

Representation of AC Hysteretic Characteristics of Silicon Steel Sheet Using Simple Excess Eddy-Current Loss Approximation

Tetsuji Matsuo, *Member, IEEE*, Yasushi Terada, and Masaaki Shimasaki, *Member, IEEE*

Department of Electrical Engineering, Graduate School of Engineering, Kyoto University, Kyoto 615-8510, Japan

A stop model is combined with a one-dimensional finite-difference method or a homogenization method to represent ac hysteretic characteristics of a silicon steel sheet. Eddy-current analysis without excess eddy-current loss fails to give an accurate eddy-current field. Representation of ac B - H loops is improved by increased conductivity that is obtained from an excess loss evaluation by the Pry and Bean model.

Index Terms—AC hysteresis, anomaly factor, excess eddy-current loss, homogenization, Pry and Bean model, stop model.

I. INTRODUCTION

GRAIN-ORIENTED silicon steel sheets are widely used as a core material, not only for transformers, but also for segmented core motors. The grain-oriented steel sheets have a large excess (or anomaly) eddy-current loss [1] because of their relatively large magnetic domain size compared with nonoriented steel sheets [2]. An anomaly factor [2] is often used to evaluate the excess eddy-current loss, which does not require an accurate B - H field distribution inside the steel sheet.

Recent development of computer technology enables us to simulate detailed B - H fields in electrical machines. However, accurate computation of the B - H field is not yet easy because of complex magnetic properties of the steel sheet including the excess eddy-current loss, hysteresis [1], and a vector property [3].

On the other hand, several precise dc hysteresis models have been developed, such as play and stop models [4], [5]. Those models are sufficiently efficient to be applied to magnetic field analyses. For example, a stop model having an input-dependent shape function [6], [7] has been proposed to accurately represent dc hysteretic characteristics of silicon steel sheets.

This study combines the stop model with a one-dimensional (1-D) finite-difference method or a homogenization method to represent ac hysteretic characteristics of a grain-oriented silicon steel sheet. A homogenized model is derived from the Pry and Bean model [8], where the effect of excess eddy-current loss is evaluated simply by an anomaly factor. This simple loss evaluation is also applied to the 1-D finite-difference analysis.

II. EDDY-CURRENT ANALYSIS WITHOUT EXCESS LOSS

This study analyzes an eddy-current field in a grain-oriented silicon steel sheet (JIS: 30P105). This steel sheet has electric conductivity σ of 2×10^6 S/m.

One-dimensional magnetic properties of the silicon steel sheet are measured using a single sheet tester [9] that applies the magnetic flux sinusoidally.

One-dimensional eddy-current fields are described by

$$\frac{\partial^2 H}{\partial y^2} = \sigma \frac{\partial B}{\partial t} \quad (1)$$

where the y direction is perpendicular to the rolling and transverse directions of the silicon steel sheet.

The finite-difference method is used to simulate the eddy-current field, where a stop model having the input-dependent shape function [6], [7] represents the hysteretic relation $H = H(B)$. The backward Euler time-difference scheme leads to

$$\frac{H(B_{i+1}^n) - 2H(B_i^n) + H(B_{i-1}^n)}{(\Delta y)^2} = \sigma \frac{B_i^n - B_i^{n-1}}{\Delta t} \quad (2)$$

where $B_i^n = B|_{y=i\Delta y, t=n\Delta t}$. The surface magnetic field is given by the measured data as the boundary condition. The computed B is averaged along the y direction for comparison with measured data.

Simulated ac B - H loops along the rolling and transverse directions are plotted in Figs. 1 and 2, respectively. The exciting frequencies are set at 20 and 50 Hz. Figs. 1 and 2 show that the simulated loops disagree with measured ones because this analysis neglects the excess eddy-current loss [1] that is attributable to the concentration of eddy currents around the magnetic domain walls.

III. PRY AND BEAN MODEL

Pry and Bean [8] have evaluated the excess eddy-current loss assuming a 1-D periodic magnetic domain structure. Fig. 3 illustrates this structure: $2L$ is the average domain width, x_W is the wall displacement from the demagnetized position, and d is the sheet thickness.

The electric current density $\mathbf{j} = (j_x, j_y)$ satisfies the following equations in the analyzed region: $-L \leq x \leq L$ and $-d/2 \leq y \leq d/2$

$$\text{div } \mathbf{j} = 0, \quad \text{curl } \mathbf{j} = 0 \quad (x \neq x_W). \quad (3)$$

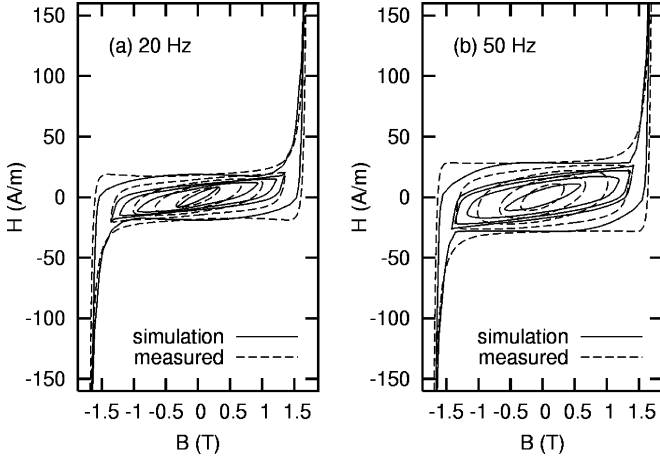


Fig. 1. AC B - H loops along the rolling direction given by eddy-current analysis.

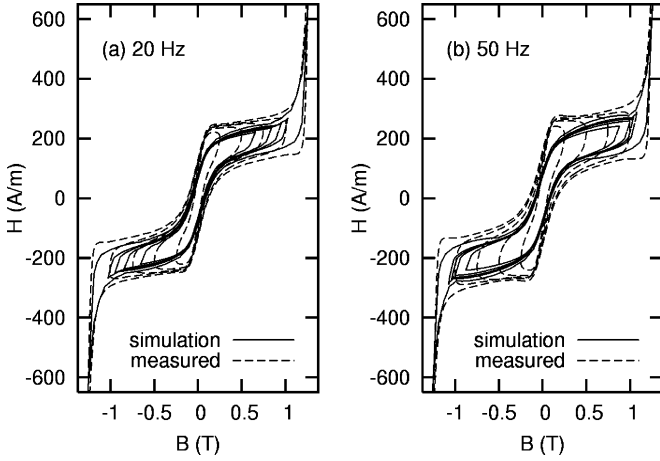


Fig. 2. AC B - H loops along the transverse direction given by eddy-current analysis.

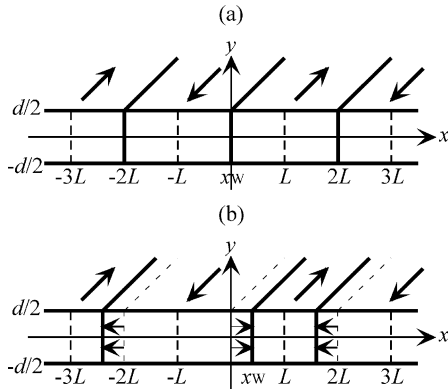


Fig. 3. One-dimensional periodic magnetic domain structure. (a) Demagnetized. (b) Magnetized.

The boundary condition is given as

$$\begin{aligned} j_x|_{x=x_w+0} &= j_x|_{x=x_w-0}, \\ j_y|_{x=x_w+0} - j_y|_{x=x_w-0} &= -2\sigma M_S v_W \end{aligned} \quad (4)$$

$$j_y = 0 \quad (x = \pm L), \quad j_y = 0 \quad \left(y = \pm \frac{d}{2}\right). \quad (5)$$

where M_S is the saturation magnetization and $v_W = dx_W/dt$ is the wall velocity.

Thereby, the instantaneous power loss per unit volume $P (= \int \int (j_x^2 + j_y^2) / \sigma dxdy / \int \int dxdy)$ becomes

$$P = \frac{16\sigma Ld}{\pi^3} \left(\frac{dB}{dt}\right)^2 \sum_{n \text{ odd}} \frac{\cosh \frac{n\pi(L+x_W)}{d} \cosh \frac{n\pi(L-x_W)}{d}}{n^3 \sinh \frac{2n\pi L}{d}} \quad (6)$$

where $B = M_S x_W / L$ is the average magnetic flux density. When $L \ll d$, P is reduced to the classical eddy-current loss P_C given by

$$P_C = \frac{\sigma d^2}{12} \left(\frac{dB}{dt}\right)^2. \quad (7)$$

On the other hand, when B is small ($x_W \ll L$), P is approximated as

$$P = \frac{8\sigma Ld}{\pi^3} \left(\frac{dB}{dt}\right)^2 \sum_{n \text{ odd}} \frac{1}{n^3} \coth \frac{n\pi L}{d} = \sigma \frac{k_E d^2}{12} \left(\frac{dB}{dt}\right)^2 \quad (8)$$

where k_E is the anomaly factor given by

$$k_E = \frac{P}{P_C} = \frac{96L}{\pi^3 d} \sum_{n \text{ odd}} \frac{1}{n^3} \coth \frac{n\pi L}{d}. \quad (9)$$

For simplicity, this paper uses this factor to approximate P as $P = k_E P_C$, independently of B .

IV. HOMOGENIZED MODEL

By setting

$$H_E = k_E \frac{\sigma d^2}{12} \frac{dB}{dt} \quad (10)$$

the power loss P is factorized as

$$P = H_E \frac{dB}{dt}. \quad (11)$$

The field H_E can be regarded as an averaged magnetic field at the domain walls [1], which is required to move the domain walls against the counter force by the eddy current. Accordingly, the applied field H_a is decomposed into

$$H_a = H_E + H_{DC} \quad (12)$$

where H_{DC} is a rate-independent field [1] that causes a dc hysteresis.

Consequently, a homogenized model including the excess eddy-current loss is given as

$$H_a(t) = H_{DC}(B(t)) + k_E \frac{\sigma d^2}{12} \frac{dB}{dt} \quad (13)$$

where $H_{DC}(B)$ represents the dc hysteresis property.

Several homogenized models having similar forms to (13) have already been presented in the literature. For example, homogenization without the excess loss ($k_E = 1$) is given in [10]. A more mathematically general form than (13) is discussed in [11].

AC B - H loops of the grain-oriented silicon steel sheet are simulated by the homogenized model (13), where the applied

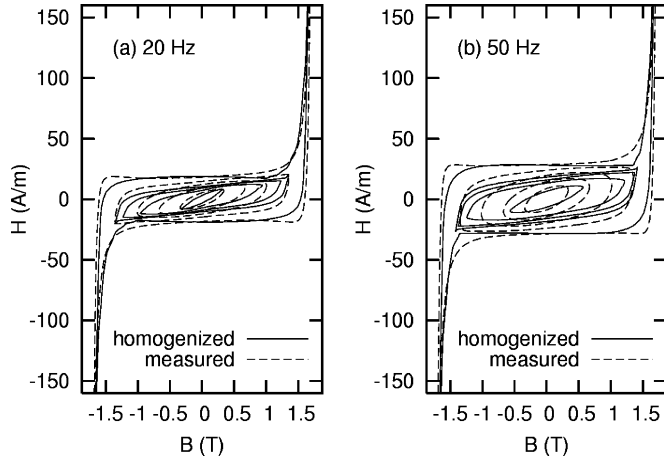


Fig. 4. AC B - H loops along the rolling direction given by the homogenized model with $k_E = 1$.

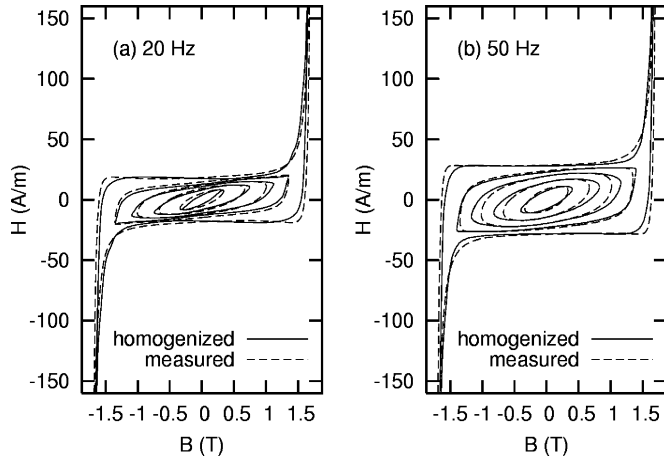


Fig. 5. AC B - H loops along the rolling direction given by the homogenized model with $k_E = 2.5$.

field $H_a(t)$ is given by the measured data for comparison with the finite-difference eddy-current analysis.

Figs. 4 and 5 show simulated B - H loops along the rolling direction, where $k_E = 1$ and 2.5 are used for the homogenized model, respectively. The factor $k_E = 2.5$ corresponds to $2L/d \approx 1.5$. Fig. 4 shows that homogenization without the excess eddy-current loss engenders a large discrepancy between the simulated and measured loops in the same way as the 1-D finite-difference analysis failed. Fig. 5 shows that the factor $k_E = 2.5$ improves the representation of ac B - H loops effectively.

Figs. 6 and 7 show simulated ac B - H loops along the transverse direction, where $k_E = 1$ and 10 (corresponding to $2L/d \approx 6$) are used, respectively. Fig. 6 shows that the simulation without the excess eddy-current loss fails to yield accurate B - H loops. The factor $k_E = 10$ improves the representation of ac B - H loops. The anomaly factor required for the transverse direction is much larger than that for the rolling direction because of the large domain size. However, a large discrepancy remains between the simulated and measured loops mainly because the Pry and Bean model cannot describe the 90° domain wall motion that dominates the magnetic property along the transverse direction.

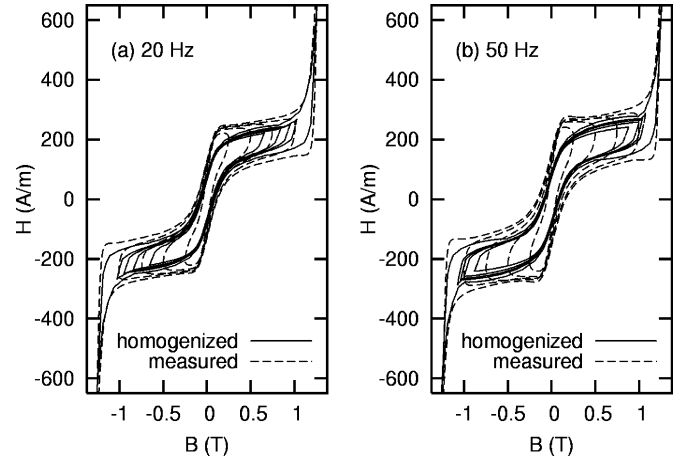


Fig. 6. AC B - H loops along the transverse direction given by the homogenized model with $k_E = 1$.

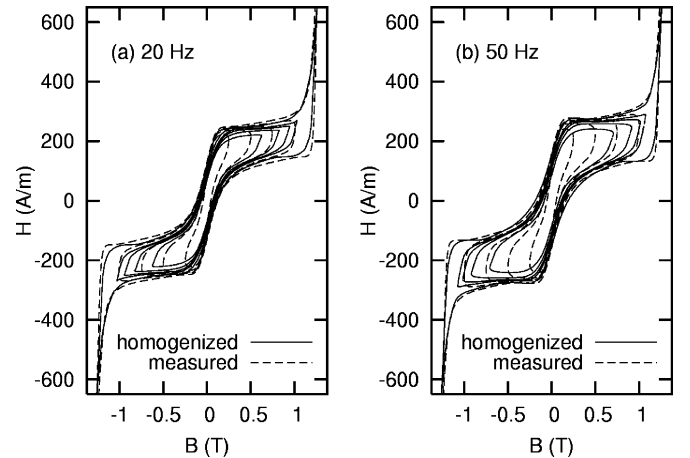


Fig. 7. AC B - H loops along the transverse direction given by the homogenized model with $k_E = 10$.

V. FINITE-DIFFERENCE ANALYSIS INCLUDING EXCESS LOSS

This paper simply multiplies electric conductivity by the factor k_E for the 1-D finite-difference analysis to approximate the effect of excess eddy-current loss in the same way as in the homogenized model

$$\frac{\partial^2 H}{\partial y^2} = \sigma' \frac{\partial B}{\partial t}, \quad \sigma' = k_E \sigma. \quad (14)$$

Fig. 8 shows simulated B - H loops along the rolling direction obtained by analysis using $k_E = 2.5$, where the increased conductivity improves the representation of the B - H loops compared with Fig. 1. Table I lists the discrepancy (%) between the simulated and measured power loss $W = \oint H dB$ of five B - H loops at 50 Hz, where $B_{\max}$ denotes the maximum magnetic field of each measured B - H loop. Table I shows that the factor $k_E = 2.5$ greatly improves the power loss evaluation. However, more than 10% of the discrepancy remains between the computed and measured losses because the excess loss evaluation by the constant anomaly factor is too simple to represent an accurate eddy-current field. Table I and the comparison between Figs. 5 and 8 show that the finite-difference model does

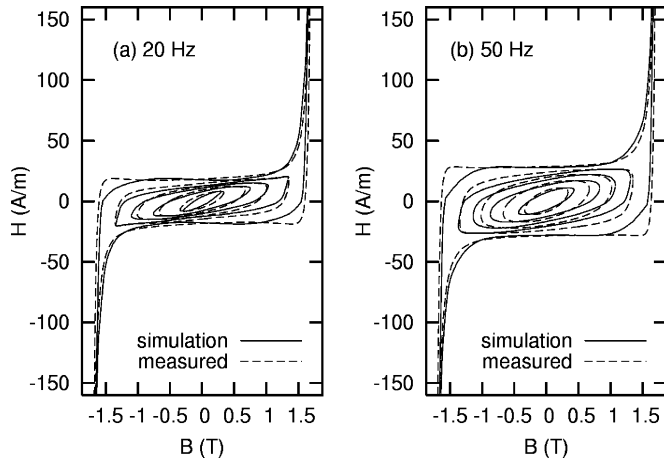


Fig. 8. AC B - H loops along the rolling direction given by eddy-current analysis with $k_E = 2.5$.

TABLE I
DISCREPANCY (%) BETWEEN THE SIMULATED AND MEASURED POWER LOSS

$B_{\max}$ (T)	finite difference		homogenization	
	$k_E = 1$	$k_E = 2.5$	$k_E = 1$	$k_E = 2.5$
0.34	+22.1	+0.6	+18.4	-3.8
0.67	+58.6	+17.7	+75.5	+17.8
1.01	+10.4	-2.0	+17.0	+11.3
1.34	-24.2	-14.5	-20.1	-2.2
1.68	-19.1	-8.6	-14.3	-8.9

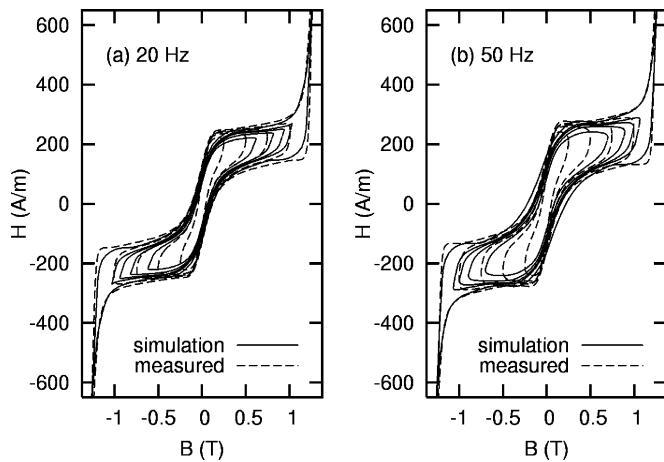


Fig. 9. AC B - H loops along the transverse direction given by eddy-current analysis with $k_E = 10$.

not greatly improve the representation accuracy of the eddy-current field compared with the homogenization model. There is little improvement because the excitation frequency is too low for the skin effect to become sufficiently large.

Fig. 9 shows B - H loops along the transverse direction simulated with $k_E = 10$, where the representation of the B - H loops

is improved compared with Fig. 2. However, a large discrepancy remains between the simulated and measured small B - H loops. This fact means that the excess loss approximation by the constant anomaly factor is not valid for the transverse direction because of the 90° domain wall motion that the Pry and Bean model cannot describe.

VI. CONCLUSION

This paper combines a 1-D finite-difference method or a homogenization method with a stop model to describe ac hysteretic properties of a grain-oriented silicon steel sheet. A homogenized model is derived from the Pry and Bean model, where the excess eddy-current loss is evaluated by an anomaly factor. An increased electric conductivity by the anomaly factor improves the representation of ac B - H loops by the 1-D finite-difference method as effectively as by the homogenization method.

ACKNOWLEDGMENT

This work was supported in part by the Kansai Research Foundation for Technology Promotion.

REFERENCES

- [1] G. Bertotti, *Hysteresis in Magnetism*. San Diego, CA: Academic, 1998.
- [2] C. Kaido and T. Wakisaka, "Effect of material parameters on iron losses in nonoriented electrical steel sheets," *Trans. Inst. Elect. Eng. Jpn. A*, vol. 117, pp. 685–690, Jul. 1997.
- [3] M. Enokizono, T. Todaka, S. Kanao, and J. Sievert, "Two-dimensional magnetic properties of silicon steel sheet subjected to a rotating field," *IEEE Trans. Magn.*, vol. 29, no. 6, pp. 3550–3552, Nov. 1993.
- [4] M. A. Krasnosel'skii and A. V. Pokrovskii, *Systems With Hysteresis*. Berlin, Germany: Springer-Verlag, 1989.
- [5] S. Bobbio, G. Miano, C. Serpico, and C. Visone, "Models of magnetic hysteresis based on play and stop hysterons," *IEEE Trans. Magn.*, vol. 33, no. 6, pp. 4417–4426, Nov. 1997.
- [6] T. Matsuo, Y. Terada, and M. Shimasaki, "Stop model with input-dependent shape function and its identification methods," *IEEE Trans. Magn.*, vol. 40, no. 4, pp. 1776–1783, Jul. 2004.
- [7] T. Matsuo and M. Shimasaki, "Representation theorems for stop and play models with input-dependent shape functions," *IEEE Trans. Magn.*, to be published.
- [8] R. H. Pry and C. P. Bean, "Calculation of the energy loss in magnetic sheet materials using a domain model," *J. Appl. Phys.*, vol. 29, pp. 532–533, Mar. 1958.
- [9] T. Nakase, M. Nakano, K. Fujiwara, and N. Takahashi, "Method of digital waveform control for measuring magnetic properties by means of a single sheet tester," *Trans. Inst. Elect. Eng. Jpn. A*, vol. 119, pp. 1019–1025, Jul. 1999.
- [10] A. J. Bergqvist and S. G. Engdahl, "A homogenization procedure of field quantities in laminated electric steel," *IEEE Trans. Magn.*, vol. 37, no. 5, pp. 3329–3331, Sep. 2001.
- [11] S. E. Zirka, Y. I. Moroz, P. Marketos, and A. J. Moses, "Dynamic hysteresis modeling," *Physica B*, vol. 343, pp. 90–95, 2004.