

# Generalization of geometry-induced effect noted by Takagi and Tanzawa

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and

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The formulation on a particle motion in  $n$ -dimensional curved manifold  $M_n$  embedded in  $p$ -dimensional Euclidean space  $R_p$  is summarized, and the geometry-induced gauge structure is explained. Next we examine the scalar field theory with a soliton solution, and point out that in spite of the infinite degrees of freedom such a field theory has the same mathematical structure as a particle motion in  $M_n \subset R_p$ , and our formalism affords a clearer view of understanding physical contents of such a field theory.

*1. Introduction* Quantum theory on a curved manifold has been investigated from various points of view [1,2,3]. Quantum treatment of soliton such as Skyrminion provides a typical example of quantum theory on a curved manifold [4]. We consider the motion of a particle on an  $n$ -dimensional curved manifold  $M_n$  embedded in a  $p$ -dimensional Euclidean space  $R_p$ , and the particle motion is thought to be confined by some confining potential. Then a correction term with the order  $\hbar^2$  in the effective Hamiltonian on  $M_n$  appears as a quantum effect due to a particle motion in the directions perpendicular to  $M_n$  [2]. Such an effect is dropped from the beginning when we apply simply Dirac method for constrained dynamical systems.

Two years ago, Takagi and Tanzawa [5] have pointed out that , for a particle motion

in a thin tube (in  $\mathbf{R}_3$ ) forming a closed loop, in the effective Hamiltonian on  $M_1$  there appears an effective vector potential, which depends on the geometry of  $M_1$ , and that there exists a complete analogy with Aharonov-Bohm effect, called a geometry-induced AB effect. One of the present authors (K.F.) and N.Ogawa [6] generalized this result to the case of a particle motion in a thin neighborhood of  $M_n$  embedded in  $\mathbf{R}_p$ . The aim of the present report is to apply this formalism to a scalar field theory which allows a classical solution, and to examine the correspondence to the field theory of the extended object given by Sakita and others [7].

In the following, we first summarize the formalism in case of a particle motion on  $M_n \subset \mathbf{R}_p$  (in Sec.2), and extend it to the case of a scalar field theory with a classical solution (in Sec.3). The last section is devoted to discussions and summary of remaining tasks.

## 2. Particle motion in a thin layer along $M_n$ embedded in $\mathbf{R}_n$

2-1. Basic relations As in [6], a set of coordinates  $\{X^A; A = 1, \dots, p\}$  of a point in a thin-layer neighbourhood of  $M_n$  is expressed as

$$X^A(q^\beta) = x^A(q^b) + \sum_{U=n+1}^p q^U N_U^A(q^b), \quad s = 1, \dots, p; \quad (2.1)$$

$\{q^\beta, \beta = 1, \dots, n, n+1, \dots, p\}$  consists of two parts; the first part  $\{q^b, b = 1, \dots, n\}$  is a set of curvilinear coordinates on  $M_n$  and the remaining part is  $\{q^U, U = n+1, \dots, p\}$ .

$N_U^A(q^b)$  is a unit normal vector to  $M_n$  at a point  $x^A(q^b)$ . For simplicity we omit  $\sum_{U=n+1}^p$  for a dummy index  $U$  and write e.g.  $x^A(q^b)$  as  $x^A(q)$ . The metric tensor in  $\mathbf{R}_n$ , written as  $\tilde{G}_{\alpha\beta}(q, q^U)$  is given by

$$\tilde{G}_{\alpha\beta} = \tilde{B}_\alpha^A \eta_{AD} \tilde{B}_\beta^A \quad \text{with} \quad \tilde{B}_\beta^A(q, q^U) = \frac{\partial X^A(q, q^U)}{\partial q^\beta}, \quad (2.2a)$$

and the metric tensor on  $M_n$  as

$$g_{ab}(q) = B_a^A \eta_{AD} B_b^D \quad \text{with} \quad B_b^A(q) = \frac{\partial x^A(q)}{\partial q^b}. \quad (2.2b)$$

Since  $B_b^A$  is tangent to  $M_n$ , we have  $B_b^A \eta_{AD} N_V^D = 0$ . Note that we have  $\tilde{B}_\beta^A = (\tilde{B}_b^A, \tilde{B}_V^A) = (B_b^A + \partial_b N_W^A \cdot q^W, N_V^A)$ .

The fundamental equations for  $B_b^A$  and  $N_V^A$  are

$$\partial_a B_b^A = \Gamma_{ab}^d B_d^A + H_{Wab} \eta^{WU} N_U^A, \quad (2.3a)$$

$$\partial_a N_V^A = -H_{V_a}^d B_d^A - T_{VW,a} \eta^{WU} N_U^A, \quad (2.3b)$$

where  $\Gamma_{ab}^d$  is Christoffel symbol constructed in terms of  $g_{bd}$  and  $H_{Wab} = H_{Wba} = H_{Wb}^e g_{ea}$ ,  $T_{VW,a} = -T_{WV,a}$ .

Concrete forms of  $\tilde{G}_{\alpha\beta}$  and its inverse  $\tilde{G}^{\alpha\beta}$  are given as follows:

$$[\tilde{G}_{\alpha\beta}] = \begin{bmatrix} \tilde{G}_{ab} & \tilde{G}_{aU} \\ \tilde{G}_{Vb} & \tilde{G}_{VU} \end{bmatrix} \quad (2.4a)$$

with  $\tilde{G}_{ab} = \lambda_{ab} + \tilde{G}_{aX} \eta^{XY} \tilde{G}_{bY}$ ,  $\lambda_{ab} = g_{ab} - 2H_{Wab} q^W + q^X q^Y H_{Xa}^e H_{Yeb}$ ,  
 $\tilde{G}_{aU} = \tilde{G}_{Ua} = T_{UX,a} q^X$ ,  $\tilde{G}_{VU} = \eta_{VU}$ ;

$$[\tilde{G}^{\alpha\beta}] = \begin{bmatrix} \tilde{G}^{ab} & \tilde{G}^{aU} \\ \tilde{G}^{Vb} & \tilde{G}^{VU} \end{bmatrix} \quad (2.4b)$$

with  $\tilde{G}^{ab} = \lambda^{ab}$ ,  $\lambda^{ab} \lambda_{bd} = \delta^a_d$ ,  $\tilde{G}^{aU} = \tilde{G}^{Ua} = -\lambda^{ad} T_{XW,d} q^W \eta^{XU}$ ,  
 $\tilde{G}^{VU} = \eta^{VU} + \eta^{VX} T_{XW,b} q^W \lambda^{bd} T_{YZ,d} q^Z \eta^{YU}$ .

From the condition  $\partial_a B_b^A = \partial_b B_a^A$ , one obtains the curvature tensor on  $M_n$  given as

$$R_{ab,cd} = \eta_{AB} (H^A_{ac} H^B_{bd} - H^A_{ad} H^B_{bc}) \quad (2.5a)$$

with  $H^A_{ab} = H_{Vab} \eta^{VW} N_W^A$  (Euler-Schouten tensor);

hence, the tensor  $R_{bc} = g^{ad} R_{ab,cd} = \eta_{AB} (H^A_b{}^d H^B_{dc} - H^A_a{}^d H^B_{bc})$ , and scalar curvature

$$R = g^{bc} R_{bc} = \eta_{AB} (H^A_b{}^d H^B_d{}^b - H^A_a{}^a H^B_b{}^b). \quad (2.5b)$$

From  $\partial_a \partial_b N_V^A - \partial_b \partial_a N_V^A = 0$ , one obtains

$$R_{da,VW} \eta^{VW} = \eta^{VW} (-H_{Vd}^b H_{Wba} + H_{Va}^b H_{Wbd}), \quad (2.5c)$$

$$\text{where } R_{da,VW} \equiv -\partial_d T_{VW,a} + \partial_a T_{VW,d} + T_{XV,d} \eta^{XY} T_{YW,a} - T_{XV,a} \eta^{XY} T_{YW,d}. \quad (2.5d)$$

Using the extrinsic mean curvature  $H$  defined by  $H \equiv [\eta_{ab} H^A_b{}^b H^B_d{}^d]^{1/2} / n$ , one obtains

$$R = H^A_b{}^d \eta_{AB} H^B_d{}^b - n^2 H^2. \quad (2.6)$$

## 2-2. Canonical quantization and form of kinetic energy

We examine the form of

kinetic energy

$$\tilde{K} = \frac{1}{2} \dot{X}^A \eta_{AB} \dot{X}^B, \quad \dot{X}^A = \frac{dX^A}{d\tau}, \quad (2.7)$$

expressed in terms of  $q^\beta$ -variables. In order to perform the quantum-mechanical calculations from the outset, we adopt the procedure which is consistent for the transformation from Euclidean coordinates to curvilinear ones as in the present case. We assume

$$[q^\beta, \dot{q}^\delta] = i\hbar f^{\beta\delta}(q^\gamma), \quad [q^\beta, q^\delta] = 0, \quad (2.8a)$$

where  $f^{\beta\delta}$  is a function of only  $(q^\gamma)$ . When we require the canonical commutation relations

$$[q^\alpha, p_\beta] = i\hbar \delta^\alpha_\beta, \quad [p_\alpha, p_\beta] = 0 \quad (2.8b)$$

$$\text{for } p_\beta \equiv \frac{\partial \tilde{K}}{\partial \dot{q}^\beta} = \frac{1}{2} \{ \tilde{G}_{\beta\delta}, \dot{q}^\delta \} \equiv \langle \tilde{G}_{\beta\delta}, \dot{q}^\delta \rangle, \quad (2.8c)$$

we obtain  $f^{\alpha\beta} \tilde{G}_{\beta\delta} = \delta^\alpha_\delta$ , i.e.  $f^{\alpha\beta}$  is the inverse of  $\tilde{G}_{\beta\delta}$ .

Now we rewrite  $\tilde{K}$  (2.7) in the covariant form

$$\tilde{K} = \frac{1}{2} \tilde{G}^{-1/4} p_\alpha \tilde{G}^{-1/2} \tilde{G}^{\alpha\beta} p_\beta \tilde{G}^{-1/4}, \quad \tilde{G} = |\det \tilde{G}_{\alpha\beta}|. \quad (2.9a)$$

We obtain by noting  $\tilde{G} = |\det \lambda_{ab} \cdot \det \eta_{VW}| = |\det \lambda_{ab}| \equiv \lambda$ ,

$$\tilde{K} = \frac{1}{2} \lambda^{-1/4} \Pi_a \lambda^{1/2} \lambda^{ab} \Pi_b \lambda^{-1/4} + \frac{1}{2} \lambda^{-1/4} p_V \lambda^{1/2} \eta^{VW} p_W \lambda^{-1/4}. \quad (2.9b)$$

Here,  $\Pi_a$  is defined by

$$\Pi_a \equiv p_a + \frac{1}{2} T_{VW,a} L^{VW}, \quad (2.9c)$$

$$L^{WX} \equiv q^W \eta^{XV} p_V - q^X \eta^{WV} p_V = \eta^{XV} p_V q^W - \eta^{WV} p_V q^X. \quad (2.9d)$$

$L^{WX}$  satisfies the commutation relation

$$[L^{VX}, L^{WY}] = i\hbar (\eta^{VW} L^{XY} + \eta^{XY} L^{VW} - \eta^{VY} L^{XW} - \eta^{XW} L^{VY}). \quad (2.9e)$$

In the thin layer approximation  $|H_{Uab} q^U| \ll 1$  and  $|T_{UV,b} q^U| \ll 1$  [5,6], one obtains

$$\tilde{K} \xrightarrow{\text{thin layer}} K^* = K + \frac{1}{2} p_V \eta^{VW} p_W + \Delta V^*, \quad (2.10a)$$

$$\text{where } K = \frac{1}{2}g^{-1/4}\Pi_a g^{1/2}g^{ab}\Pi_b g^{-1/4}, \quad \Delta V^* = \frac{\hbar^2}{2}\left[-\frac{R}{2} - \frac{1}{4}n^2 H^2\right]. \quad (2.10b)$$

$\Delta V^*$  comes from the last term in (2.9b). It may be worthy of noting that we have

$$\Delta V^* = -\frac{1}{2}[\tilde{Y}(q, q^V) - Y(q)]|_{thin \text{ layer}}; \quad (2.11a)$$

$$\text{where } -\frac{1}{2}\tilde{Y}(q, q^V) = \hbar^2\left[\frac{1}{4}\partial_\delta(\tilde{G}^{\delta\beta}\tilde{\Gamma}_\beta + \frac{1}{8}\tilde{G}^{\delta\beta}\tilde{\Gamma}_\delta\tilde{\Gamma}_\beta)\right], \tilde{\Gamma}_\alpha \equiv \tilde{\Gamma}_{\alpha\beta}^\beta = \tilde{G}^{\gamma\beta}\partial_\alpha\tilde{G}_{\gamma\beta}/2.$$

$Y(q)$  is the quantity constructed in terms of  $g_{ab}$  corresponding to  $\tilde{Y}$ ;

$$-\frac{1}{2}Y/\hbar^2 = \frac{1}{4}\partial_b(g^{bd}\Gamma_d) + \frac{1}{8}g^{bd}\Gamma_b\Gamma_d = \frac{1}{8}[R + g^{ab}\Gamma_a^c{}_d\Gamma_{be}^d]. \quad (2.11b)$$

Using (2.11b), we can rewrite (2.10b) as

$$K(2.10b) = \frac{1}{2}\Pi_a g^{ab}\Pi_b - \frac{1}{2}Y(q). \quad (2.11c)$$

### 2-3. Commutator $[\Pi_a, \Pi_b]$

Utilizing some relations given in 2-1, we obtain

$$[\Pi_b, \Pi_d] = \frac{i\hbar}{2}R_{bd, VW}L^{VW}. \quad (2.12a)$$

This is analogous to a charged particle moving in magnetic field  $\vec{H}$ , in which we have

$$[\Pi_j, \Pi_k] = i\hbar\frac{e}{c}F_{jk}; \quad \Pi_j = p_j - \frac{e}{c}A_j; \quad j, k = 1, 2, 3; \quad F_{jk} = \frac{1}{2}\epsilon_{jkl}H^l. \quad (2.12b)$$

We see that the field  $T_{VW,a} = N_V^A\partial_a N_W^B\eta_{AB}$  plays a role of gauge potential. The gauge property including the non-Abelian one is seen as follows: When the total Hamiltonian  $\tilde{H} = \tilde{K}(2.9b) + V$  has the part of potential which confines the particle motion to  $\mathbf{M}_n$ , and is invariant under rotation of the set of  $\{N_U^A, U = n+1, \dots, p\}$  such as

$$N_V'^A(q) = N_W^A(q)\Lambda^W{}_V(q), \quad \eta_{XY}\Lambda^X{}_W\Lambda^Y{}_V = \eta_{WV}, \quad (2.13a)$$

we obtain

$$T_{WV,b} \longrightarrow T'_{WV,b} = (\Lambda^{-1})_W^X T_{XY,b} \Lambda^Y{}_V + (\Lambda^{-1})_{WX} \partial_b \Lambda^X{}_V. \quad (2.13b)$$

$T_{WV,b}$  cannot be eliminated globally. In case of a tube embedded in  $\mathbf{R}_3$ [5],  $\Pi_b$  reduces to

$$\Pi_1 = p_1 + T_{23}L^{23} \equiv p - \omega L. \quad (2.14a)$$

It is pointed out in Ref.[5] due to multivaluedness of triangular function

$$\int_0^l \omega(q) dq = \int_0^l \tau(q) dq \pmod{2\pi} \quad (2.14b)$$

is obtained, where  $l$  is the length of center line of the tube;  $\tau$  is the torsion appearing in Frenet-Serret equation in  $\mathbf{R}_3$ .

### 3. Application to field theory of extended object

**3-1. Purpose** In this section we extend the formalism given in Section 2 to field theory. For simplicity, we examine the scalar field theory which allows a soliton solution. We consider the Lagrangian expressed as

$$L = -\frac{1}{2}\partial_\mu\phi^A(x)\eta_{AB}\partial^\mu\phi^B(x) - V(\phi(x)), \quad (3.1)$$

where  $(x^\mu)$  is a space-time coordinate; its metric is  $\eta_{\mu\nu}$  with  $\text{diag}(\eta_{\mu\nu}) = (- + + \dots)$ ; the upper index A of  $\phi^A$  denotes the internal degrees of freedom.

The field operator  $\phi^A(\vec{x}, x^0)$  is assumed to be expanded as [7]

$$\phi^B(\vec{x}, x^0) = \phi_0^B(\vec{x}, q^b) + \sum_U \phi_U^B(\vec{x}, q^b) q^U, \quad (3.2a)$$

where  $\{q^b, b = 1, \dots, n\}$  denotes a set of collective coordinates representing the center of mass coordinates of the classical soliton, the orientation on the internal space and so on;  $\phi_0^B$  is the soliton solution satisfying

$$-\frac{\partial}{\partial \vec{x}} \frac{\partial}{\partial \vec{x}} \phi_0^B(\vec{x}, q^b) + \frac{\partial V}{\partial \phi_0^A(\vec{x}, q^b)} \eta^{AB} = 0. \quad (3.2b)$$

$\partial \phi_0^B(\vec{x}, q)/\partial q^b$  satisfies

$$[-\vec{\nabla}^2 \delta_D^B + \frac{\partial^2 V}{\partial \phi_0^D(\vec{x}, q) \partial \phi_0^A(\vec{x}, q)} \eta^{AB}] \partial_b \phi_0^D(\vec{x}, q) = 0; \quad (3.2c)$$

$\phi_U^D(\vec{x}, q)$ 's are non-zero-mode solutions corresponding to (3.2b), and are normalized to

$$\int d\vec{x} \phi_V^B(\vec{x}, q) \eta_{BD} \phi_W^D(\vec{x}, q) = \eta_{VW}, \quad \int d\vec{x} \partial_b \phi_0^B(\vec{x}, q) \eta_{BD} \phi_W^D(\vec{x}, q) = 0. \quad (3.2d)$$

(Hereafter we write a function of  $q^b$ 's,  $f(q^b)$ , as  $f(q)$  for simplicity.)

The expression (3.2a) with the properties (3.2d) is completely analogous with (2.1). With the aim of making clear the analogy, we write (3.2a) and (3.2d) as

$$\phi^{Bx}(x^0) = \phi_0^{Bx}(q) + N_U^{Bx}(q) \cdot q^U, \quad (3.3a)$$

$$N_V^{Bx}(q) \eta_{Bx, Dy} N_U^{Dy}(q) = \eta_{VW}, \quad B_b^{Bx}(q) \eta_{Bx, Dy} N_U^{Dy}(q) = 0, \quad (3.3b)$$

where  $\eta_{Bx, Dy} = \eta_{BD} \delta(\vec{x} - \vec{y})$ ;  $B_b^{Bx}(q) = \partial_b \phi_0^{Bx}(q)$ . The following subsections is devoted to investigate the role of this analogy in constructing the field theory of extended object.

## 3-2. Fundamental relations and the metric

Various relations given in the previous section remain to hold if the index A (representing the vector property in  $R_p$ ) is changed to Ax; from (2.3a) and (2.3b) one obtains

$$\partial_a B_b^{Ax}(q) = \Gamma_{ab}^d B_d^{Ax} + H_{Wab} \eta^{WU} N_U^{Ax}, \quad (3.4a)$$

$$\partial_a N_V^{Ax}(q) = -B_b^{Ax} H_{Va}^b - T_{VW,a} \eta^{WU} N_U^{Ax}. \quad (3.4b)$$

As to  $\tilde{B}_\beta^{Ax} \equiv \partial \phi^{Ax} / \partial q^\beta$ , we have

$$\tilde{B}_b^{Ax}(q) = B_b^{Ax}(q) + \partial_b N_V^{Ax} \cdot q^V, \quad \tilde{B}_V^{Ax}(q) = N_V^{Ax}(q); \quad (3.5a)$$

and the small interval

$$ds^2 = d\phi^{Bx} \eta_{Bx,Dy} d\phi^{Dy} = \tilde{G}_{\alpha\beta}(q, q^V) dq^\alpha dq^\beta \quad \text{with} \quad \tilde{G}_{\alpha\beta} = \tilde{B}_\alpha^A \eta_{AB} \tilde{B}_\beta^B. \quad (3.5b)$$

## 3-3. Canonical quantization and Hamiltonian form

From Lagrangian (3.1), one obtains the momentum operator  $p_{Ax}$ , conjugate to  $\phi^{Ax}$ , defined by

$$p_{Ax} \equiv \frac{\partial L}{\partial \partial \phi^{Ax} / \partial x^0} = \frac{\partial \phi^{Dy}}{\partial x^0} \eta_{Dy,Ax} \equiv \dot{\phi}^{Dy} \eta_{Dy,Ax}. \quad (3.6a)$$

Utilizing

$$\dot{\phi}^{Bx} = (\tilde{B}_\beta^{Bx} \cdot \dot{q}^\beta + \dot{q}^\beta \cdot \tilde{B}_\beta^{Bx}) / 2 \equiv \langle \tilde{B}_\beta^{Bx}, \dot{q}^\beta \rangle, \quad (3.6b)$$

and following the quantization procedure described in 2-2, one obtains

$$p_\beta \equiv \frac{\partial L}{\partial \dot{q}^\beta} = \langle \tilde{G}_{\beta\alpha}, \dot{q}^\alpha \rangle, \quad (3.6c)$$

$$p_{Ax} = \langle \eta_{Ax,By} \tilde{B}_\beta^{By}, \dot{q}^\beta \rangle = \langle \eta_{Ax,By} \tilde{B}_\beta^{By} \tilde{G}^{\beta\delta}, p_\delta \rangle. \quad (3.6d)$$

From the commutation relations (2.8b), we obtain the equal-time ones as

$$[\phi^{Bx}, \phi^{Dy}] = 0, \quad [\phi^{Bx}, p_{Dy}] = i\hbar \delta^{Bx}_{Dy}, \quad [p_{Bx}, p_{Dy}] = 0. \quad (3.6e)$$

Hamiltonian  $H[\phi, p]$  is defined by

$$H[\phi, p] \equiv \frac{1}{2} \{p_{Ax}, \dot{\phi}^{Ax}\} - \int d\vec{x} L, \quad (3.7a)$$

which is expressed as

$$H[\phi, p] = \frac{1}{2} p_{Ax} \eta^{Ax,By} p_{By} + \frac{1}{2} (\vec{\nabla} \phi)^{Ax} \eta_{Ax,By} (\vec{\nabla} \phi)^{By} + \int V(\phi) d\vec{x}. \quad (3.7b)$$

This is expressed in terms of  $(q^\alpha, p_\beta)$  variables as

$$H[\phi, p] = \tilde{K}(2.9b) + \int V(\phi) d\vec{x}, \quad (3.7c)$$

$$\tilde{K}(2.9b) = \frac{1}{2}\Pi_a \lambda^{ab} \Pi_b + 1/2 p_V \eta^{VW} p_W - \frac{1}{2} \tilde{Y}(q, q^U). \quad (3.7d)$$

Here, the last term  $-\frac{1}{2}\tilde{Y}$  is given by (2.11a) and can be rewritten as

$$-\frac{1}{2}\tilde{Y} = \frac{\hbar^2}{8} [\tilde{R} + \tilde{\Gamma}_{\alpha\gamma}^{\delta} \tilde{G}^{\alpha\beta} \tilde{\Gamma}_{\beta\delta}^{\gamma}]; \quad (3.7e)$$

in the present case,  $\tilde{R} = 0$ . We will examine the correspondence of the above expressions to those derived by Gervais and others [7] in the next subsection.

3-4. *Momentum operators* From (3.6c) and (3.6d), one obtains

$$p_{\beta} = \langle \tilde{B}_{\beta}^{Ax}, p_{Ax} \rangle, \quad i.e. \quad p_U = \langle N_U^{Ax}, p_{Ax} \rangle, \quad p_b = \langle \tilde{B}_b^{Bx}, p_{Bx} \rangle. \quad (3.8)$$

We define

$$\Pi_{Bx} \equiv \eta_{Bx, Dy} N_V^{Dy} \eta^{VW} p_W. \quad (3.9a)$$

By employing the second relation of (3.8), one obtains

$$\Pi_{Bx} = \langle \eta_{Bx, Dy} N_V^{Dy} \eta^{VW} N_W^{Ez}, p_{Ez} \rangle. \quad (3.9b)$$

When we define  $p_{0, Bx} \equiv p_{Bx} - \Pi_{Bx}$ , we obtain

$$p_{0, Bx} = \langle \delta_{Bx}^{Ez} - \eta_{Bx, Dy} N_V^{Dy} \eta^{VW} N_W^{Ez}, p_{Ez} \rangle = \langle \eta_{Bx, Dy} B_b^{Dy} g^{bd} B_d^{Ez}, p_{Ez} \rangle; \quad (3.9c)$$

$$N_V^{Bx} \cdot \Pi_{Bx} = p_V, \quad B_b^{Bx} \cdot \Pi_{Bx} = 0 \quad (3.9d)$$

$$\langle N_V^{Bx}, p_{0, Bx} \rangle = 0, \quad \langle B_b^{Bx}, p_{0, Bx} \rangle = \langle B_b^{Bx}, p_{Bx} \rangle. \quad (3.9e)$$

Therefore, we write  $\phi^{Bx}$  and  $p_{Bx}$  as

$$\phi^{Bx} = \phi_0^{Bx} + \chi^{Bx}, \quad p_{Bx} = p_{0, Bx} + \Pi_{Bx}, \quad (3.10a)$$

with  $\chi^{Bx} = N_V^{Bx} q^V$ ,  $\Pi_{Bx}$  = (3.9a), from which we have

$$\langle \Pi_{Bx}, \partial_b \phi^{Bx} \rangle = -\frac{1}{2} T_{WV, b} L^{WV}. \quad (3.10b)$$

Thus we see  $\Pi_a$  in (3.7d) is expressed as

$$\Pi_a = p_a - \langle \Pi_{Bx}, \partial_a \phi^{Bx} \rangle, \quad (3.10c)$$

which is the same as given by Gervais and others [7].

Next we examine the form of

$$\langle p_{Bx}, \dot{\phi}^{Bx} \rangle = \int d\vec{x} L - H[\phi, p] \quad (3.11a)$$

in the c-number theory. We obtain

$$p_{Bx} \dot{\phi}^{Bx} = p_\beta \dot{q}^\beta = p_{0,Bx} \dot{\phi}^{Bx} + \Pi_{Bx} \dot{\chi}^{Bx}, \quad (3.11b)$$

$$p_{0,Bx} \dot{\phi}^{Bx} = p_d \dot{q}^d - p_V T_{XW,dq}^W \eta^{XV} \dot{q}^d = \Pi_d \dot{q}^d. \quad (3.11c)$$

4. *Discussions and conclusion* In sec.3, we examine the scalar field theory which allows a soliton solution with the aim of establishing the correspondence between such a field theory and the formalism of one-particle motion on a curved manifold  $M_n$  embedded in  $R_p$ . Our formulation seems helpful to see general mathematical structure of the field theory with soliton and fluctuation around it. A noteworthy point in the present formalism is that there appears a geometry-induced gauge structure which corresponds to that giving rise to the geometry-induced Aharonov-Bohm effect for a particle moving in a thin-tube in  $R_3$ .

One of the future tasks is to investigate about what physical effects such a geometry-induced structure brings. It may be interesting to examine the possibility of inducing the gauge field as an independent dynamical degree of freedom after compactifying  $q^U$ -space. From such a point of view, the recent works presented by Kikkawa and others [8] seems interesting, on which we shall discuss elsewhere.

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