/3} R_0^{1/3}, \quad (2.10)$$

such that $f(L) = 0$. In this form, the metric resembles the solution of a de Sitter black hole written in Schwarzschild coordinates, but there are several important differences. The first one is that the $x_\perp$ part of the metric represents a plane and it does not have a spherical symmetry as in the usual black hole geometries. This explains why there is no constant term in (2.9). Furthermore this is a time-dependent solution so, as it is, it only describes a region similar to the Schwarzschild-de Sitter (SdS) solutions beyond the cosmological horizon. Finally, let us look at the relative sign of each of the terms on the function $f(T)$. The first term describes the existence of a positive cosmological constant in our energy-momentum tensor as we should since we want our metric to approach de Sitter space asymptotically but the factor R_0 seems to represent a negative mass term.² In this new coordinate system, the $t = 0$ region is mapped into a finite time $T = L$,³ which in this language corresponds to the horizon of this geometry. In fact, it is the analog of the cosmological horizon in the SdS geometry.

²The case with positive mass term R_0 does not have any horizon but only the singularity at $T = 0$. This is in fact, the negative branch solution mentioned earlier.

³We can also set the value of R_0 to exactly match the solution found in the previous form.

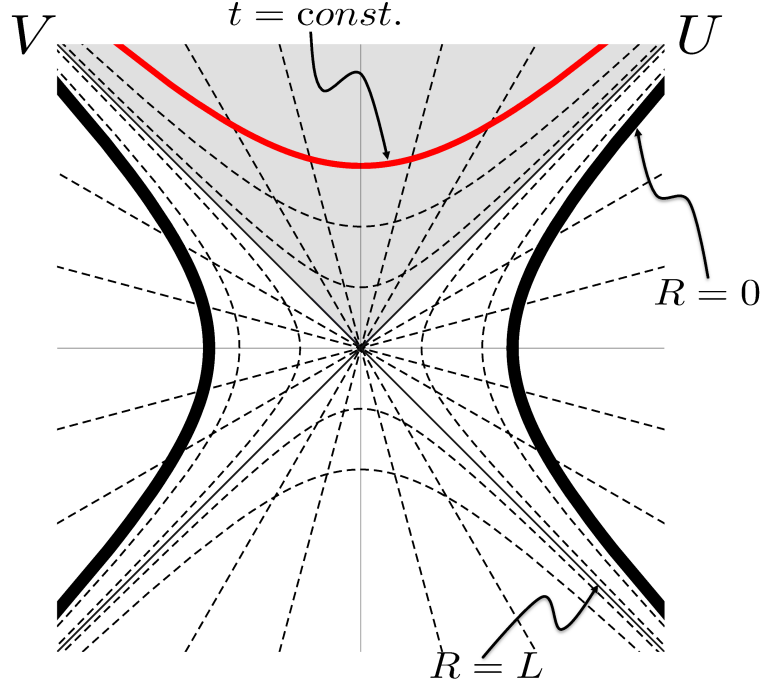


Figure 1. Kruskal diagram for the maximally extended Kasner-de Sitter solution. The shaded region is covered by the original Bianchi I metric (2.5) (and also by (2.8)). The lightlike lines at $R = L$ and the thick solid timelike curves at $R = 0$ correspond to the cosmological horizon and the timelike singularity, respectively. The red curve denotes the constant time hypersurface in the metric given by eq. (2.5). Each point in this diagram corresponds to a $2d$ plane.

In summary, this metric can be thought of as describing the region behind a cosmological horizon of a planar black hole of negative mass embedded in de Sitter space. The detailed coordinate transformations between the two descriptions of the Kasner-de Sitter metric (2.5) and (2.8) are explained in the appendix B.

The important point of having this representation for our spacetime is that it becomes now clear how to interpret the region *before* $t = 0$. One should do what is normally done in the black hole geometries to obtain the part of the spacetime beyond the horizon so that T becomes spacelike and r becomes timelike in the region where $T < L$. Replacing T and r with the new coordinates R and τ , respectively, we can write the metric in the form,

$$ds^2 = -\tilde{f}(R)d\tau^2 + \frac{dR^2}{\tilde{f}(R)} + R^2 dx_\perp^2 \quad (2.11)$$

where $0 < R < L$, so that

$$\tilde{f}(R) := H^2 \left(\frac{L^3}{R} - R^2 \right) > 0. \quad (2.12)$$

The other difference with the black hole case is that $\tilde{f}(R)$ does not have another horizon, there is no other root of this function so the metric plunges directly into a singularity at $R = 0$.

The relevant question for us is to what extent this singularity could affect the initial conditions in our universe; the initial conditions for our anisotropic inflation. This is a question about the causal structure of this spacetime which is better addressed in a Kruskal

diagram. In order to do this we introduce the new coordinate system given by,⁴

$$ds^2 = -\mathcal{F}(R)dUdV + R^2 dx_\perp^2, \quad (2.13)$$

where

$$\mathcal{F}(R) := \left(\frac{2}{3HL}\right)^2 \exp\left[-\sqrt{3}\arctan\left(\frac{L+2R}{\sqrt{3}L}\right)\right] \frac{(R^2 + LR + L^2)^{\frac{3}{2}}}{R}, \quad (2.14)$$

and the coordinates U and V read

$$V := \exp\left(-\frac{3H^2L}{2}\tau\right)\mathcal{H}(R)^{\frac{1}{2}}, \quad U := -\exp\left(\frac{3H^2L}{2}\tau\right)\mathcal{H}(R)^{\frac{1}{2}}, \quad (2.15)$$

with

$$\mathcal{H}(R) := \exp\left[\sqrt{3}\arctan\left(\frac{L+2R}{\sqrt{3}L}\right)\right] \left(\frac{L-R}{\sqrt{R^2 + LR + L^2}}\right), \quad (2.16)$$

and where R is obtained from the relation,

$$UV = \mathcal{H}(R). \quad (2.17)$$

Using this coordinate system we see that nothing special happens at the horizon where $R = L$. On the other hand, there is a real singularity that appears at $R = 0$, a timelike singularity that lies on a hyperbolic line on the $U - V$ plane. Finally, hypersurfaces of constant T (or constant t in the original metric (2.8)) in the time-dependent part of the spacetime are given by hyperbolas in the interior of the lightcone.

Following a similar procedure as one does in the Schwarzschild case one can find the maximal extension of this geometry. We show in figure 1 the Kruskal diagram of this maximal extension. Its structure is similar to the Schwarzschild diagram rotated by 90 degrees.

We show in red a typical hypersurface of constant time in the cosmological part of the spacetime described by eq. (2.5). This can be thought of a constant time hypersurface at the beginning of inflation. It is clear that the past lightcone of any point in this hypersurface would intersect the timeline singularity (the thick black curves in the spacelike part of the geometry) and therefore one cannot disregard its possible effect on the quantum state of the perturbations in our current universe.

2.3 Quantum initial state

The study of cosmological perturbations in our background is complicated by the fact that we are evolving not in an FRW universe but in an anisotropic Bianchi I universe. This brings the additional complication of the mixing of scalar- and tensor-type perturbations during the initial anisotropic stage of the universe [17, 18]. In the following we will concentrate on the study of the perturbations of a massless scalar field and its evolution on this background as a simplified model for perturbations. This is of course an approximation and one should really perform the correct evolution of perturbations along the lines of [17, 18]. We leave this for future work.

The lesson drawn from the previous section is that to study the quantum state of the universe in the cosmological region (the shaded region in figure 1), one should first understand the form of the metric in the spacelike region outside of the horizon. In order to do that let

⁴Look at the appendix A for the details of this coordinate transformation.

us start by writing the extension of the solution given by eq. (2.5) beyond the horizon in a similar gauge, namely,

$$ds^2 = - \left(\frac{2}{3} H^{-1} \sin \frac{3H\rho}{2} \left(\cos \frac{3H\rho}{2} \right)^{-\frac{1}{3}} \right)^2 d\tau^2 + d\rho^2 + \left(\cos \frac{3H\rho}{2} \right)^{\frac{4}{3}} dx_\perp^2. \quad (2.18)$$

There are two relevant regions in this metric. The near horizon part of the geometry where $\rho \approx 0$ and the metric approaches the Minkowski space in a Rindler form,

$$ds^2 = -\rho^2 d\tau^2 + d\rho^2 + dx_\perp^2 \quad (2.19)$$

and the singularity region at $\rho = \rho_{\max} := \frac{\pi}{3H}$ where the metric approaches the Taub geometry [27]

$$ds^2 = -(\rho_{\max} - \rho)^{-2/3} d\tau^2 + d\rho^2 + (\rho_{\max} - \rho)^{4/3} dx_\perp^2. \quad (2.20)$$

Not all timelike singularities are harmful and the quantization of a scalar field in this background could be possible if the information of the singularity were to be shielded from the actual cosmological observers inside of the horizon. In order to investigate this possibility we study the quantization of a massless scalar field in the vicinity of this Taub timelike singularity. Following [28], it would be useful to write this metric in the following gauge $\xi := (\rho_{\max} - \rho)^{4/3}$

$$ds^2 = -\xi^{-1/2} (d\tau^2 - d\xi^2) + \xi dx_\perp^2 \quad (2.21)$$

where the singularity occurs at $\xi \rightarrow 0$. To understand the behavior of the massless scalar field modes in this background, we start by decomposing the scalar field as

$$\phi_{k_\perp, E} = \xi^{-1/2} \psi_{k_\perp, E}(\xi) e^{-ik_\perp x_\perp} e^{-iE\tau} \quad (2.22)$$

so the equation for the field $\psi_{k_\perp, E}$ takes the form,

$$\left[-\frac{d^2}{d\xi^2} + V_{k_\perp}(\xi) \right] \psi_{k_\perp, E} = E^2 \psi_{k_\perp, E} \quad (2.23)$$

where

$$V_{k_\perp}(\xi) := -\frac{1}{4} \xi^{-2} + k_\perp^2 \xi^{-3/2}. \quad (2.24)$$

This is a Schrödinger type equation for the field ψ with a divergent potential near $\xi = 0$ where $V_{k_\perp}(\xi) \approx -(2\xi)^{-2}$. Potentials of this type have been discussed in the literature in [28, 29] where it was argued that this potential would lead to 2 normalizable solutions near the singularity. This means that we would have to impose some sort of boundary condition at the singularity, making the solution for the scalar field and, in turn, our quantum state unpredictable.

Note that the coefficient in front of the $1/\xi^2$ term in the potential is rather special since a slightly more negative value would lead to a much more serious problem with an ill-defined quantum mechanical problem [30]. We would of course like to have a different type of singularity where the potential is not attractive but repulsive so that the perturbations are uniquely specified by their asymptotic value far away from the singularity. Taking into account the backreaction of the scalar field in this background may improve the behavior of the fluctuations near the singularity and achieve such a repulsive potential. This is precisely what one finds in another closely related singular instanton, the so-called Hawking-Turok instanton [31]. In this case one can show that the situation improves dramatically taking

into account backreaction. See the discussion in [32]. On the other hand, to study this in our case would require a careful treatment of the anisotropic nature of the scalar perturbations in this part of the geometry using a decomposition of the form described in [17, 18]. We leave the investigation of this point for future work.

3 Anisotropic inflation as a result of quantum tunneling

The arguments presented in the previous section show that one should take into account the spacelike region of the geometry in order to describe the quantum state of the cosmological fluctuations in the timelike region. On the other hand, the extension of the simple Kasner-de Sitter geometry across the horizon yields a timelike singularity that is “visible” from our universe, spoiling the predictability of this spacetime. One could think of different ways to improve on this situation, either by introducing some regulating procedure, by cutting out entirely the singularity from the spacetime⁵ or making it “invisible” by adding some other form of matter that dominates near the singularity.⁶

In this paper we would like to take a different approach and give a new interpretation to the anisotropic Kasner-de Sitter solution as the outcome of a quantum tunneling process of a previously compactified space. In order to make the connection to this interpretation we should first imagine that the coordinates $x_{\perp}$ are not infinite but compact such that collectively represent a $2d$ torus, T_2 . This does not change anything in terms of the solutions presented earlier, they are still solutions of Einstein’s equations with a pure cosmological constant. The difference is that we should think of the spatial topology of the universe as $R \times T_2$ instead of R^3 . This does not suppose a radical change at least for the timelike part of the geometry where these directions are expanding and this new view will only impose a minor restriction on the size of these extra dimensions over our past cosmological history in order for them to be compatible with phenomenology. Looking at the form of the solution near the $t = 0$ we see that these extra dimensions approach a static configuration. This suggests a possible modification of the region of the space across the horizon that considers this spacelike region as a part of the universe where the extra-dimensions were static. These two regions together would therefore describe a decompactification transition.

These type of transdimensional transitions were first discussed in models of flux compactification in [36, 37] in the context of a higher-dimensional landscape of multiple vacua. In [38–40] the authors discussed another example of decompactification from a lower dimensional spacetime very similar to the one we have now. The difference between those models and the present work is the symmetry of the space. In their case, the spacetime had anisotropic spatial curvature (it had either open or closed subspaces) that led to a lower bound in the number of e-foldings after the anisotropic initial expansion of the transition.⁷ This requirement made it difficult to get an observational effect in the spectrum of cosmological perturbations since, as we explained earlier, a large number of e-foldings would move all the effects from primordial anisotropic inflation out of our present horizon. Here on the other hand, we do not have this problem since both sections of spacetime are spatially flat and the number of e-folds could be as low as 60.

⁵Similarly what is done for the Hawking-Turok instanton in [32, 33].

⁶Another possibility would be to allow the spacetime to end similarly to what happens in the “bubble from nothing” geometry [34, 35].

⁷This is due to the limits on the induced quadrupole generated by the late time anisotropic expansion due to curvature. See [38–41] for details.

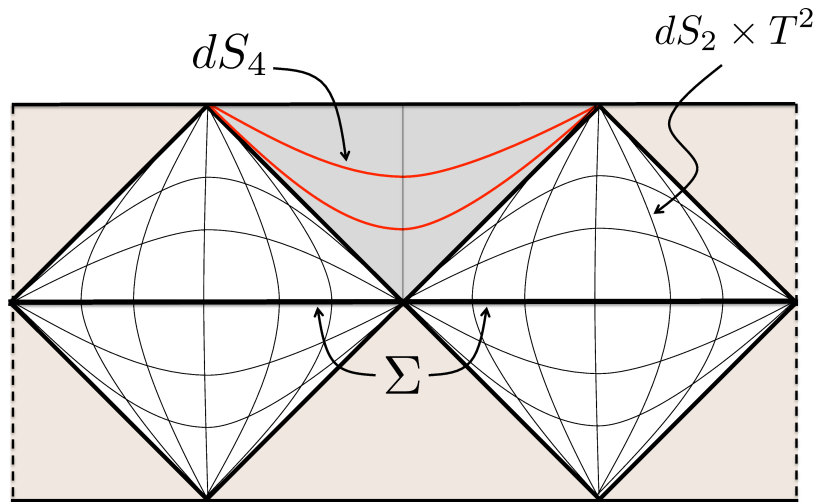


Figure 2. The Penrose diagram for the Kasner-de Sitter bubble nucleation from a de Sitter₂ × T₂ parent vacuum. Each point in this figure corresponds to a 2d torus, the T₂. The region of the spacetime shaded in gray is the part of the metric described by eq. (2.5). The white region corresponds to compactified parent vacuum described by the metric in (3.1). The asymptotic constant time slices represented in red rapidly approach an isotropic de Sitter space in 4d.

The decompactification solutions presented in [38, 40] were described by instantons that also mediated the creation of black objects in de Sitter space similar to the ones presented in [37, 42]. In those cases the solutions had two different horizons, a cosmological horizon that led to the anisotropic universe dominated by the cosmological constant and the “black hole” horizon that would lead to a spacetime resembling a lower dimensional compactified universe. In our case, we only have one horizon, the cosmological horizon. The difference is again the lack of curvature in our spacetime, so we need to supply the solution with some extra ingredient that allows for this other horizon that would substitute and regularize the geometry in the spacelike region. One can try to do this by adding an electric charge to this solution. This can be accomplished easily in the *Schwarzschild-like* gauge by introducing a new term in the solution for $f(T)$ proportional to Q^2/T^2 . We discuss this possibility in the appendix C where we show that this geometry also leads to a single horizon and an again a visible singularity. The timelike continuation of this spacetime has been considered recently in [43, 44]. Much of our discussion in this paper applies to these solutions as well.

It is interesting to note that one could, in principle, obtain an exact solution with the properties we are looking for by changing the sign of the coefficient in front of the kinetic term for the Maxwell field. This is a rather exotic possibility and we will not consider it further in this paper.

A much more interesting possibility was discussed in [45] where the authors found the instanton transitions we are interested in considering the contribution to the geometry from the Casimir type of calculation. It is tempting to think of this geometry as the quantum corrected geometry of the singular classical toroidal black hole in de Sitter space we have been discussing.

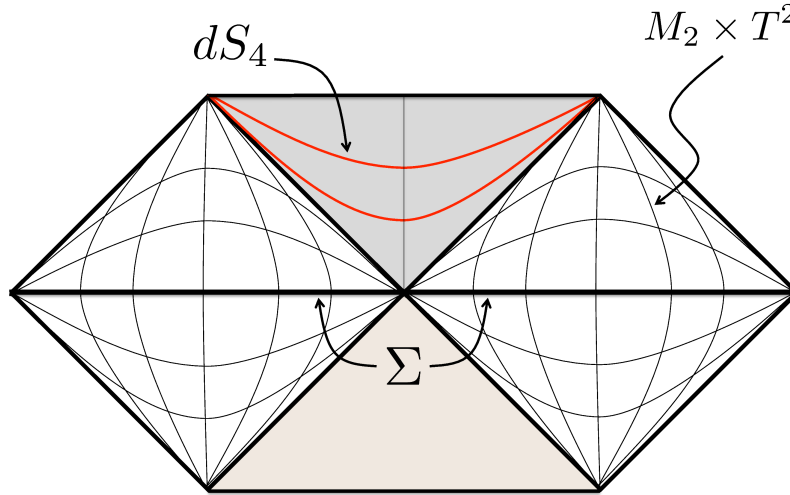


Figure 3. The Penrose diagram for the Kasner-de Sitter bubble nucleation from a Minkowski₂ × T₂ parent vacuum. Each point in this figure corresponds to a 2d torus, the T₂.

The Penrose diagram of this type of solution is shown in figure 2 where we denote by Σ the Cauchy surface for this geometry. In the following we will approximate the geometry in this spacelike region by the simpler $dS_2 \times T_2$ solution. This corresponds, in fact, to the Hawking-Moss limit of the instanton transition, where the size of the extra dimension does not change in the region between the horizons. Written in this gauge the solution becomes,

$$ds^2 = \left[-\frac{1}{H_{2d}^2} \sin(H_{2d} r)^2 d\tau^2 + dr^2 \right] + dx_\perp^2, \quad (3.1)$$

which clearly takes the correct Rindler form given by eq. (2.19) to match to the cosmological Kasner-de Sitter solution across the lightcone.

Another possibility is to assume that the initial state of the universe was in a static $M_2 \times T_2$ configuration right before its decompactification transition. The metric outside of horizon will now be given by $M_2 \times T_2$ in Rindler coordinates,

$$ds^2 = -r^2 d\tau^2 + dr^2 + dx_\perp^2. \quad (3.2)$$

This initial state is also compatible with the boundary conditions of the cosmological evolution inside of our bubble and it is therefore worth considering even if its interpretation as a tunneling process is not so clear in this case. A diagram of such a transition is given by figure 3.

Here we do not specify the matter content that could give rise to these parent compactified states and simply assume that they exist. It is also important to stress that in order to identify a model for this setup one would also have to study its perturbative stability which in many simple models would not be easy to achieve either for $dS_2 \times T_2$ or $M_2 \times T_2$.

Finally there is one more appealing point for this new interpretation of the non-singular Kasner-de Sitter solution. From the point of view of a generic Kasner solution the spacetime

we discussed seems to be fine-tuned to avoid the initial Big-Bang singularity. On the other hand, the interpretation of the metric as a tunneling transition gives an explanation for this rather unnatural initial conditions. The regularity of the instanton enforces the form of the solution around the lightcone and therefore its regularity is necessary in order to be able to have a transition.

This new interpretation of the spacelike region of our solution will allow us to set up the initial quantum state on the Σ hypersurfaces. This is what we do next.

4 Quantization of a scalar field

We are interested in understanding the spectrum of perturbations in this background geometry. As a first approximation we will study the spectrum of fluctuations for a massless scalar field φ minimally coupled to gravity:

$$S_\varphi = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right). \quad (4.1)$$

Although we do not specify a concrete origin of this massless scalar field, one can imagine that this is a simplified analog of the fluctuation of the inflaton field in this anisotropic background [17, 18] or the isocurvature fluctuation of a subdominant light degree of freedom during inflation which could be converted into the adiabatic perturbation after inflation [38].

The background metric in the shaded region of figure 1 can be written in several different gauges as we described earlier. For our purposes in this section it will be useful to express it in the following form⁸

$$ds^2 = -\frac{d\eta^2}{\sinh^2(-H_{2d}\eta)} + \alpha^4 \frac{e^{4H_{2d}\eta/3}}{\sinh^{2/3}(-H_{2d}\eta)} dr^2 + \alpha^{-2} \frac{e^{-2H_{2d}\eta/3}}{\sinh^{2/3}(-H_{2d}\eta)} dx_\perp^2 \quad (4.2)$$

where $-\infty < \eta < 0$ and we have introduced the definitions, $\alpha = 2^{1/3}$ and $H_{2d} = 3H$.

The expansion of the quantized scalar field in this region of spacetime can be given in general by,

$$\phi(\eta, r, x_\perp) = \int dk \sum_{k_\perp} \left[\frac{1}{(2\pi)^{3/2}} \tilde{a}_{k_\perp, k} f_{k_\perp, k}(\eta) e^{ik_\perp x_\perp} e^{-ikr} + \text{h.c.} \right], \quad (4.3)$$

where the evolution equations of each mode functions are,

$$\left[\frac{d^2}{d\eta^2} + \Omega^2(k_\perp, k, \eta) \right] f_{k_\perp, k}(\eta) = 0 \quad (4.4)$$

with,

$$\Omega^2(k_\perp, k, \eta) = \alpha^{-4} \sinh^{-4/3}(-H_{2d}\eta) e^{2H_{2d}\eta/3} (\alpha^6 k_\perp^2 + e^{-2H_{2d}\eta} k^2). \quad (4.5)$$

We have not been able to solve these equations analytically so we will integrate them numerically for each value of k as well as $k_\perp$. This will allow us to compute the power spectrum of the perturbations outside of the horizon. In order to do that we need to set up a vacuum state or an initial form of the mode functions $f_{k_\perp, k}(\eta)$ in the limit of $\eta \rightarrow -\infty$.

⁸Look at the appendix B for a detailed explanation of the change of coordinates among the different gauges used throughout this paper.

4.1 Choice of a parent vacuum

Several groups have tackled the quantization of a scalar field in this geometry using numerical [17] as well as analytic techniques [23–26, 46–48]. The important difference with our present work is the choice of the vacuum state which is now dictated by the parent vacuum before decompactification. Previous computations followed the conventional approach of most inflationary models and assume that one could set the initial vacuum state by looking at the form of the metric at $\eta \rightarrow -\infty$. Taking the $\eta \rightarrow -\infty$ limit in eq. (4.2) one arrives at,

$$ds^2 = \frac{4}{e^{-2H_{2d}\eta}} (-d\eta^2 + dr^2) + dx_\perp^2 \quad (4.6)$$

which is nothing more than our metric near the horizon given by eq. (2.7) but written in a conformal gauge. In other words this is the conformal form of the spacetime given by $\text{Milne}_2 \times T_2$. This is just a confirmation that indeed our metric becomes very anisotropic as one goes far into the past as it is supposed to.

The use of H_{2d} in this metric is arbitrary in this limit and can be reabsorbed in the definition of the coordinates. We use the $2d$ subscript to make the connection to the other possible parent vacua, namely the $dS_2 \times T_2$ case whose open slicing metric would take exactly the same form as the Milne case near the horizon, namely,

$$ds^2 = \frac{1}{\sinh^2(-H_{2d}\eta)} (-d\eta^2 + dr^2) + dx_\perp^2 \rightarrow \frac{4}{e^{-2H_{2d}\eta}} (-d\eta^2 + dr^2) + dx_\perp^2. \quad (4.7)$$

This is again nothing surprising, we are just saying that an open universe slicing of dS_2 should not know about the cosmological constant at early times, so it should behave as a spatially-flat universe dominated by curvature, a Milne universe in 2 dimensions in this case.

Let us discuss the two different vacua separately and identify the correct mode functions for each case.

4.1.1 $M_2 \times T_2$

One can write the equations of motion for the perturbations near the lightcone to obtain

$$\left[\frac{d^2}{d\eta^2} + (4e^{2H_{2d}\eta} k_\perp^2 + k^2) \right] f_{k_\perp, k}(\eta) = 0, \quad (4.8)$$

which can be thought of as the equations for a massive scalar field in $1+1$ Milne spacetime where the mass scale is given by mode number along the internal dimensions, $k_\perp$. The general solution of this equation can be written in terms of a combination of the Bessel functions of the form, $J_{\pm i\tilde{k}}(2\tilde{k}_\perp e^{H_{2d}\eta})$, where we have introduced the definitions, $\tilde{k} := \frac{k}{H_{2d}}$ and $\tilde{k}_\perp := \frac{k_\perp}{H_{2d}}$. Imposing that the mode functions behave as,

$$\lim_{\eta \rightarrow -\infty} f_{k_\perp, k}(\eta) \propto \frac{1}{\sqrt{2k}} e^{-ik\eta} \quad (4.9)$$

as one approaches $\eta \rightarrow -\infty$ (in other words $t \rightarrow 0$) and normalizing them on any constant time slice inside of the lightcone,⁹ one arrives at the following form for the mode functions,

$$f_{k_\perp, k}^{(c)}(\eta) = \sqrt{\frac{\pi}{2H_{2d} \sinh(\pi\tilde{k})}} J_{-i\tilde{k}}(2\tilde{k}_\perp e^{H_{2d}\eta}), \quad (4.10)$$

⁹One can see by looking at figure 3 that these constant time slices are, in this case, Cauchy surfaces for the whole spacetime.

where the superscript (c) refers to the fact that one can identify this choice of mode functions as the usual conformal vacua of a Milne spacetime [49]. The authors in [23, 46–48] found that this vacuum leads to a divergent behavior in the limit of $k \ll k_\perp$, the so-called planar regime. Furthermore it has also been shown that this vacuum induces a severe backreaction problem in this limit as well [26]. All these constraints make it difficult to consider this vacuum state as the relevant one for our period of primordial anisotropic inflation.

It is also clear that one cannot consider this vacuum as the one arising from a tunneling transition where the universe decompactifies from $M_2 \times T_2$ since things would blow up at the horizon [49] preventing the existence of the instanton itself. In fact, our interpretation of the anisotropic geometry in eq. (2.5) as the interior of a bubble created within a previously existing spacetime forces us to take the vacuum state of the parent vacuum as the correct state for scalar perturbations. In our case, this is nothing more than the usual Minkowski two-dimensional vacuum written in this Milne coordinate system. This corresponds to the mode functions of the form,

$$f_{k_\perp, k}^{(M)}(\eta) = \frac{1}{2} \sqrt{\frac{\pi}{H_{2d}}} e^{\pi \tilde{k}/2} H_{i\tilde{k}}^{(2)} \left(2\tilde{k}_\perp e^{H_{2d}\eta} \right) \quad (4.11)$$

where $H_{i\tilde{k}}^{(2)}$ denotes the Hankel functions of the second kind. One can show that these mode functions are related to the previous ones by a Bogoliubov transformation of the form,

$$f_{k_\perp, k}^{(M)} = \alpha_k f_{k_\perp, k}^{(c)} + \beta_k \left(f_{k_\perp, k}^{(c)} \right)^* \quad (4.12)$$

where

$$\alpha_k = \frac{e^{\pi \tilde{k}/2}}{\sqrt{e^{\pi \tilde{k}} - e^{-\pi \tilde{k}}}} \quad \beta_k = -\frac{e^{-\pi \tilde{k}/2}}{\sqrt{e^{\pi \tilde{k}} - e^{-\pi \tilde{k}}}}. \quad (4.13)$$

We show in the appendix D how one can obtain the explicit form of this vacuum state by studying the behavior of the mode functions on a Cauchy surface (Σ) on the previous vacuum and propagating them to the interior of the bubble.

4.1.2 $dS_2 \times T_2$

We will also consider the possibility that our parent vacuum was $dS_2 \times T_2$. Following techniques similar to the ones used in open inflation [50, 51] one can show that the correct vacuum state inside of our bubble is given by the analytic continuation of the appropriate solutions in the spacelike region of the $dS_2 \times T_2$ geometry.¹⁰ We show the details of the calculation in the appendix E. The resulting vacuum is given by,

$$\phi(\eta, r, x_\perp) = \int dk \sum_{k_\perp, i} \left[\frac{1}{(2\pi)^{3/2}} \tilde{a}_{k_\perp, k, i} f_{k_\perp, k}^{(i)}(\eta) e^{ik_\perp x_\perp} e^{-ikr} + \text{h.c.} \right], \quad (4.14)$$

where

$$f_{k_\perp, k}^{(1)}(\eta) = \frac{1}{\sqrt{2k}} \frac{e^{\pi k/2 H_{2d}}}{\sqrt{2 \sinh(\pi k/H_{2d})}} N(k, k_\perp) \tilde{f}_{k_\perp, k}^{(1)}(\eta) \quad (4.15)$$

$$f_{k_\perp, k}^{(2)}(\eta) = \frac{1}{\sqrt{2k}} \frac{e^{\pi k/2 H_{2d}}}{\sqrt{2 \sinh(\pi k/H_{2d})}} \left(L(k, k_\perp) \tilde{f}_{k_\perp, k}^{(1)}(\eta) + e^{-\pi k/H_{2d}} \tilde{f}_{k_\perp, k}^{(2)}(\eta) \right) \quad (4.16)$$

¹⁰This is 1+1 dimensional analogue of the Bunch-Davies vacuum in the open chart of de Sitter found in [52].

and we have introduced the functions,

$$\tilde{f}_{k_\perp, k}^{(1)}(\eta) = e^{-ik\eta} F\left[-\nu, \nu+1, 1-\mu, \frac{1+\xi_i}{2}\right] \quad (4.17)$$

$$\tilde{f}_{k_\perp, k}^{(2)}(\eta) = e^{ik\eta} F\left[-\nu, \nu+1, 1+\mu, \frac{1+\xi_i}{2}\right] \quad (4.18)$$

with

$$\xi_i = \coth(H_{2d}\eta); \quad \mu = i\left(\frac{k}{H_{2d}}\right); \quad \nu(\nu+1) = -\left(\frac{k_\perp}{H_{2d}}\right)^2 \quad (4.19)$$

and F denotes the generalized hypergeometric function so that $F[a, b, c, x] = {}_2F_1[a, b, c, x]$. Finally the normalization factors are given by,

$$N(k, k_\perp) = \frac{\Gamma(1+\nu-\mu)\Gamma(-\mu-\nu)}{\Gamma(1-\mu)\Gamma(-\mu)} \quad (4.20)$$

$$L(k, k_\perp) = -\frac{\Gamma(1+\mu)\Gamma(1+\nu-\mu)\Gamma(-\mu-\nu)}{\Gamma(1-\mu)\Gamma(-\nu)\Gamma(1+\nu)}. \quad (4.21)$$

5 The power spectrum

The results of the previous section give us the initial state of the scalar field modes right inside of the lightcone at the beginning of the bubble. One can then take these mode functions and evolve them forward in time numerically using eq. (4.4).

Here we present the results for the two cases we have studied, the $M_2 \times T_2$ and the $dS_2 \times T_2$ parent vacua. For each case we give the power spectrum

$$\mathcal{P} = \frac{1}{2\pi^2} (\alpha^{-4}k^2 + \alpha^2k_\perp^2)^{\frac{3}{2}} \times \begin{cases} |f_{k_\perp, k}^{(M)}(\eta \rightarrow 0)|^2 & (M_2 \times T_2) \\ \sum_{i=1}^2 |f_{k_\perp, k}^{(i)}(\eta \rightarrow 0)|^2 & (dS_2 \times T_2) \end{cases} \quad (5.1)$$

for several different values of the angle θ as the function of $\bar{k}$, which we define by the prescription, $k = \bar{k} \cos \theta$ and $k_\perp = \bar{k} \sin \theta$ in the momentum space.

We see that, as expected, the power spectrum becomes scale invariant and isotropic when $\bar{k}$ becomes a few times larger than the corresponding comoving wavenumber associated with the comoving Hubble radius at the onset of the inflationary regime. These roughly correspond to the comoving momentum of the modes that leave the horizon when the universe starts to become isotropic and inflationary. To simplify our notation, in the following, we normalize the comoving wavenumbers simply dividing them by H .

This is basically due to the fact that the large $\bar{k}$ modes leave the horizon when the universe is already expanding isotropically so one should recover in this case the usual isotropic scale invariant spectrum of dS_4 .¹¹ Furthermore, the power in this case is not divergent as one approaches the planar limit, $k \ll k_\perp$, as it is the case in the conformal vacuum.

In case of $M_2 \times T^2$ parent vacuum, it is interesting to note that the power spectrum is suppressed at large scales, low $\bar{k}$ and at the same time is slightly anisotropic in this regime, where the spectrum is more sensitive to the initial anisotropic state as well as the anisotropic evolution.

¹¹In a realistic model one should include a potential energy instead of a pure cosmological constant. That would introduce to a small tilt in the scalar power spectrum.

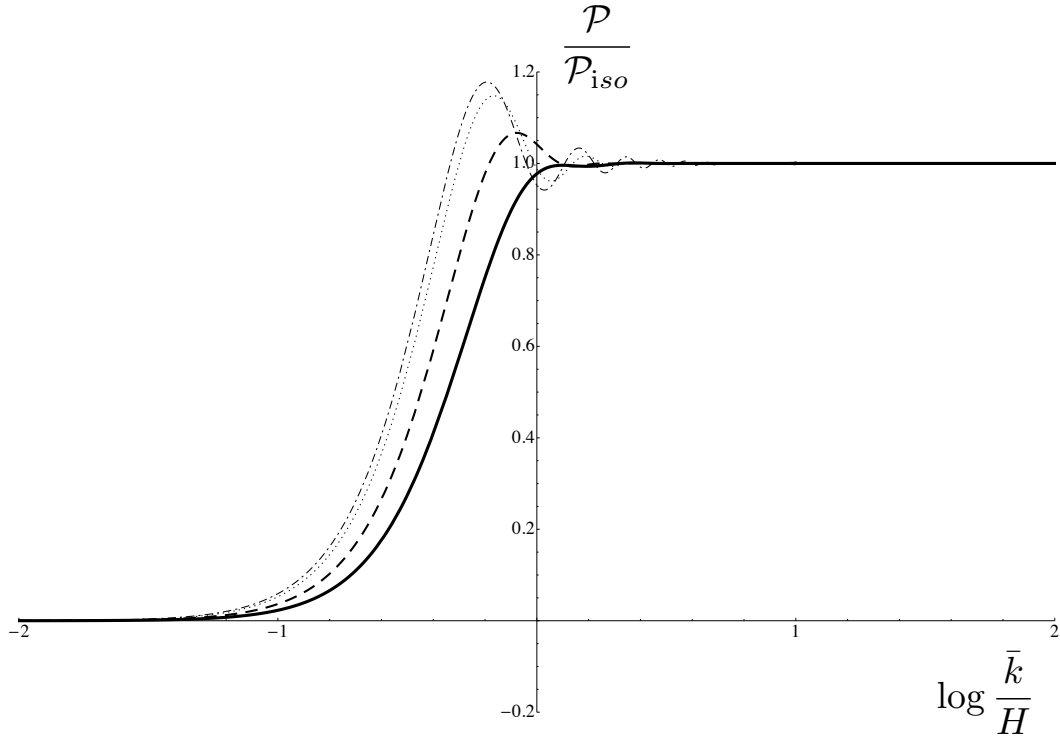


Figure 4. Power Spectrum for the $M_2 \times T_2$. We plot the ratio of the power spectrum to the power in the isotropic limit as the function of $\bar{k}$. We plot the following angles, $\theta = \pi/8, \pi/4, 3\pi/8, \pi/2$, which correspond to the solid, dashed, dotted and dot-dashed lines respectively.

The situation for the $dS_2 \times T_2$ parent vacuum is different and one finds a diverging power in the planar regime for low $\bar{k}$. This does not signal the presence of any singularity behavior at the lightcone, since the power spectrum is exactly the one that we will get for a set of massive scalar fields in a pure dS_2 background. On the other hand, from the observational point of view this type of spectrum with a large enhancement of power at low $\bar{k}$ seems to be in contradiction with the CMB observations. This suggests that we should take the $\log \bar{k} \approx 0$ point in the figure to be associated with the largest observable scales pushing all the extra power outside of our horizon today. Unfortunately this also means that it would be very challenging to distinguish between this decompactification model from any other model of nearly scale invariant isotropic spectrum in $4d$.

6 Conclusions

We have investigated the power spectrum of a massless scalar field in a model of primordial anisotropic inflation given by the Kasner-de Sitter solution of Einstein's equations in the presence of a pure cosmological constant. At early times, this geometry approaches a state of high anisotropy where part of the metric is described by a $1+1$ dimensional Milne universe while the other two spatial directions remain static. Without any other sources present to preserve the anisotropy, the spacetime quickly becomes the isotropic dS_4 making this metric a good candidate for a primordial anisotropic stage in the early universe before inflation. Any

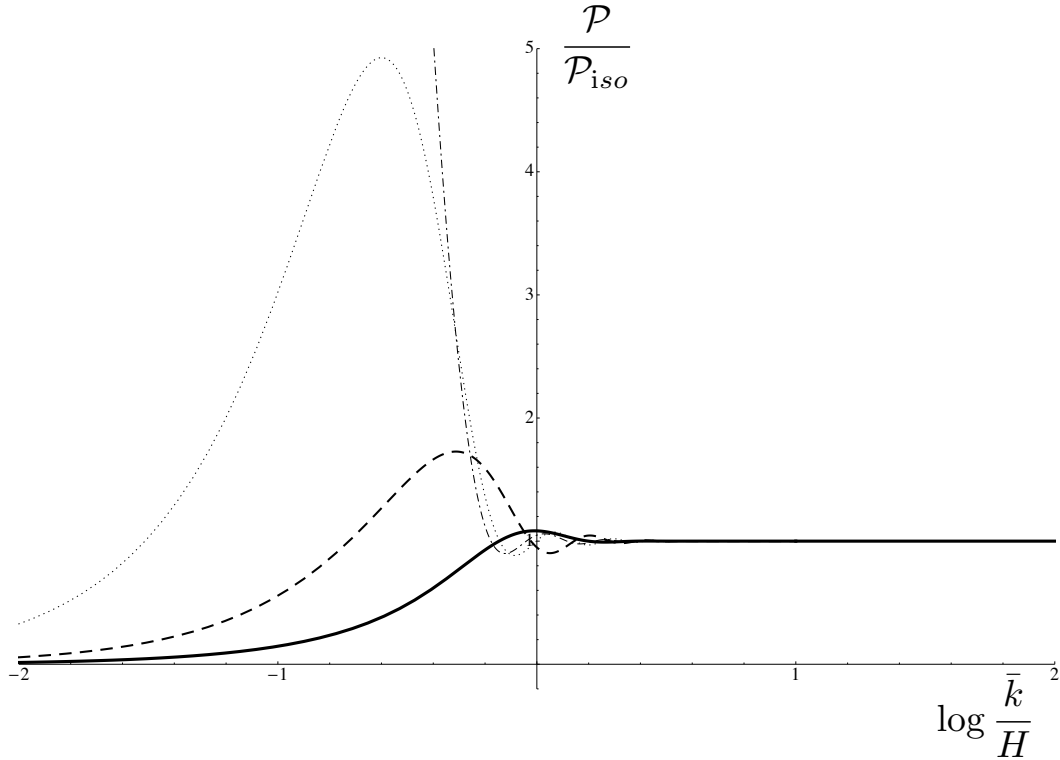


Figure 5. Power Spectrum for the $dS_2 \times T_2$. We use the same set of angles as in the previous figure.

observable effect that one can obtain from this initial period would therefore be encoded in the large scale today, the scales that leave the horizon during the anisotropic era. This means that these observables would be really sensitive to the initial vacuum state of the fluctuations.

Previous attempts to study the fluctuations in this geometry rely on the idea of identifying a vacuum state as the positive frequency modes for the initial Milne₂ state. This led to several divergences on the power spectrum that make this choice of initial state questionable.

Here we present an alternative view on this Kasner-de Sitter geometry that comes from the realization that the surface of the Big Bang for this spacetime is in fact a coordinate singularity and nothing more than the lightcone of the Milne slicing of Minkowski space. Extending the geometry beyond this $t = 0$ surface one encounters a timelike singularity that would be visible for observers in our cosmological spacetime making necessary to regulate the spacetime somehow before we can identify a vacuum state for our perturbations.

We propose to give a different interpretation to the Kasner-de Sitter metric as the outcome of a decompactification transition. We assume that our parent vacuum state was described by a cosmological $1 + 1$ dimensional spacetime compactified over a $2d$ internal space that we take to be a flat torus, T_2 . For simplicity we take the parent vacua to be either de Sitter₂ $\times$ T_2 or Minkowski₂ $\times$ T_2 such that they can be matched to the Kasner-de Sitter metric along the lightcone. Taking into account the full geometry of the decay process, one can identify a global Cauchy surface for these spacetimes and obtain a suitable vacuum state for the scalar field perturbations.

We calculated the power spectrum for a massless scalar field for different orientations of the wavevector of the perturbation. We find that, as expected, the power spectrum is isotropic

and scale invariant at small scales, since by the time that these modes leave the horizon the universe is pretty much isotropic. On the other hand, the spectra vary substantially from these results for the large scales. We find that the spectrum for $dS_2 \times T_2$ presents an important enhancement of its power at large scales. This seems to be in contradiction with current CMB observations that do not see this increase in power at large scales but the opposite. One can of course assume that the number of e-folds inside of our bubble was larger than 60, which will push the cosmological wavelengths associated with these modes outside of the current horizon making their effects almost irrelevant for us.

We have also computed the power spectrum for a transition from $M_2 \times T_2$. The results in this case are much more encouraging. We find that a transition of this kind leads to a suppression in the power spectrum at large scales as well a small variation on the power with the angle, a small anisotropic effect. These features could be related to some of the low- ℓ anomalies recently reported by the CMB collaborations.

In order to make a more precise comparison with the data, and test the presence of detectable anisotropy in the power spectrum one would have to compute the multiple correlators, the $C_{\ell\ell'mm'}$, looking in particular for signals that could set this model apart from other similar scenarios. Furthermore one should also include the proper treatment of metric perturbations in these backgrounds using the results in [17, 18] and study the effect of considering new vacuum states coming from decompactification. We leave these considerations for future research.

Acknowledgments

It is a pleasure to thank Jaume Garriga, Manuel Valle and Kepa Sousa for interesting discussions. We also want to give special thanks to Mike Salem for collaboration in the early stages of this work and his invaluable guidance with the details of the calculation. J.J. B.-P. would also like to acknowledge the hospitality of the Tufts Institute of Cosmology and the Yukawa Institute for Theoretical Physics where part of this work was conducted. M. M. wishes to thank the University of the Basque Country for their hospitality. This work is supported in part by IKERBASQUE, the Basque Foundation for Science and the Spanish Ministry of Science and Technology under Grant FPA2012-34456 (J.J. B.-P), the Yukawa Fellowship, the Grant-in-Aid for Young Scientists (B) of JSPS Research under Contract No. 24740162 and the FCT-Portugal through Grant No. SFRH/BPD/88299/2012 (M.M.).

A Kruskal extension of the Kasner-de Sitter universe

Starting from the original metric

$$ds^2 = -H^2 \left(\frac{L^3}{R} - R^2 \right) d\tau^2 + H^{-2} \left(\frac{L^3}{R} - R^2 \right)^{-1} dR^2 + R^2 dx_\perp^2, \quad (\text{A.1})$$

where L was defined in (2.10), we can introduce the following *tortoise coordinate*

$$dR_* = \frac{dR}{H^2 \left(\frac{L^3}{R} - R^2 \right)}, \quad (\text{A.2})$$

to arrive to the metric of the form,

$$ds^2 = H^2 \left(\frac{L^3}{R} - R^2 \right) (-d\tau^2 + dR_*^2) + R^2 dx_\perp^2, \quad (\text{A.3})$$

where the relation between R and R_* can be written explicitly as

$$R_* = \frac{1}{3LH^2} \left[-\sqrt{3} \arctan \left(\frac{1 + \frac{2R}{L}}{\sqrt{3}} \right) + \ln \left\{ \frac{\sqrt{R^2 + LR + L^2}}{L - R} \right\} \right]. \quad (\text{A.4})$$

The Kruskal extension can be obtained as follows: define new null coordinates

$$V = e^{k(\tau + R_*)}, \quad U = -e^{-k(\tau - R_*)} \rightarrow -dUdV = k^2(-d\tau^2 + dR_*^2)e^{2kR_*}. \quad (\text{A.5})$$

Choosing k so that

$$\frac{2k}{3LH^2} = -1,$$

then

$$k^2 e^{2kR_*} = \left(\frac{3H^2 L}{2} \right)^2 \exp \left[\sqrt{3} \arctan \left(\frac{1 + \frac{2R}{L}}{\sqrt{3}} \right) \right] \frac{L - R}{\sqrt{R^2 + LR + L^2}}. \quad (\text{A.6})$$

which brings the metric (A.3) to its final form, (2.13)–(2.16).

$$ds^2 = -\mathcal{F}(R)dUdV + R^2 dx_\perp^2, \quad (\text{A.7})$$

where

$$\mathcal{F}(R) := \left(\frac{2}{3HL} \right)^2 \exp \left[-\sqrt{3} \arctan \left(\frac{L + 2R}{\sqrt{3}L} \right) \right] \frac{(R^2 + LR + L^2)^{\frac{3}{2}}}{R}, \quad (\text{A.8})$$

and the coordinates U and V satisfy,

$$UV = \mathcal{H}(R). \quad (\text{A.9})$$

with

$$\mathcal{H}(R) := \exp \left[\sqrt{3} \arctan \left(\frac{L + 2R}{\sqrt{3}L} \right) \right] \left(\frac{L - R}{\sqrt{R^2 + LR + L^2}} \right), \quad (\text{A.10})$$

B The Kasner-de Sitter metric in several gauges

As we discussed in the main part of the text, perhaps the simplest version of the Kasner-de Sitter solution can be written in a Schwarzschild-like metric of the form,

$$ds^2 = -\frac{dT^2}{f(T)} + f(T)d\tilde{r}^2 + T^2 d\tilde{x}_\perp^2 \quad (\text{B.1})$$

where,

$$f(T) = H^2 \left(T^2 - \frac{L^3}{T} \right), \quad (\text{B.2})$$

and L was introduced in (2.10).

We can now go to a proper time coordinate by defining,

$$t = \frac{2}{3} H^{-1} \log \left[\left(\frac{T}{L} \right)^{3/2} + \sqrt{\left(\frac{T}{L} \right)^3 - 1} \right] \quad (\text{B.3})$$

to arrive at the metric,

$$ds^2 = -dt^2 + (HL)^2 \left(\sinh \frac{3Ht}{2} \left(\cosh \frac{3Ht}{2} \right)^{-\frac{1}{3}} \right)^2 d\tilde{r}^2 + L^2 \left(\cosh \frac{3Ht}{2} \right)^{\frac{4}{3}} dx_\perp^2. \quad (\text{B.4})$$

Rescaling the spacelike coordinates we arrive to the desired form of the metric, given in the main text, namely

$$ds^2 = -dt^2 + \left(\frac{2}{3} H^{-1} \sinh \frac{3Ht}{2} \left(\cosh \frac{3Ht}{2} \right)^{-\frac{1}{3}} \right)^2 dr^2 + \left(\cosh \frac{3Ht}{2} \right)^{\frac{4}{3}} dx_\perp^2. \quad (\text{B.5})$$

Finally we can also introduce another form of the metric by changing the time coordinate according to the identification,

$$\sinh(3Ht) = \frac{1}{\sinh(-3H\eta)} \quad (\text{B.6})$$

which brings the metric to the form,

$$ds^2 = -\frac{d\eta^2}{\sinh^2(-3H\eta)} + \alpha^4 \frac{e^{4H\eta}}{\sinh^{2/3}(-3H\eta)} dr^2 + \alpha^{-2} \frac{e^{-2H\eta}}{\sinh^{2/3}(-3H\eta)} dx_\perp^2. \quad (\text{B.7})$$

C Adding an electric field

The form of our anisotropic metric in its Schwarzschild gauge suggests an extension of this family of solutions that includes the possibility of an electromagnetic field, in other words, there may be solutions of the following action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \Lambda \right] \quad (\text{C.1})$$

that respect our Bianchi I form and that have some sort of electromagnetic field turned on in its background. Similarly to what one does in the case of black holes with spherical horizon we take the ansatz

$$ds^2 = -f(T)^{-1} dT^2 + f(T) dr^2 + T^2 dx_\perp^2 \quad (\text{C.2})$$

where the solution we are looking for looks like,

$$f(T) = H^2 T^2 - \frac{R_0}{T} - \frac{Q^2}{T^2}. \quad (\text{C.3})$$

This is indeed a solution of Einstein's equations for a configuration with a cosmological constant, $\Lambda = 3H^2$ and an electric field along the r direction, namely

$$F_{Tr} = \frac{Q}{T^2}. \quad (\text{C.4})$$

The solutions of this family have many similarities to the ones presented in the main part of the text and lead to the same kind of observational signatures. It is easy to see that this is indeed the case if we take the particular case where $R_0 = 0$ and bring the metric on a more cosmological form, namely,

$$ds^2 = -dt^2 + a^2(t) dr^2 + b^2(t) dx_\perp^2 \quad (\text{C.5})$$

where in this case we have the scale factors,

$$a(t) = 2^{\frac{1}{2}} e^{Ht_b} \sinh(2H(t - t_b)) [\cosh(2H(t - t_b))]^{-\frac{1}{2}}, \quad b(t) = 2^{\frac{1}{2}} e^{Ht_b} [\cosh(2H(t - t_b))]^{\frac{1}{2}}.$$

where $t_b = \frac{1}{2H} \log\left(\frac{Q}{2H}\right)$. We can bring the metric to a form much closer to the Kasner-de Sitter metric used in the text by shifting the time as well as rescaling the coordinates to arrive to:

$$ds^2 = -dt^2 + \left(\frac{1}{2H} \sinh(2Ht) [\cosh(2Ht)]^{-\frac{1}{2}}\right)^2 dr^2 + \cosh(2Ht) dx_{\perp}^2, \quad (C.6)$$

which clearly has the same qualitative behavior as one approaches the lightcone, at $t = 0$, as our Kasner-de Sitter geometry. In this regard, this metric is another example of a primordial stage of anisotropic inflation.

Note that this is the same metric as the one that has recently been discussed in the context of anisotropic inflation in [43, 44] where the form of the scale factors in this case were given in a slightly different form,

$$a(t) = A(t) \frac{1 - \frac{Q^2}{4A(t)^4 H^2}}{\sqrt{1 + \frac{Q^2}{4A(t)^4 H^2}}}, \quad b(t) = A(t) \sqrt{1 + \frac{Q^2}{4A(t)^4 H^2}}, \quad (C.7)$$

with $A(t) = e^{Ht}$. One can investigate the Kruskal diagram for this metric following exactly the same steps as we have done in this paper and see that the structure for this metric is identical to the Kasner-de Sitter case (See figure 1). This also implies that one should look beyond the lightcone to set the initial quantum state of the perturbations.

Studying the perturbations of a scalar field near the singularity one can check that the coefficient of the ξ^{-2} term in (2.24) is now replaced with $-\frac{2}{9}$. This again admits the existence of 2 normalizable solutions near the singularity which implies that this is a visible singularity for observers in our anisotropic universe.

We can include both effects in the metric, namely a mass and an electric field but the conclusions would be unchanged so it does not seem possible to regularize this singularity adding just an electromagnetic field. One should therefore think about a consistent way to specify the vacuum state in these kinds of models [43, 44] as well.

D The $M_2 \times T_2$ parent vacuum

D.1 Rindler and Milne spaces

The full Minkowski space in $1 + 1$ dimensions can be split into 4 regions, 2 Rindler wedges and 2 Milne wedges. In this section we present the metric appropriate for each region, their relation to the usual Minkowski metric and the analytic continuations that allow us to go between the different wedges.

The two Rindler wedges can be found by the embedding in Minkowski space by the following expressions,

$$T = \frac{2}{H_{2d}} e^{H_{2d}\eta_R} \sinh(H_{2d}r_R), \quad X = \frac{2}{H_{2d}} e^{H_{2d}\eta_R} \cosh(H_{2d}r_R) \quad (D.1)$$

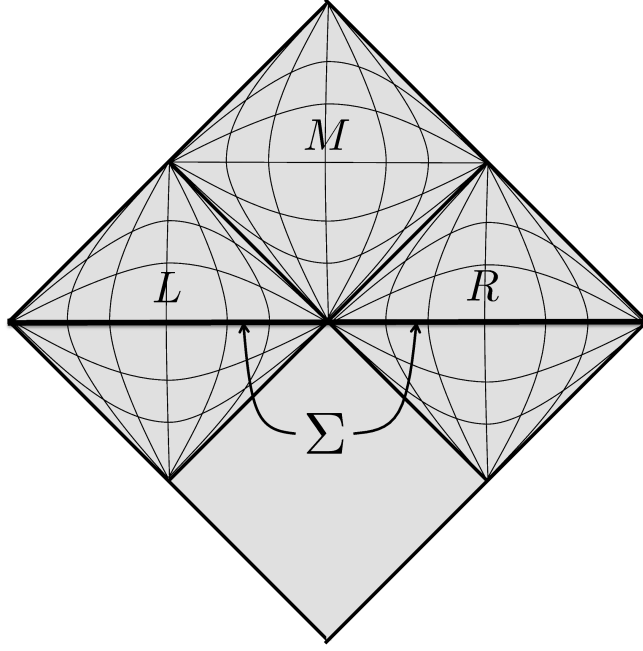


Figure 6. The full Minkowski space in $1 + 1$ dimensions split into 4 regions, 2 Rindler (R and L) wedges and 2 Milne wedges (the future wedge is M). We also show the Cauchy surface Σ where we will quantize the field.

where $-\infty < \eta_R < \infty$ and $-\infty < r_R < \infty$.

$$T = -\frac{2}{H_{2d}} e^{H_{2d}\eta_L} \sinh(H_{2d}r_L), \quad X = -\frac{2}{H_{2d}} e^{H_{2d}\eta_L} \cosh(H_{2d}r_L) \quad (\text{D.2})$$

where $-\infty < \eta_L < \infty$ and $-\infty < r_L < \infty$.

Furthermore the Milne part can also be found by the expression,

$$T = \frac{2}{H_{2d}} e^{H_{2d}\eta_M} \cosh(H_{2d}r_M), \quad X = \frac{2}{H_{2d}} e^{H_{2d}\eta_M} \sinh(H_{2d}r_M) \quad (\text{D.3})$$

where $-\infty < \eta_M < \infty$ and $-\infty < r_M < \infty$.

We can go from the R region to the M region by the following analytic continuation,

$$\eta_R = \eta_M + \frac{\pi}{2H_{2d}}i, \quad r_R = r_M - \frac{\pi}{2H_{2d}}i, \quad (\text{D.4})$$

similarly for the L region we have,

$$\eta_L = \eta_M + \frac{\pi}{2H_{2d}}i, \quad r_L = r_M + \frac{\pi}{2H_{2d}}i. \quad (\text{D.5})$$

D.2 Scalar field quantization in $M_2 \times T_2$

The form of the metric for the $M_2 \times T_2$ parent vacuum state can be written for the region outside of the lightcone as the so-called Rindler wedge,

$$ds^2 = 4e^{2H_{2d}\eta_R} (d\eta_R^2 - dr_R^2) + dx_\perp^2. \quad (\text{D.6})$$

This metric however only covers half of the spacetime outside of the bubble, and one needs to supplement it with the analogous *left* wedge (L). We will see shortly the relevance of these 2 different parts of the spacetime, but for the time being let us discuss the quantization of a massless scalar field on the *right* wedge.

Using the same expansion for the scalar field as before,

$$\phi(\eta_R, r_R, x_\perp) = \int dk \sum_{k_\perp} \left[\frac{1}{(2\pi)^{3/2}} \tilde{c}_{k_\perp, k} h_{k_\perp, k}(\eta_R) e^{ik_\perp x_\perp} e^{-ikr_R} + \text{h.c.} \right], \quad (\text{D.7})$$

we get,

$$h''_{k_\perp, k} + (k^2 - 4e^{2H_{2d}\eta_R} k_\perp^2) h_{k_\perp, k} = 0. \quad (\text{D.8})$$

One can think of this equation as the Schrödinger equation for an exponential potential and an energy state denoted by k^2 . The solutions of these mode functions are therefore suppressed for $\eta \rightarrow \infty$. One can show that the solutions that fulfill this requirement are given by,

$$h_{k_\perp, k}^{(R)}(\eta_R) = \sqrt{\frac{2 \sinh(\pi k / H_{2d})}{\pi H_{2d}}} K_{ik/H_{2d}} \left(\frac{2k_\perp e^{H_{2d}\eta_R}}{H_{2d}} \right), \quad (\text{D.9})$$

where we have fixed the normalization so that the functions are normalized on the Rindler wedge. Putting this together with the rest of the spatial dependence one arrives at,

$$\phi^{(R)}(\eta_R, r_R, k, k_\perp) = \sqrt{\frac{2 \sinh(\pi k / H_{2d})}{\pi H_{2d}}} K_{ik/H_{2d}} \left(\frac{2k_\perp e^{H_{2d}\eta_R}}{H_{2d}} \right) e^{-ikr_R}. \quad (\text{D.10})$$

One can also define the corresponding functions on the left wedge by making the substitution $R \rightarrow L$ with $e^{-ikr_R} \rightarrow e^{ikr_L}$, as in the left wedge time runs in the opposite way and so the positive frequency mode functions have the opposite sign in the exponent. Each of these functions only has support on their respective wedge but one can define a new mode function defined over the entire Cauchy surface, Σ , that one obtains by merging the $r_R = 0$ and $r_L = 0$ hypersurfaces (See figure 6). These new functions are given by [53],

$$h_{k_\perp, k}^{(M)} = \frac{1}{\sqrt{2 \sinh(\pi k / H_{2d})}} \left[e^{\pi k / 2 H_{2d}} \phi^R(\eta_R, r_R, k, k_\perp) + e^{-\pi k / 2 H_{2d}} (\phi^L(\eta_L, r_L, k, k_\perp))^* \right]. \quad (\text{D.11})$$

Where the overall factor has been chosen so that the new modes are properly normalized on Σ . Furthermore due to the nice analytic properties of these new functions one can analytically continue them into the interior of the lightcone to obtain the final form of our mode functions, namely,

$$f_{k_\perp, k}^{(M)}(\eta_M) = \frac{1}{2} \sqrt{\frac{\pi}{H_{2d}}} e^{\pi k / 2 H_{2d}} H_{ik/H_{2d}}^{(2)} \left(\frac{2k_\perp e^{H_{2d}\eta_M}}{H_{2d}} \right). \quad (\text{D.12})$$

These are in fact the mode functions that describe the usual Minkowski vacuum in the Milne coordinates so the expression for the initial state near the lightcone should be given by,

$$\phi(\eta_M, r_M, x_\perp) = \int dk \sum_{k_\perp} \left[\frac{1}{(2\pi)^{3/2}} \tilde{a}_{k_\perp, k} f_{k_\perp, k}^{(M)}(\eta_M) e^{ik_\perp x_\perp} e^{-ikr_M} + \text{h.c.} \right]. \quad (\text{D.13})$$

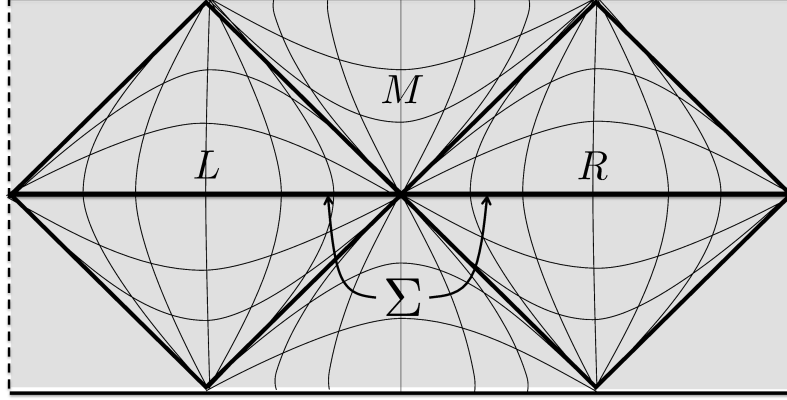


Figure 7. The Penrose diagram for the full de Sitter space in 1 + 1 dimensions split into the different regions discussed in the text.

E The $dS_2 \times T_2$ parent vacuum

E.1 dS_2 metrics

The Euclidean description of dS_2 is given by a 2 sphere which can be embedded in $3d$ by the equation $X_0^2 + X_1^2 + X_2^2 = H_{2d}^{-2}$, which we can parametrize by,

$$X_0 = H_{2d}^{-1} \cos \tau \cos \rho, \quad X_1 = H_{2d}^{-1} \sin \tau, \quad X_2 = H_{2d}^{-1} \cos \tau \sin \rho, \quad (\text{E.1})$$

giving the metric

$$ds_E^2 = H_{2d}^{-2} (d\tau^2 + \cos^2 \tau d\rho^2). \quad (\text{E.2})$$

We can do the following Wick rotation,

$$\tau = -iH_{2d}t_M + \frac{\pi}{2}, \quad \rho = -iH_{2d}r_M \quad (\text{E.3})$$

to obtain the embedding of part of dS_2 in $3d$ Minkowski space given by,

$$\begin{aligned} X_0 &= H_{2d}^{-1} \cosh(H_{2d}r_M) \sinh(H_{2d}t_M), & X_1 &= H_{2d}^{-1} \cosh(H_{2d}t_M), \\ X_2 &= H_{2d}^{-1} \sinh(H_{2d}t_M) \sinh(H_{2d}r_M), \end{aligned} \quad (\text{E.4})$$

so the induced metric becomes,

$$ds_M^2 = -dt_M^2 + \sinh^2(H_{2d}t_M) dr_M^2. \quad (\text{E.5})$$

This slicing of dS_2 covers the region that we denote by M in figure [7¹²](#)

¹²The label of this region as M comes from the fact that indeed this metric approaches the Milne one in the limit of $t_M \rightarrow 0$.

On the other hand, we can also do another Wick rotation,

$$\tau = H_{2d}t_R, \quad \rho = -iH_{2d}r_R + \frac{\pi}{2} \quad (\text{E.6})$$

to obtain,

$$\begin{aligned} X_0 &= H_{2d}^{-1} \cos(H_{2d}t_R) \sinh(H_{2d}r_R), & X_1 &= H_{2d}^{-1} \sin(H_{2d}t_R), \\ X_2 &= H_{2d}^{-1} \cos(H_{2d}t_R) \cosh(H_{2d}r_R), \end{aligned} \quad (\text{E.7})$$

that gives the metric

$$ds_R^2 = dt_R^2 - \cos^2(H_{2d}t_R) dr_R^2. \quad (\text{E.8})$$

which covers the region R in figure 7. We can also find the analogue of this coordinate system for the L part as well.

One can change from one chart to the other by doing,

$$t_R = -it_M + \frac{\pi}{2H_{2d}}, \quad r_R = r_M - i\frac{\pi}{2H_{2d}}. \quad (\text{E.9})$$

Also, we can introduce the a new coordinate system, by the following change of variables

$$\sinh(H_{2d}t_M) = \frac{1}{\sinh(-H_{2d}\eta_M)} \quad (\text{E.10})$$

with $0 \leq t_R \leq \infty$ being mapped to $-\infty \leq \eta_R \leq 0$, to get,

$$ds_M^2 = \frac{1}{\sinh^2(-H_{2d}\eta_M)} (-d\eta_M^2 + dr_M^2) \quad (\text{E.11})$$

which is the metric induced by the following embedding,

$$X_0 = H_{2d}^{-1} \cosh(H_{2d}r_M) \operatorname{csch}(-H_{2d}\eta_M) \quad (\text{E.12})$$

$$X_1 = H_{2d}^{-1} \cosh(H_{2d}t_M) \quad (\text{E.13})$$

$$X_2 = H_{2d}^{-1} \sinh(H_{2d}t_M) \operatorname{csch}(-H_{2d}\eta_M). \quad (\text{E.14})$$

Similarly, one can change coordinates outside of the horizon using the following relation,

$$\cos(H_{2d}t_R) = \frac{1}{\cosh(H_{2d}\eta_R)} \quad (\text{E.15})$$

with $-\frac{\pi}{2H_{2d}} \leq t_R \leq \frac{\pi}{2H_{2d}}$ being mapped to $-\infty \leq \eta_R \leq \infty$, so we get,

$$ds_R^2 = \frac{1}{\cosh^2(H_{2d}\eta_R)} (d\eta_R^2 - dr_R^2). \quad (\text{E.16})$$

This form of the metric is given by the embedding,

$$X_0 = H_{2d}^{-1} \operatorname{sech}(H_{2d}\eta_R) \sinh(H_{2d}r_R) \quad (\text{E.17})$$

$$X_1 = H_{2d}^{-1} \tanh(H_{2d}\eta_R) \quad (\text{E.18})$$

$$X_2 = H_{2d}^{-1} \operatorname{sech}(H_{2d}\eta_R) \cosh(H_{2d}r_R). \quad (\text{E.19})$$

We can relate both charts by the following analytic continuation,

$$r_R = r_M - i\frac{\pi}{2H_{2d}}, \quad \eta_R = -\eta_M - i\frac{\pi}{2H_{2d}}. \quad (\text{E.20})$$

E.2 Scalar field quantization in $dS_2 \times T_2$

E.2.1 Outside of the horizon

As we described in the main part of the text, we will define our vacuum state on the Cauchy surface Σ outside of the horizon of the bubble that describes the tunneling process (See figure 2). Our assumption is that the spacetime in this region is well approximated by the R and L patches of $dS_2 \times T^2$ where one can identify the analogous Cauchy surface. (See figure 7.) Following the description in the previous appendix E.1. one can write the metric of this part of dS_2 in the following way,

$$ds^2 = \frac{1}{\cosh^2(H_{2d}\eta_R)}(-dr_R^2 + d\eta_R^2) + dx_\perp^2. \quad (\text{E.21})$$

In order to describe the vacuum state for a massless scalar field in this geometry, we expand the field in the following form,

$$\phi(\eta_R, r_R, x_\perp) = \int dk \sum_{k_\perp, i} \left[\frac{1}{(2\pi)^{3/2}} \tilde{c}_{k_\perp, k, i} h_{k_\perp, k}^{(i)}(\eta_R) e^{ik_\perp x_\perp} e^{-ikr_R} + \text{h.c.} \right], \quad (\text{E.22})$$

so the equations of motion for the mode functions become,¹³

$$\left[-\frac{d^2}{d\eta_R^2} + \left(\frac{k_\perp^2}{\cosh^2(H_{2d}\eta_R)} - k^2 \right) \right] h_{k_\perp, k}^{(i)}(\eta_R) = 0. \quad (\text{E.23})$$

We can write two independent solutions of this equation in terms of Hypergeometric functions of the form,

$$\tilde{h}_{k_\perp, k}^{(1)} = \left(\frac{\xi_o + 1}{1 - \xi_o} \right)^{\mu/2} F \left[-\nu, \nu + 1, 1 - \mu, \frac{1 - \xi_o}{2} \right] \quad (\text{E.24})$$

$$\tilde{h}_{k_\perp, k}^{(2)} = \left(\frac{\xi_o + 1}{1 - \xi_o} \right)^{-\mu/2} F \left[-\nu, \nu + 1, 1 + \mu, \frac{1 - \xi_o}{2} \right] \quad (\text{E.25})$$

where we have defined,

$$\xi_o = \tanh(H_{2d}\eta_R), \quad \mu = i \left(\frac{k}{H_{2d}} \right), \quad \nu(\nu + 1) = - \left(\frac{k_\perp}{H_{2d}} \right)^2, \quad (\text{E.26})$$

and we have simplified the notation by defining the generalized hypergeometric function simply by $F[a, b, c, x] = {}_2F_1[a, b, c, x]$. One now needs to find the correct combination of these functions that are Klein-Gordon normalized on our Cauchy surface. As a first step in this direction, we identify the asymptotic form for each of the mode functions.

In the $\eta_R \rightarrow \infty$ limit, the $\xi_o \rightarrow 1$ one simply finds,

$$\tilde{h}_{k_\perp, k}^{(1)} \rightarrow \left(\frac{\xi_o + 1}{1 - \xi_o} \right)^{\mu/2} = e^{ik\eta_R}, \quad \tilde{h}_{k_\perp, k}^{(2)} \rightarrow \left(\frac{\xi_o + 1}{1 - \xi_o} \right)^{-\mu/2} = e^{-ik\eta_R}. \quad (\text{E.27})$$

To calculate the $\xi_o \rightarrow -1$ limit we use the relation of the hypergeometric functions,

$$\begin{aligned} F[a, b, c, x] &= \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} F[a, b, a+b-c+1, 1-x] \\ &+ (1-x)^{c-a-b} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} F[c-a, c-b, c-a-b+1, 1-x]. \end{aligned} \quad (\text{E.28})$$

¹³Note that this is analogous to the Schrödinger equation for a $k_\perp^2/\cosh^2(x)$ potential.

so the mode functions in the $\eta_R \rightarrow -\infty$ ($\xi_o \rightarrow -1$) limit become,

$$\begin{aligned} \tilde{h}_{k_\perp, k}^{(1)} &\rightarrow \left(\frac{\xi_o + 1}{1 - \xi_o}\right)^{\mu/2} \left[\frac{\Gamma(1 - \mu)\Gamma(-\mu)}{\Gamma(1 + \nu - \mu)\Gamma(-\mu - \nu)} + \frac{\Gamma(1 - \mu)\Gamma(\mu)}{\Gamma(-\nu)\Gamma(1 + \nu)} \left(\frac{1 + \xi_o}{2}\right)^{-\mu} \right] \\ &\approx \frac{\Gamma(1 - \mu)\Gamma(-\mu)}{\Gamma(1 + \nu - \mu)\Gamma(-\mu - \nu)} e^{ik\eta_R} + \frac{\Gamma(1 - \mu)\Gamma(\mu)}{\Gamma(-\nu)\Gamma(1 + \nu)} e^{-ik\eta_R} \end{aligned} \quad (\text{E.29})$$

as well as,

$$\begin{aligned} \tilde{h}_{k_\perp, k}^{(2)} &\rightarrow \left(\frac{\xi_o + 1}{1 - \xi_o}\right)^{-\mu/2} \left[\frac{\Gamma(\mu)\Gamma(1 + \mu)}{\Gamma(1 + \nu + \mu)\Gamma(\mu - \nu)} + \frac{\Gamma(1 + \mu)\Gamma(-\mu)}{\Gamma(-\nu)\Gamma(1 + \nu)} \left(\frac{1 + \xi_o}{2}\right)^{\mu} \right] \\ &\approx \frac{\Gamma(\mu)\Gamma(1 + \mu)}{\Gamma(1 + \nu + \mu)\Gamma(\mu - \nu)} e^{-ik\eta_R} + \frac{\Gamma(1 + \mu)\Gamma(-\mu)}{\Gamma(-\nu)\Gamma(1 + \nu)} e^{ik\eta_R}. \end{aligned} \quad (\text{E.30})$$

Looking at these asymptotic expansions one can then identify the correct combination of the mode functions that are normalized in other words that satisfy the conditions,

$$\int_{-\infty}^{\infty} d\eta \, h_{k_\perp, k}^{(i)} \left(h_{k'_\perp, k'}^{(i')} \right)^* = 2\pi \, \delta(k - k') \delta_{i, i'}. \quad (\text{E.31})$$

Using techniques borrowed from the 1d quantum mechanics problems [50, 51] one can find the correct combination to be,¹⁴

$$h_{k_\perp, k}^{(1)} = N(k, k_\perp) \tilde{h}_{k_\perp, k}^{(1)}, \quad h_{k_\perp, k}^{(2)} = L(k, k_\perp) \tilde{h}_{k_\perp, k}^{(1)} + \tilde{h}_{k_\perp, k}^{(2)} \quad (\text{E.32})$$

where we have introduced the coefficients,

$$N(k, k_\perp) = \frac{\Gamma(1 + \nu - \mu)\Gamma(-\mu - \nu)}{\Gamma(1 - \mu)\Gamma(-\mu)} \quad (\text{E.33})$$

$$L(k, k_\perp) = -\frac{\Gamma(1 + \mu)\Gamma(1 + \nu - \mu)\Gamma(-\mu - \nu)}{\Gamma(1 - \mu)\Gamma(-\nu)\Gamma(1 + \nu)}. \quad (\text{E.34})$$

E.2.2 Normalizing the mode functions

The general expression for the mode decomposition in the R-region, outside of the bubble, is given by,

$$\begin{aligned} \phi(\eta_R, r_R, x_\perp) &= \int dk \sum_{k_\perp, i} \left[\tilde{c}_{k_\perp, k, i} \phi_{k_\perp, k}^{(i)}(\eta_R, r_R, x_\perp) + \text{h.c} \right] \\ &= \int dk \sum_{k_\perp, i} \left[\tilde{c}_{k_\perp, k, i} \mathcal{N}_{k_\perp, k}^{(i)} h_{k_\perp, k}^{(i)}(\eta_R) e^{-ikr_R} e^{ik_\perp x_\perp} + \text{h.c} \right] \end{aligned} \quad (\text{E.35})$$

in order to quantize this model we need to normalize these modes using the Klein-Gordon normalization given by,

$$\left(\phi_{k_\perp, k}^{(i)}, \phi_{k'_\perp, k'}^{(i')} \right) = -i \int d\Sigma_\mu g^{\mu\nu} \left(\phi_{k_\perp, k}^{(i)} \partial_\nu \left(\phi_{k'_\perp, k'}^{(i')} \right)^* - \partial_\nu \phi_{k_\perp, k}^{(i)} \left(\phi_{k'_\perp, k'}^{(i')} \right)^* \right). \quad (\text{E.36})$$

¹⁴In particular, one can find this solution for the $1/\cosh^2(x)$ potential in [30].

Taking the Cauchy surface as the hypersurface of nucleation Σ with normal vector in the R-region

$$n^\mu = \cosh(H_{2d}\eta_R) (1, 0, 0, 0) \quad (\text{E.37})$$

we find that $d\Sigma^\mu = \cosh(H_{2d}\eta_R)\delta_0^\mu d\Sigma = d^3x\delta_0^\mu$. Inserting this into the normalization expression, we get

$$\begin{aligned} (\phi_{k_\perp, k}^{(i)}, \phi_{k'_\perp, k'}^{(i')}) &= -i \int d^3x \left(\phi_{k_\perp, k}^{(i)} \partial_{r_R} (\phi_{k'_\perp, k'}^{(i')})^* - \partial_{r_R} \phi_{k_\perp, k}^{(i)} (\phi_{k'_\perp, k'}^{(i')})^* \right) \\ &= -i \mathcal{N}_{k_\perp, k}^{(i)} \mathcal{N}_{k'_\perp, k'}^{(i')*} \left[2ik \left(\int d^2x_\perp e^{i(k_\perp - k'_\perp)x_\perp} \right) \left(\int d\eta_R h_{k_\perp, k}^{(i)} (h_{k'_\perp, k'}^{(i')})^* \right) \right] \\ &= 2k (2\pi)^3 \left| \mathcal{N}_{k_\perp, k}^{(i)} \right|^2 \delta(k_\perp - k'_\perp) \delta(k - k') \delta_{i, i'}, \end{aligned} \quad (\text{E.38})$$

where to get to the last line, we use,

$$\int d^2x_\perp e^{i(k_\perp - k'_\perp)x_\perp} = (2\pi)^2 \delta(k'_\perp - k_\perp) \quad (\text{E.39})$$

as well as,

$$\int_{-\infty}^{\infty} d\eta_C h_{k_\perp, k}^{(i)} (h_{k'_\perp, k'}^{(i')})^* = 2\pi \delta(k - k') \delta_{i, i'}. \quad (\text{E.40})$$

This means that we should take $\mathcal{N}_{k_\perp, k}^{(i)} = \left((2\pi)^{3/2} \sqrt{2k} \right)^{-1}$, so that our normalized functions become,

$$\phi_{k_\perp, k}^{(i)} = \frac{1}{(2\pi)^{3/2} \sqrt{2k}} h_{k_\perp, k}^{(i)}(\eta_R) e^{-ikr_R} e^{ik_\perp x_\perp}. \quad (\text{E.41})$$

This calculation shows that the final expansion of the quantized field in the R region should be of the form,

$$\phi(\eta_R, r_R, x_\perp) = \int dk \sum_{k_\perp, i} \left[\frac{1}{(2\pi)^{3/2} \sqrt{2k}} \tilde{c}_{k_\perp, k, i} h_{k_\perp, k}^{(i)}(\eta_R) e^{-ikr_R} e^{ik_\perp x_\perp} + \text{h.c} \right] \quad (\text{E.42})$$

where $h_{k_\perp, k}^{(i)}(\eta_R)$ are given by eqs. (E.32).

One can carry out the same type of computations in the L -wedge of the space-time arriving to the analogous normalized mode functions in the L section of the Σ surface.

E.2.3 Constructing the vacuum state inside of the lightcone

Following [54] one can define similarly to what we did in the Minkowski case, a new set of mode functions that are analytic over the whole Cauchy surface, Σ . To do that, we introduce the following normalized functions,

$$h_{k_\perp, k}^{(i)} = \frac{1}{\sqrt{2 \sinh(\pi k / H_{2d})}} \left[e^{\pi k / 2 H_{2d}} (h_{k_\perp, k}^{(i)}(\eta_R) e^{-ikr_R}) + e^{-\pi k / 2 H_{2d}} ((h_{k_\perp, -k}^{(i)}(\eta_L))^* e^{-ikr_L}) \right]$$

where $i = 1, 2$ run over the 2 independent solutions previously found in each $(dS_2)_{R,L}$ wedges.

One can find the form of the vacuum state inside of the light cone, in region M, by the following analytic continuations of the coordinates,

$$r_R = r_M - i \frac{\pi}{2H_{2d}}, \quad \eta_R = -\eta_M - i \frac{\pi}{2H_{2d}}, \quad (\text{E.43})$$

as well as the analogous one for the L coordinates.

Performing this analytic continuation we arrive at

$$\phi(\eta_M, r_M, x_\perp) = \int dk \sum_{k_\perp, i} \left[\frac{1}{(2\pi)^{3/2}} \tilde{a}_{k_\perp, k, i} f_{k_\perp, k}^{(i)}(\eta_M) e^{-ikr_M} e^{ik_\perp x_\perp} + \text{h.c.} \right], \quad (\text{E.44})$$

where

$$f_{k_\perp, k}^{(1)}(\eta_M) = \frac{1}{\sqrt{2k}} \frac{e^{\pi k/2H_{2d}}}{\sqrt{2 \sinh(\pi k/H_{2d})}} N(k, k_\perp) \tilde{f}_{k_\perp, k}^{(1)}(\eta_M) \quad (\text{E.45})$$

$$f_{k_\perp, k}^{(2)}(\eta_M) = \frac{1}{\sqrt{2k}} \frac{e^{\pi k/2H_{2d}}}{\sqrt{2 \sinh(\pi k/H_{2d})}} \left(L(k, k_\perp) \tilde{f}_{k_\perp, k}^{(1)}(\eta_M) + e^{-\pi k/H_{2d}} \tilde{f}_{k_\perp, k}^{(2)}(\eta_M) \right) \quad (\text{E.46})$$

where we have defined,

$$\tilde{f}_{k_\perp, k}^{(1)}(\eta_M) = e^{-ik\eta_M} F \left[-\nu, \nu + 1, 1 - \mu, \frac{1 + \xi_i}{2} \right], \quad (\text{E.47})$$

$$\tilde{f}_{k_\perp, k}^{(2)}(\eta_M) = e^{ik\eta_M} F \left[-\nu, \nu + 1, 1 + \mu, \frac{1 + \xi_i}{2} \right], \quad (\text{E.48})$$

with

$$\xi_i = \coth(H_{2d}\eta_M) \quad ; \quad \mu = i \left(\frac{k}{H_{2d}} \right) \quad ; \quad \nu(\nu + 1) = - \left(\frac{k_\perp}{H_{2d}} \right)^2. \quad (\text{E.49})$$

and where we have simplified the notation by denoting, $F[a, b, c, x] = {}_2F_1[a, b, c, x]$.

As we explained in the previous section of this appendix doing this analytic continuation to our metric bring us to the other patch of dS_2 , the one inside of the lightcone, namely,

$$ds_M^2 = \frac{1}{\sinh^2(-H_{2d}\eta_M)} (-d\eta_M^2 + dr_M^2). \quad (\text{E.50})$$

On the other hand, this metric has the same asymptotic behavior in the $\eta_M \rightarrow -\infty$ limit as the $2d$ part of our anisotropic de Sitter (Kasner-de Sitter) metric so we can take the analytic continuation of our vacuum as the right initial conditions for the mode functions inside of the decompactification bubble. This is what we do in the main part of the text.

References

- [1] WMAP collaboration, G. Hinshaw et al., *Nine-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: cosmological parameter results*, *Astrophys. J. Suppl.* **208** (2013) 19 [[arXiv:1212.5226](#)] [[INSPIRE](#)].
- [2] PLANCK collaboration, P.A.R. Ade et al., *Planck 2013 results. XXII. Constraints on inflation*, *Astron. Astrophys.* **571** (2014) A22 [[arXiv:1303.5082](#)] [[INSPIRE](#)].
- [3] G. Hinshaw et al., *2-point correlations in the COBE DMR 4-year anisotropy maps*, *Astrophys. J.* **464** (1996) L25 [[astro-ph/9601061](#)] [[INSPIRE](#)].
- [4] M. Tegmark, A. de Oliveira-Costa and A. Hamilton, *A high resolution foreground cleaned CMB map from WMAP*, *Phys. Rev. D* **68** (2003) 123523 [[astro-ph/0302496](#)] [[INSPIRE](#)].
- [5] D.J. Schwarz, G.D. Starkman, D. Huterer and C.J. Copi, *Is the low- ℓ microwave background cosmic?*, *Phys. Rev. Lett.* **93** (2004) 221301 [[astro-ph/0403353](#)] [[INSPIRE](#)].

- [6] K. Land and J. Magueijo, *Is the universe odd?*, *Phys. Rev. D* **72** (2005) 101302 [[astro-ph/0507289](#)] [[INSPIRE](#)].
- [7] H.K. Eriksen, F.K. Hansen, A.J. Banday, K.M. Gorski and P.B. Lilje, *Asymmetries in the cosmic microwave background anisotropy field*, *Astrophys. J.* **605** (2004) 14 [Erratum *ibid.* **609** (2004) 1198] [[astro-ph/0307507](#)] [[INSPIRE](#)].
- [8] Y. Akrami et al., *Power asymmetry in WMAP and Planck temperature sky maps as measured by a local variance estimator*, *Astrophys. J.* **784** (2014) L42 [[arXiv:1402.0870](#)] [[INSPIRE](#)].
- [9] L. Ackerman, S.M. Carroll and M.B. Wise, *Imprints of a primordial preferred direction on the microwave background*, *Phys. Rev. D* **75** (2007) 083502 [Erratum *ibid.* **D 80** (2009) 069901] [[astro-ph/0701357](#)] [[INSPIRE](#)].
- [10] M.-A. Watanabe, S. Kanno and J. Soda, *Inflationary universe with anisotropic hair*, *Phys. Rev. Lett.* **102** (2009) 191302 [[arXiv:0902.2833](#)] [[INSPIRE](#)].
- [11] J.D. Barrow and S. Hervik, *Simple types of anisotropic inflation*, *Phys. Rev. D* **81** (2010) 023513 [[arXiv:0911.3805](#)] [[INSPIRE](#)].
- [12] R. Emami, H. Firouzjahi, S.M. Sadegh Movahed and M. Zarei, *Anisotropic inflation from charged scalar fields*, *JCAP* **02** (2011) 005 [[arXiv:1010.5495](#)] [[INSPIRE](#)].
- [13] N. Bartolo, S. Matarrese, M. Peloso and A. Ricciardone, *Anisotropy in solid inflation*, *JCAP* **08** (2013) 022 [[arXiv:1306.4160](#)] [[INSPIRE](#)].
- [14] J. Ohashi, J. Soda and S. Tsujikawa, *Anisotropic non-Gaussianity from a two-form field*, *Phys. Rev. D* **87** (2013) 083520 [[arXiv:1303.7340](#)] [[INSPIRE](#)].
- [15] R.M. Wald, *Asymptotic behavior of homogeneous cosmological models in the presence of a positive cosmological constant*, *Phys. Rev. D* **28** (1983) 2118 [[INSPIRE](#)].
- [16] J. Soda, *Statistical anisotropy from anisotropic inflation*, *Class. Quant. Grav.* **29** (2012) 083001 [[arXiv:1201.6434](#)] [[INSPIRE](#)].
- [17] A.E. Gumrukcuoglu, C.R. Contaldi and M. Peloso, *Inflationary perturbations in anisotropic backgrounds and their imprint on the CMB*, *JCAP* **11** (2007) 005 [[arXiv:0707.4179](#)] [[INSPIRE](#)].
- [18] C. Pitrou, T.S. Pereira and J.-P. Uzan, *Predictions from an anisotropic inflationary era*, *JCAP* **04** (2008) 004 [[arXiv:0801.3596](#)] [[INSPIRE](#)].
- [19] P. Saunders, *Observations in some simple cosmological models with shear*, *Mon. Not. Roy. Astron. Soc.* **142** (1969) 213.
- [20] G.F.R. Ellis and M.A.H. MacCallum, *A class of homogeneous cosmological models*, *Commun. Math. Phys.* **12** (1969) 108 [[INSPIRE](#)].
- [21] H. Sato, *Dynamics of higher dimensional universe models*, *Prog. Theor. Phys.* **72** (1984) 98 [[INSPIRE](#)].
- [22] S. Hervik, *The Bianchi type-I minisuperspace model*, *Class. Quant. Grav.* **17** (2000) 2765 [[gr-qc/0003084](#)] [[INSPIRE](#)].
- [23] H.-C. Kim and M. Minamitsuji, *Revisiting the spectrum of a scalar field in an anisotropic universe*, *J. Korean Phys. Soc.* **64** (2014) 140 [[arXiv:1211.1132](#)] [[INSPIRE](#)].
- [24] A. Dey, E.D. Kovetz and S. Paban, *Power spectrum and non-Gaussianities in anisotropic inflation*, *JCAP* **06** (2014) 025 [[arXiv:1311.5606](#)] [[INSPIRE](#)].
- [25] A. Dey, E. Kovetz and S. Paban, *Non-Gaussianities in the cosmological perturbation spectrum due to primordial anisotropy II*, *JCAP* **10** (2012) 055 [[arXiv:1205.2758](#)] [[INSPIRE](#)].
- [26] A. Dey and S. Paban, *Non-Gaussianities in the cosmological perturbation spectrum due to primordial anisotropy*, *JCAP* **04** (2012) 039 [[arXiv:1106.5840](#)] [[INSPIRE](#)].

- [27] A. Taub, *Empty space-times admitting a three parameter group of motions*, *Annals Math.* **53** (1951) 472 [[INSPIRE](#)].
- [28] M. Blau, D. Frank and S. Weiss, *Scalar field probes of power-law space-time singularities*, *JHEP* **08** (2006) 011 [[hep-th/0602207](#)] [[INSPIRE](#)].
- [29] G.T. Horowitz and D. Marolf, *Quantum probes of space-time singularities*, *Phys. Rev. D* **52** (1995) 5670 [[gr-qc/9504028](#)] [[INSPIRE](#)].
- [30] R. Landau, *Quantum mechanics. Non-relativistic theory*, Wiley, New York U.S.A. (1990).
- [31] S.W. Hawking and N. Turok, *Open inflation without false vacua*, *Phys. Lett. B* **425** (1998) 25 [[hep-th/9802030](#)] [[INSPIRE](#)].
- [32] J. Garriga, *Open inflation and the singular boundary*, *Phys. Rev. D* **61** (2000) 047301 [[hep-th/9803210](#)] [[INSPIRE](#)].
- [33] R. Bousso and A. Chamblin, *Open inflation from nonsingular instantons: wrapping the universe with a membrane*, *Phys. Rev. D* **59** (1999) 063504 [[hep-th/9805167](#)] [[INSPIRE](#)].
- [34] J.J. Blanco-Pillado, H.S. Ramadhan and B. Shlaer, *Bubbles from nothing*, *JCAP* **01** (2012) 045 [[arXiv:1104.5229](#)] [[INSPIRE](#)].
- [35] J. Garriga, *Smooth ‘creation’ of an open universe in five-dimensions*, [hep-th/9804106](#) [[INSPIRE](#)].
- [36] J.J. Blanco-Pillado, D. Schwartz-Perlov and A. Vilenkin, *Quantum tunneling in flux compactifications*, *JCAP* **12** (2009) 006 [[arXiv:0904.3106](#)] [[INSPIRE](#)].
- [37] S.M. Carroll, M.C. Johnson and L. Randall, *Dynamical compactification from de Sitter space*, *JHEP* **11** (2009) 094 [[arXiv:0904.3115](#)] [[INSPIRE](#)].
- [38] J.J. Blanco-Pillado and M.P. Salem, *Observable effects of anisotropic bubble nucleation*, *JCAP* **07** (2010) 007 [[arXiv:1003.0663](#)] [[INSPIRE](#)].
- [39] P.W. Graham, R. Harnik and S. Rajendran, *Observing the dimensionality of our parent vacuum*, *Phys. Rev. D* **82** (2010) 063524 [[arXiv:1003.0236](#)] [[INSPIRE](#)].
- [40] J. Adamek, D. Campo and J.C. Niemeyer, *Anisotropic Kantowski-Sachs universe from gravitational tunneling and its observational signatures*, *Phys. Rev. D* **82** (2010) 086006 [[arXiv:1003.3204](#)] [[INSPIRE](#)].
- [41] M. Demianski and A.G. Doroshkevich, *Extension of the standard cosmological model: anisotropy, rotation and magnetic field*, *Phys. Rev. D* **75** (2007) 123517 [[astro-ph/0702381](#)] [[INSPIRE](#)].
- [42] J.J. Blanco-Pillado, D. Schwartz-Perlov and A. Vilenkin, *Transdimensional tunneling in the multiverse*, *JCAP* **05** (2010) 005 [[arXiv:0912.4082](#)] [[INSPIRE](#)].
- [43] X. Chen and Y. Wang, *Relic vector field and CMB large scale anomalies*, *JCAP* **10** (2014) 027 [[arXiv:1305.4794](#)] [[INSPIRE](#)].
- [44] X. Chen and Y. Wang, *Non-Bunch-Davies statistical anisotropies*, *JCAP* **07** (2014) 004 [[arXiv:1306.0609](#)] [[INSPIRE](#)].
- [45] J.M. Arnold, B. Fornal and K. Ishiwata, *Finite temperature structure of the compactified standard model*, *JHEP* **08** (2011) 030 [[arXiv:1103.0002](#)] [[INSPIRE](#)].
- [46] M. Minamitsuji, *Higher order spectra from an initially anisotropic universe*, *Phys. Rev. D* **85** (2012) 103526 [[arXiv:1205.5600](#)] [[INSPIRE](#)].
- [47] H.-C. Kim and M. Minamitsuji, *An analytic approach to perturbations from an initially anisotropic universe*, *JCAP* **03** (2011) 038 [[arXiv:1101.0329](#)] [[INSPIRE](#)].
- [48] H.-C. Kim and M. Minamitsuji, *Scalar field in the anisotropic universe*, *Phys. Rev. D* **81** (2010) 083517 [*Erratum ibid.* **82** (2010) 109904] [[arXiv:1002.1361](#)] [[INSPIRE](#)].

- [49] N. Birrell and P. Davies, *Quantum field theory in curved spacetime, spacetime formalism*, Cambridge Monogr. Math. Phys., Cambridge U.K. (1982).
- [50] J. Garriga, X. Montes, M. Sasaki and T. Tanaka, *Canonical quantization of cosmological perturbations in the one bubble open universe*, *Nucl. Phys. B* **513** (1998) 343 [[astro-ph/9706229](#)] [[INSPIRE](#)].
- [51] J. Garriga, X. Montes, M. Sasaki and T. Tanaka, *Spectrum of cosmological perturbations in the one bubble open universe*, *Nucl. Phys. B* **551** (1999) 317 [[astro-ph/9811257](#)] [[INSPIRE](#)].
- [52] M. Sasaki, T. Tanaka and K. Yamamoto, *Euclidean vacuum mode functions for a scalar field on open de Sitter space*, *Phys. Rev. D* **51** (1995) 2979 [[gr-qc/9412025](#)] [[INSPIRE](#)].
- [53] W.G. Unruh, *Notes on black hole evaporation*, *Phys. Rev. D* **14** (1976) 870 [[INSPIRE](#)].
- [54] S.A. Fulling, *Alternative vacuum states in static space-times with horizons*, *J. Phys. A* **10** (1977) 917 [[INSPIRE](#)].