

Stochastic Optimal Control Models for Management
of *Plecoglossus altivelis* under Predation Pressure
from *Phalacrocorax carbo*

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CHAPTER 1

Introduction

1.1 Research background

Management of animal populations is an important ecological problem. Such examples are fishery resources [35], stock-farming [78], endangered species [54] and so on. Management of *Plecoglossus altivelis* (*P. altivelis*, Ayu) [50] is very important in Japan. *P. altivelis* is one of the most economically and culturally important inland fishery resources in Japan [1, 85]. Its fish catch in 2018 accounts for 7.9% in Japanese inland fisheries [62] and has served as their main source of income. Total fish catch of *P. altivelis* in Japan has gradually been decreasing in recent years, which is considered due to the decrease of the number of fishermen and anglers, demand of the fish, and its population. There are several causes of the population decline. They include natural factors such as climate changes and artificial factors including manipulation of river environment through the installation of physical barriers like dams and weirs that destroy habitats and fragment migration routes of *P. altivelis*. Another most possible cause is increasing predation pressure from *Phalacrocorax carbo* (*P. carbo*, Great Cormorant) [99, 100, 28]: a fish-eating waterfowl reigning as a top predator in Japanese river ecosystems. Yamamoto (2008, 2009) [99, 100] reviewed biology and ecology of *P. carbo* and pointed out that this waterfowl has a sufficiently high exercise capacity to catch most of the fish species living in rivers; *P. altivelis* is not an exception. The population of *P. carbo* has been increasing in Japan since the 1970' s due to the inappropriate conservation policy of the

government of the country [29, 41]. Before 1970, the number of *P. carbo* decreased due to mainly hunting. After 1970, the number of *P. carbo* began to increase due to the conservation of the bird and water quality improvement. For decreasing the increased number of the bird, expulsions of the bird were performed, which led to the dispersion of *P. carbo* to the whole country and the number of *P. carbo* burgeoned. One of the other causes of the increase of the bird is the releases of fishes into rivers by local fishery cooperatives. This release events provided a large number of fishes to *P. carbo*. In order to reduce the predation pressure, local fishery cooperatives and municipalities have been taking various countermeasures. However, plans for taking countermeasures are determined according to their experience and are not based on scientific methods. In some cases, conversely, taking countermeasures led to the dispersion and the increase of *P. carbo* [99, 100]. Thus, scientific-based management plans for taking countermeasures that can effectively reduce the predation pressure from *P. carbo* and sustainably maintain the population of *P. carbo* is required.

Mathematical modeling of animal population dynamics would help to resolve the severe situation currently faced by inland fisheries in Japan. By using mathematical models, a variety of hypothetical and real conditions can be examined. There are many kinds of mathematical models that can treat management problems of animal population: discrete-time model [21], multi-agent model [10], continuous-time model [57], matrix model [46], and so on. Mathematical models based on difference and differential equations are indispensable tools not only for comprehending and assessing the population dynamics of animals but also for establishing their management policies [16]. Kar and Matsuda (2008) [38] presented a bio-economic model based on ordinary differential equations, which describes single-species population dynamics in a marine fishery resource. They mathematically analyzed its equilibrium stability considering the impacts of allocating reserve areas. Differential equation models have also provided optimal solutions to wildlife management problems [58]. Especially, stochastic control theory is one of the most useful tools when the dynamics involve uncertainty. Dynamics of animal populations can be interpreted as stochastic, thus the

stochastic control theory can be considered an appropriate tool for analyzing the dynamics. Various kinds of controls can be considered by using stochastic control theory: regular control, singular control, impulse control and so on [66, 70, 86]. They are different in constraints of the controls and should be applied properly according to the setting of problems. The optimal management plans can be obtained by solving degenerate elliptic or parabolic partial differential equations (PDEs) called as Hamilton-Jacobi-Bellman equations (HJBs) [66, 70, 86].

1.2 Research issues

Current issues on optimal management plans of *P. altivelis* under the predation pressure from *P. carbo* are presented in this section, which are listed up as follows.

- Management plans for taking countermeasures are determined according to the experience of decision-makers and are often not based on scientific methods.
- Management plans for taking countermeasures that can effectively reduce the predation pressure from *P. carbo* to *P. altivelis* have not been investigated theoretically.
- Management plans for taking countermeasures that consider the sustainability of the population of *P. carbo* are not sufficient.

1.3 Research objectives

This thesis has three objectives. The first objective is to formulate stochastic control models for optimal management plans of *P. altivelis* under the predation pressure from *P. carbo*. The second objective is to propose numerical schemes that can numerically solve resulting HJBs. The third objective of this thesis is to numerically compute the optimal management strategy of released *P. altivelis* for various situations (parameters and boundary and terminal conditions) and compare the current management plans with the results by the established mathematical models.

In order to achieve the objectives, this thesis

1. proposes stochastic control models that derive optimal management plans of *P. altivelis* under the predation pressure from *P. carbo*,
2. presents numerical schemes that can solve governing equations of the stochastic control models (Hamilton-Jacobi-Bellman equation, Hamilton-Jacobi-Bellman variational inequality, and Hamilton-Jacobi-Bellman quasi-variational inequality),
3. presents management plans for taking countermeasures that can effectively reduce the predation pressure from *P. carbo* and sustainably maintain the population of *P. carbo*, and
4. reconsider the current management plans for taking countermeasures by using the established mathematical models.

1.4 Structure of this thesis

This thesis consists of 7 chapters. Chapter 1 presents the background and objectives of this thesis. Chapter 2 provides literature reviews on feeding damage from *P. carbo* to *P. altivelis*, and stochastic control theory. Chapter 3 proposes a regular stochastic control model for the management of *P. altivelis*. The regular stochastic control model is the first step for modeling management of *P. altivelis* by stochastic control theory. Chapter 4 develops a stochastic singular control model for the management of *P. carbo*. In Lake Biwa, threshold-type management of *P. carbo* is performed. For modeling this situation, a stochastic singular control model is employed. The control rule of singular control models is a threshold-type control. Chapter 5 develops a stochastic impulse control model for the management of *P. carbo* and numerical schemes that can solve Hamilton-Jacobi-Bellman quasi-variational inequalities. Stochastic impulse control models are an extension of stochastic singular control models, and fixed costs, an important factor for the application of models to the real problems, can be included. Chapter 6 establishes a coupling model for the management of *P. altivelis*

under the predation pressure from *P. carbo*, which is a stochastic impulse control model. The difference between this model and that of Chapter 5 is that this model considers both the population dynamics of *P. carbo* and *P. altivelis*. The decision-maker can decide management plans depending on the day, the number of *P. carbo* and *P. altivelis*. Chapter 7 is the summary and future perspectives. Note that characters (parameters and variables) are defined in each chapter and some characters have different meanings in different chapters.

CHAPTER 2

Literature review

2.1 Introduction

This chapter provides literature reviews on feeding damage from *P. carbo* to *P. altivelis* and stochastic optimal control theory.

2.2 Literature review on feeding damage from *P. carbo* to *P. altivelis*

P. altivelis is one of the most economically and culturally important inland fishery resources in Japan [1, 85]. Its fish catch in 2018 accounts for 7.9% in Japanese inland fisheries [62] and has served as their main source of income. *P. altivelis* plays an important role in Japanese culture. For example, there are many Japanese traditional dishes made of *P. altivelis*, such as Ayu-no-shioyaki (baked fish with salt) and Ayu-meshi (boiled rice with the fish and soy sauce). Moreover, *P. altivelis* is a key aquatic species in Japanese river environment and ecosystems, and its population dynamics significantly affects a variety of aquatic species, such as insects, algae, and other fishes [40, 87]. Inland fishery cooperatives would encounter management difficulties without this fish species. For maintaining the populations of *P. altivelis*, inland fishery cooperatives have released the farmed fish in rivers. Natural *P. altivelis* is an annual and diadromous fish having a unique one-year life history [80, 82]. The adult fishes spawn eggs during autumn in downstream reaches of a river in which they live and die soon afterwards.

There exist specialized traps to catch spawning *P. altivelis* called Yana [55] that is a weir-shaped trap made of bamboo. Hatched larvae descend to coastal areas of a downstream water body of the river: the sea or an estuary. In the coming spring, grown fishes ascend into the midstream of the river to mature until the coming autumn. Life history of *P. altivelis* released by local inland fishery cooperatives differs from that of the natural ones in that the former is released into the midstream from a breeding facility during spring. Total fish catch of *P. altivelis* in Japan has gradually been decreasing in recent years, which is considered due to the decrease in the number of fishermen, the demand of the fish, and its population. There are several causes of the population decrease. They possibly include natural factors such as climate changes and artificial factors including manipulation of river environment through the installation of physical barriers like dams and weirs that destroy habitats and fragment migration routes of *P. altivelis*. Another most possible cause is increasing predation pressure from *P. carbo* [99, 100]: a fish-eating waterfowl reigning as a top predator in Japanese river ecosystems. Yamamoto (2008, 2009) [99, 100] reviewed biology and ecology of *P. carbo*. *P. carbo* is a colonial fish-eating bird with worldwide distribution [59] and they live in Japan [59, 36], Europe [89], North America [20], and so on. *P. carbo* is physically adapted to outswim most of the riverine fish, and an individual adult bird eats 500 (g) of fish per day [36]. During the 1960s–70s, populations of *P. carbo* in Japan significantly decreased due to human activities, which deteriorated the foraging environment and habitats. However, these populations have quickly recovered in recent years, owing to intensive fish releases by Japanese fishery cooperatives and inadequate human interventions [28, 41]. Consequently, *P. carbo* populations have dispersed throughout the whole country [49]. The excessive predation of riverine fish by burgeoning *P. carbo* populations is among the most serious problems for current inland fisheries in Japan [36]. For example, Lake Biwa houses the largest *P. carbo* colony in the country and its total population of *P. carbo* has continuously increased in recent years. Local fishery cooperatives and governments have empirically taken various countermeasures against this situation, such as gun shooting (sharpshooting, especially at

Lake Biwa), expulsion by fireworks and guns, and freezing eggs using dry ice [36]. However, the situation is still severe and has not been improved. Klimaszyk and Rzymiski (2016) [43] reported rapidly increasing *P. carbo* populations in other regions as well, such as Europe [89], North America [20], and Greenland [90]. To explain these widespread increases, they suggested a high degree of ecological adaptation, lack of regular predators in the environment, and high foraging ability of *P. carbo*, along with surface water eutrophication (which increases the fish biomass) and global climate changes. Besides feeding damage, *P. carbo* is causing serious environmental and ecological problems, such as excessive emission of nitrous oxide from their colonies [41]. Critically, however, the bird is a native species, at least in Japan [36]. Therefore, its utter extermination should not be a candidate goal of management policies. The birds also offer ecological benefits such as nutrient recycling (unless excessive) [37], and enhanced water transparency, the latter by controlling the benthivorous fish populations [31].

2.3 Literature review on stochastic control theory

2.3.1 Stochastic regular control

A developed management policy should effectively handle the inherent stochasticity in animal population dynamics [24]. Many population dynamics have been analyzed by stochastic process models composed of noise-driven differential and/or difference equations [34]. Models based on stochastic processes and dynamic programming play central roles in applications such as invasive species management [71]. As discussed by Nguyen et al. (2016) [60], the management of inland fishery resources should be developed in a feed-back control context, considering both human activities and biological dynamics in the system. This implies that the management of some population dynamics can be modeled by optimal control theory, which naturally derives a feed-back type policy given the assumed population dynamics and a performance index to be maximized. Among such mathematical models, stochastic regular control models [70, 86] are eminently suitable because they efficiently describe the stochasticity in the population dynamics. In stochastic regular control theory, the constraints of the

control are bounded. A governing equation of stochastic regular control models is Hamilton-Jacobi-Bellman equations (HJBs) [70, 86]. Stochastic regular control provides mathematical tools for modeling and computation of resources and has been applied in many fields: environmental [15, 61], and population dynamics [64, 71]. Yaegashi et al. (2016, 2017) [94, 95] proposed a stochastic differential equation (SDE) [66] that governs temporal evolution of the biomass of a population of released *P. altivelis* in rivers subject to predation pressure from *P. carbo* and harvesting by a human. The SDE has control variables on the effort to reduce the predation pressure and the fishing pressure, which are optimized so that the objective function from the viewpoint of inland fishery cooperatives, the decision-maker, is maximized.

2.3.2 Stochastic singular control

In stochastic singular control theory, the control rule is usually of a threshold-type control. The decision-maker should take countermeasures only when the animal population attains a threshold so that the animal population does not exceed the threshold [70, 86]. The constraints of the control are unbounded. The governing equation of singular control models is Hamilton-Jacobi-Bellman variational inequalities (HJBVIs) [70, 86]. The singular control can be obtained if the fixed cost in the impulse control approaches 0 [86]. Stochastic singular control models have been applied in many fields. Al Motairi (2017) [2] considered an irreversible capacity expansion model and the associated optimization problem takes the form of a singular stochastic control problem that admits an explicit solution. Cadenillas (2106) [12] developed a stochastic debt control model to study the optimal government debt ceiling. Azcue (2014) [6] shows how a viscosity approach can be used to tackle control problems in insurance such as maximization of survival probability as well as the maximization of dividends. Lungu and Øksendal (1997) [48] analyzed simple resource-management policies with stochastic dynamics in both abstract settings. Yaegashi et al. (2018b) [97] used a tractable singular stochastic control model for finding a sustainable population management policy for *P. carbo*. The bird population dynamics around a habitat lake are described by a stochastic

differential equation with a singular control variable representing human interventions to suppress the predation. The model is applied to the management of *P. carbo* population around Lake Biwa, Japan, where predation of riverine fish is the most severe in the country. The current management policy around the lake is analyzed and verified in the context of the proposed model, focusing on the population-control threshold and the average population reduction per unit time.

2.3.3 Stochastic impulse control

In stochastic impulse control theory, the control rule is usually of a threshold-type control. The decision-maker should take countermeasures only when the animal population attains an upper threshold and reduce the population to the lower threshold [68, 86]. The constraints of the control are unbounded. The governing equation of stochastic impulse control models is Hamilton-Jacobi-Bellman quasi-variational inequalities (HJBQVIs) [68, 86]. Stochastic impulse control models have been applied in many fields. Ohnishi and Tsujimura (2006) [63] examined an optimal impulse control problem of a stochastic system whose state follows a geometric Brownian motion and supposed that, when an agent intervenes in the system, it requires costs consisting of a quadratic form of the system state. Besides the intervention costs, running costs are continuously incurred to the system, and they are also of a quadratic form. Cadenillas and Zapatero (1999) [13] considered a currency with exchange rate dynamics modeled as a geometric Brownian motion and the objective of the Central Bank is to keep this exchange rate as close as possible to a given target. Bensoussan et al. (2012) [9] modeled an impulse control problem when the controller's action affects the state as well as the dynamics of the state process for a random amount of time. Korn (1998) [44] presented an impulse control approach where the investor can change his portfolio only finitely often in finite time intervals and considered transaction costs including a fixed and a proportional cost component. Øksendal and Sulem (2005) [67] considered a market model with one risk-free and one risky asset, in which the dynamics of the risky asset are governed by a geometric Brownian motion

and supposed that these transfers involve a fixed transaction cost, independent of the size of the transaction, plus a cost proportional to the size of the transaction. Yaegashi et al. (2018a, 2018c) [96, 98] introduced a stochastic impulse control model is introduced for finding the cost-effective and ecologically conscious population management policy of *P. carbo*. The optimal management policy is usually of a threshold type; if the population reaches an upper threshold, then taking a countermeasure to immediately reduce the bird to a lower threshold. This optimal policy is found through solving an HJBQVI.

CHAPTER 3

A stochastic regular control model for management of *P. altivelis*

3.1 Introduction

P. altivelis [50] is one of the most economically and culturally important inland fishery resources in Japan [1, 85]. For maintaining the population of *P. altivelis*, inland fishery cooperatives have released the farmed fish in rivers. Total fish catch of *P. altivelis* in Japan has gradually been decreasing in recent years, which is considered due to the increasing predation pressure from *P. carbo* [99, 100], a fish-eating waterfowl reigning as a top predator in Japanese river ecosystems. In order to reduce the predation pressure, local fishery cooperatives and municipalities have been trying to develop countermeasures that can effectively reduce predation pressure. So far, their effectiveness and efficiency have not been systematically quantified well, which can be achieved with the help of an appropriate mathematical model; however, such approaches are still rare. Stochastic regular control theory provides mathematical tools for modelling and computation of many problems. Researchers including the author have recently asserted that a stochastic regular control theory involving a dynamical equation of the population of the fish provides an effective tool for this purpose. Yaegashi et al. (2015) [93] have proposed a stochastic regular control model, however, the model had a significant ecological drawback that the total biomass can be quantified while the number of individuals and the individual's bodyweight cannot be despite they are important quantities in practice.

Mathematical models that are more practical should be able to overcome this inconsistency. This is the main motivation of this chapter.

This chapter has two objectives. The first objective is to formulate a stochastic regular control model for optimal management plans of released *P. altivelis* that can overcome the above-mentioned drawback of the previous model [94]. The present model is based on a pure death process of the population where the mortality due to natural death, predation from *P. carbo*, and harvesting by human are considered. Population dynamics of non-renewable fishery resources with death processes have already been applied to managing fishery resources in aquaculture systems [47, 69], but its application to released fishery resources in a natural environment is still rare. The concept of the death process is appropriate for describing population dynamics of released *P. altivelis*, which is considered as a non-renewable fishery resource since its reproduction ability has been pointed to be low [79]. Based on the stochastic process model, finding an optimal management strategy reduces to solving a 2-D Hamilton-Jacobi-Bellman equation that is performed with a finite element scheme [95]. The second objective of this paper is to numerically compute optimal management plans of released *P. altivelis* along with sensitivity analysis against model parameters to demonstrate its consistency with the reality and to comprehend the dependence on the parameters.

The rest of this chapter is organized as follows. Section 2 presents the mathematical model. Section 3 provides a real application of the model to managing population dynamics of released *P. altivelis* in the Hii River, the San-in area, Japan. Sensitivity analysis of the optimal management strategies against model parameters is also performed in this section. Section 4 gives conclusions of this chapter.

3.2 Mathematical model

3.2.1 Deterministic model

A basic deterministic population dynamics model is firstly presented. Population dynamics in a river as an isolated habitat is considered. Let W_t (kg) be the body weight of individual

P. altivelis at the time t (day). Assume that the growth of individual *P. altivelis* follows the classical Verhulst model [84]

$$\frac{dW_t}{dt} = r \left(1 - \frac{W_t}{K} \right) W_t \quad (3.1)$$

where r (1/day) is the intrinsic growth rate of *P. altivelis* and $K = mW_0$ (kg) with $m > 1$ is the potential upper limit of the body weight where $0 < W_0 < K$ is the initial condition at the time $t = 0$. The Verhulst model has been employed as a base stone of the present population dynamics model since it is one of the simplest ordinary differential equation (ODE) that describes growth of individuals with a realistic sigmoid curve [83]. Let Y_t be the total number of individuals at the time t in the habitat, which is considered as a pure death process that is non-increasing in time. The process Y_t is assumed to follow the ODE

$$\frac{dY_t}{dt} = -l_t Y_t \quad (3.2)$$

with

$$l_t = R_t + k_t + c_t \quad (3.3)$$

where R_t , k_t , and c_t are the mortality rates (1/day) of *P. altivelis* due to natural death, predation from *P. carbo*, and harvesting by human, respectively. This ODE can serve as a minimal mathematical model that describes temporal evolution of the total number of individuals without reproduction in a closed environment.

The total biomass X_t of *P. altivelis* in the habitat at the time t is expressed as

$$X_t = Y_t W_t. \quad (3.4)$$

Differentiating the both sides of (3.4) with respect to t leads to the ODE of X_t as

$$\frac{dX_t}{dt} = \frac{d}{dt} (Y_t W_t) = r \left(1 - \frac{X_t}{KY_t} \right) X_t - l_t X_t. \quad (3.5)$$

The factor $(KY_t)^{-1}$ in (3.5) is derived on the basis of the assumption that the population dynamics of the released *P. altivelis* follows a pure death process. The ODE (3.5) can be seen as a Verhulst model [11] with a monotonically decreasing carrying capacity, which leads to $Y_t \rightarrow 0$ as $t \rightarrow \infty$. It is reasonable to consider that an analogous discount factor should

appear in a stochastic counterpart of the deterministic model, which is formulated in the next subsection.

3.2.2 Stochastic process model

A stochastic population dynamics model that is inspired from the deterministic model is formulated. In this present model, inherent stochastic fluctuations of the population due to unresolved environmental, ecological, and hydrological factors are considered with a lumped multiplicative noise term $\sigma X_t dB_t$ in the Itô's sense [66] where σ (1/day^{1/2}) is the noise intensity and B_t (day^{1/2}) is the 1-D standard Brownian motion [66]. This noise term leads to a minimal SDE model that can preserve non-negativity of X_t for $t \geq 0$ given $X_0 \geq 0$ [32]. The stochastic population dynamics of the released *P. altivelis* is then described with the system of Itô's SDEs

$$d \begin{pmatrix} X_t \\ Y_t \end{pmatrix} = \begin{pmatrix} r \left(1 - \frac{X_t}{KY_t}\right) X_t - l_t X_t \\ -l_t Y_t \end{pmatrix} dt + \begin{pmatrix} \sigma X_t \\ 0 \end{pmatrix} dB_t \quad (3.6)$$

subject to initial conditions Y_0 and $X_0 = Y_0 W_0$. The generator G associated with the coupled process (t, X_t, Y_t) for generic sufficiently regular function $\varphi = \varphi(s, x, y)$ is given as the second-order partial differential operator [66]

$$G\varphi = \frac{\partial \varphi}{\partial s} + \left[r \left(1 - \frac{x}{Ky}\right) x - lx \right] \frac{\partial \varphi}{\partial x} - ly \frac{\partial \varphi}{\partial y} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 \varphi}{\partial x^2} \quad \text{in } \Omega \quad (3.7)$$

where $\Omega = \{(s, x, y) \mid 0 < s < T, 0 < x < \infty, 0 < y < \infty\}$.

3.2.3 Hamilton-Jacobi-Bellman equation

It is assumed that predation pressure k_t from *P. carbo* is a function of the non-dimensional effort $u = u_t$ to reduce the predation pressure by human. The variables u_t and c_t are assumed to be adapted Markov controls adapted to a natural filtration generated from the Brownian motion B_t [66]. The ranges U and C of u_t and c_t are $U = \{u \mid 0 \leq u \leq 1\}$ and $C = \{c \mid 0 \leq c \leq c_M \chi_{\{t \geq T_c\}}\}$, respectively where T_c (day) is the opening time of harvesting *P. altivelis*, c_M (1/day) is the maximum harvesting rate, and $\chi_{\{S\}} = \chi_{\{S\}}(z)$ is the indicator function such that $\chi_{\{S\}} = 1$ for $z \in S$ and $\chi_{\{S\}} = 0$ otherwise. From the management

perspective of local fishery cooperatives, the objective function to be maximized is set as [94]

$$v(s, x, y, u, c) = E^{s,x,y} \left[\int_s^T [-\omega f(u_t) + \theta c_t X_t] dt \right] \quad (3.8)$$

where f is an increasing function of u that represents the cost to reduce the feeding damage from *P. carbo*, ω (yen/day) is the cost per unit time to carry out a countermeasure, and θ (yen/kg) is the profit by selling *P. altivelis* per unit weight. The right hand side of (3.8) represents the sum of the cost of executing countermeasures and the profit by selling the caught fish during the time interval (s, T) . The ultimate goal of the stochastic control is to find the maximizers u^* and c^* of the expectation of the objective function v :

$$V(s, x, y) = \sup_{u, c} v(s, x, y, u, c) = v(s, x, y, u^*, c^*) \quad (3.9)$$

where $E^{s,x,y}[\cdot]$ represents the expectation operator conditioned on $X_s = x$ and $Y_s = y$. Based on the dynamic programming principle for controlled stochastic processes [25], the HJBE that governs V is formally derived as

$$-\sup_{u \in U, c \in C} (GV - \omega f(u) + \theta cx) = 0 \quad \text{in } \Omega \quad (3.10)$$

which is a nonlinear degenerate parabolic equation. Although it is appropriate to analyse (3.10) from the perspective of the viscosity solutions [70], such analysis is not performed in this thesis. For treating the infinite domain Ω , the spati-temporal domain $\Omega_{\text{comp}} = \{(s, x, y) \mid 0 < s < T, 0 < x < L, 0 < y < Y_0\}$ is used where a positive constant L (kg) is a sufficiently large value to numerically truncate the unbounded domain of X_t . The terminal condition at $s = T$ is $V = 0$. The boundary conditions along $x = 0$ and $x = L$ are

$$V = 0 \quad (3.11)$$

and

$$\frac{1}{2} \sigma^2 x^2 \frac{\partial V}{\partial x} = 0, \quad (3.12)$$

respectively. The latter condition is an artificial one due to the truncation of the computational domain and assumes that the curvature of the solution is small. The natural boundary condition to be specified along $y = 0$ is

$$V = 0. \quad (3.13)$$

The boundary conditions $V = 0$ along $x = 0$ and $y = 0$ mean that the total profit is 0 when population of *P. altivelis* becomes extinct. These boundary conditions mean that extinction of the population is not beneficial for fishery cooperatives. No boundary condition is required along $y = Y_0$ since l is positive. The optimal control u^* depends on the functional forms of f and k . In this chapter, simple but realistic functional forms $k(u) = k_0(1 - \alpha u)$ and $f(u) = u$ are used [94] where k_0 (1/day) is predation pressure from *P. carbo* without any countermeasures and $0 < \alpha < 1$ is the constant that represents effectiveness of the countermeasure to reduce the predation pressure; higher (lower) α corresponds to higher (lower) effectiveness. The optimal controls u^* and c^* are expressed with partial derivatives of V . The optimal control u^* is expressed through partial differentials of the value function V as

$$u^* = \begin{cases} 1 & (\gamma \geq 0) \\ 0 & (\gamma < 0) \end{cases} \quad (3.14)$$

with

$$\gamma = \alpha k_0 \left(x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y} \right) - \omega. \quad (3.15)$$

Similarly, the optimal control c^* is expressed through partial differentials of the value function V as

$$c^* = \begin{cases} c_M \chi_{\{s \geq T_c\}} & (\kappa \geq 0) \\ 0 & (\kappa < 0) \end{cases} \quad (3.16)$$

with

$$\kappa = - \left(x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y} \right) + \theta x. \quad (3.17)$$

3.2.4 Statistical indexes for verifying the optimal plans

Once the optimal controls u^* and c^* are found over the spatio-temporal domain Ω , statistical indexes to verify sensitivity of the model are obtained considering mathematical linkages between SDEs and terminal and boundary value problems of parabolic PDEs [66]. The theory of diffusion processes governed by SDEs shows that the statistical index

$$J(s, x, y) = E^{s, x, y} \left[\int_s^T g(t, X_t) dt \right] \quad (3.18)$$

with the function $g = g(t, X_t)$ solves the Kolmogorov's Backward Equation (KBE)

$$GJ + g = 0 \quad (3.19)$$

in Ω , subject to terminal condition $J = 0$ at $s = T$ and boundary conditions

$$J = 0 \quad \text{along } x = 0 \text{ and } y = 0 \quad (3.20)$$

and

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial V}{\partial x} = 0 \quad \text{along } x = L. \quad (3.21)$$

The latter condition is an artificial one due to the truncation of the computational domain and assumes that the curvature of the solution is small. Hereafter, the notation $J_0 = J(0, X_0, Y_0)$ is used for the sake of brevity of descriptions. The index J_0 represents the expectation of the temporal integration of g from $t = 0$ to $t = T$. The indexes considered in this paper include $J_{0,f}$ ($g = f(u_t^*)$) and $J_{0,c}$ ($g = \chi_{\{t \geq T_c\}} \theta c_t^* X_t$), which are the total cost to reduce the feeding damage from *P. carbo* (yen) and the profit obtained from selling *P. altivelis* (yen), respectively. In addition, $V(0, X_0, Y_0, T) - \eta Y_0 W_0$ that represents the total profit (yen) is denoted as $J_{0,v}$ where η (yen/kg) is the cost to buy juvenile *P. altivelis* per unit weight.

3.3 Application

This section applies the presented stochastic control model to managing released *P. altivelis* in a Japanese river. The model parameters are estimated from collected ecological, economic, and environmental data. The HJBE and KBE are solved with a finite element scheme so that optimal management plans are numerically computed for a variety of parameter values.

3.3.1 Numerical scheme

The HJBE (3.10) and KBE (3.19) are numerically solved with a 2-D Petrov-Galerkin finite element method (PGFEM) [95] inspired from the 1-D counterpart that has successfully been applied to a variety of elliptic and parabolic problems [102]. Accuracy and stability of the scheme have been numerically investigated for linear and nonlinear test problems, indicating

that its convergence rate is first to second order both in space and time even for advection-dominant cases. The scheme performs a compact spatial discretization with unstructured triangular elements and exponentially fitted weight and shape functions determined through analytical solutions to local two-point boundary value problems on element edges. An edge-based approximation technique following [92] is used in the scheme to achieve unconditional stability in space with smaller numerical diffusion effects than the conventional first-order upwind scheme. The scheme satisfies the discrete maximum principle at least for linear elliptic problems, which can preserve signs of numerical solutions under appropriate boundary conditions. In addition, the 2-D PGFE scheme has already been successfully applied to numerically computing energetically optimal swimming velocity field of individual fishes in horizontally 2-D shallow water flows [103].

3.3.2 Study area

The stochastic control model is applied to the management problem of released *P. altivelis* in the Hii River, San-in area, Japan. The total length of the mainstream and the catchment area of the river are 153 (km) and 2,070 (km²), respectively [53]. The Hii River flows into a brackish lake, Lake Shinji, which is connected to a brackish lake, Lake Nakaumi. Hydrological characteristics of the Hii River are reviewed in [77]. Both of the brackish lakes have served as important habitats for migrating aquatic species whose life history involves growth and spawning in the Hii River [101, 88]. *P. altivelis* is the inland fishery resource that exhibits the largest fish catches in the Hii River [51, 52]. The union members of the Hii River Fishery Cooperatives say that the number of juvenile *P. altivelis* that ascends into the river during spring has significantly been decreasing. In every April to May, the Hii River Fishery Cooperatives performs intensive release events of juveniles of *P. altivelis*, which is transported from Lake Biwa, the largest freshwater lake in Japan where a vast number of *P. altivelis* inhabit, or other rivers in San-in area. After growing up, the released fishes spawn in downstream reaches during October to November; however, it has been thought that their

descendants do not successfully contribute to the next generation, which is considered due to the existence of environment of the Hii River with huge weirs [103] that possibly prevent them from migrating upstream from brackish lakes to the midstream reaches. Natural *P. altivelis* would also suffer from the same ecological issue. Finding an appropriate management strategy of released *P. altivelis* is a crucial issue for the Hii River Fishery Cooperatives for a sustainable inland fishery in the Hii River.

3.3.3 Parameter estimation and computational conditions

The model parameters used in this paper are summarized in Table 3.1. The terminal time $T = 180$ (day) is specified considering that the intensive release events of the juveniles are performed during May and *P. altivelis* spawn during November. The opening time of harvesting *P. altivelis* is set as $T_c = 90$ (day) in summer. The natural mortality rate of *P. altivelis* R (1/day) from May to November is 50% [56], leading to 3.9×10^{-3} (1/day). The intrinsic growth rate r (1/day) is identified with the analytical stationary solution to the ODE (3.1) assuming that the bodyweight of individual *P. altivelis* increases to 99% of the upper limit at the opening time $T_c = 90$ (day) [94]. This leads to $r = 7.1 \times 10^{-2}$ (1/day). For the previous model [94, 95], which is not based on a pure death process as in the present model, the condition that the population with neither predation nor harvesting has a non-trivial steady unimodal probability density function yields $0.36 > \sigma$ (1/day^{1/2}). The same range is considered in this paper with a sensitivity analysis, which can provide not optimistic results since the mean total biomass in the presented model is smaller than that of the previous model. The predation pressure k_0 is estimated as $k_0 \leq O(10^{-2})$ (1/day) with the assumption that all *P. carbo* comes from Lake Nakaumi where the nearest huge colony exists. The same range is considered in this paper with a sensitivity analysis. The effectiveness of the countermeasure α is estimated as 0.50 assuming that the feeding pressure is halved by using a countermeasure at a maximum rate [94].

The following parameters are estimated according to the data obtained through personal

communications with the Hii River Fishery Cooperatives. The initial bodyweight W_0 is specified as 9.4×10^{-3} (kg), m as 5.4, and $K = 5.1 \times 10^{-2}$ (kg) and the growth curve of *P. altivelis*. The number of *P. altivelis* released into a river at $t = 0$ (day) is $Y_0 = 1.9 \times 10^5$ (-), which is equivalent to 1.8×10^3 (kg). The maximum harvesting rate c_M is estimated from the data on the number of anglers as $c_M \leq O(10^{-2})$. The profit by selling *P. altivelis* per unit weight θ is set as 4.0×10^3 (yen/kg) and the cost of buying juvenile fishes per unit weight η as 4.5×10^3 (yen/kg). The order of the cost per unit time to carry out the countermeasure ω is estimated as $\omega = O(10^4)$ (yen/day) and set as $\omega = 1.0 \times 10^4$ (yen/day) [94].

For numerical computations, the length of the non-dimensional domain for population L should be sufficiently large, and thus is set as 1.8×10^4 (kg), whose validity was checked in advance. The spatial domain is discretized into a triangular mesh with 45,000 elements and 22,801 nodes. The time increment for temporal integration is 0.01 (day). An artificial diffusion term with the diffusion constant of 1.1×10^{-2} (kg²/day) has been added to the HJBE (3.10) in order to numerically circumvent the degeneration of diffusion terms. Preliminary numerical computation showed that adding the artificial diffusion term is sufficient to handle the degeneration of the diffusion terms in the HJBE and KBE.

Table. 3.1: Values of model parameters.

Parameter	Unit	Dimensional Value
T	day	180
R	1/day	3.9×10^{-3}
r	1/day	7.1×10^{-2}
θ	yen/kg	4.0×10^3
η	yen/kg	4.5×10^3
ω	yen/day	1.0×10^4
L	kg	1.8×10^4
W_0	kg	9.4×10^{-3}
K	kg	5.1×10^{-2}
m	—	5.4×10^0
Y_0	—	1.9×10^5
α	—	5.0×10^{-1}

3.3.4 Computational results

Firstly, a demonstrative computational example of the optimal management strategy for population dynamics of *P. altivelis* is presented. After that, the sensitivity analysis of the model against several parameters is carried out using the statistical indexes. The parameter values for the demonstrative computational example are set as $c_M = 1.0 \times 10^{-2}$ (1/day), $k_0 = 5.0 \times 10^{-3}$ (1/day), and $\sigma = 0.10$ (1/day^{1/2}). Figure.3.1 and Figure.3.2 show the computed optimal controls u^* at 54 (day), 126 (day), and 162 (day), and c^* at 90 (day), 126 (day), and 162 (day), respectively. According to Figure.3.1, the optimal strategy to reduce the feeding damage from *P. carbo* is taking the countermeasure before the opening time 90 (day). Until 90 (day), the optimal strategy for extermination does not depend on x but on y except for small x . Before 90 (day), conservation of the number of individuals is important because the population dynamics is a pure death process. After 90 (day), taking the countermeasure is optimal except for small x or y . The area with $u^* = 1$ in the spatial domain becomes smaller as time elapses, indicating that taking no countermeasure is optimal except for large x or y . According to Figure.3.2, until 162 (day), the optimal strategy of harvesting is to catch the fish except for the case where the bodyweight of individuals is small. On the other hand, harvesting the fish in all situations is optimal after 162 (day).

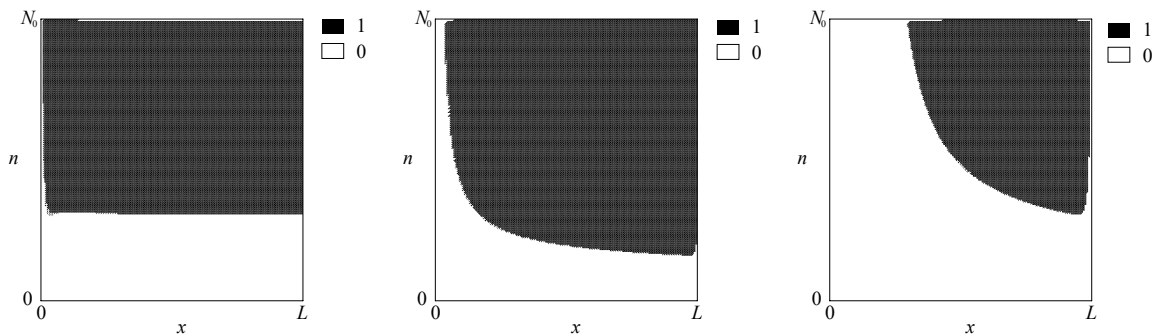


Fig. 3.1: Optimal control u^* at 54 (day) (left), 126 (day) (middle), and 162 (day) (right).

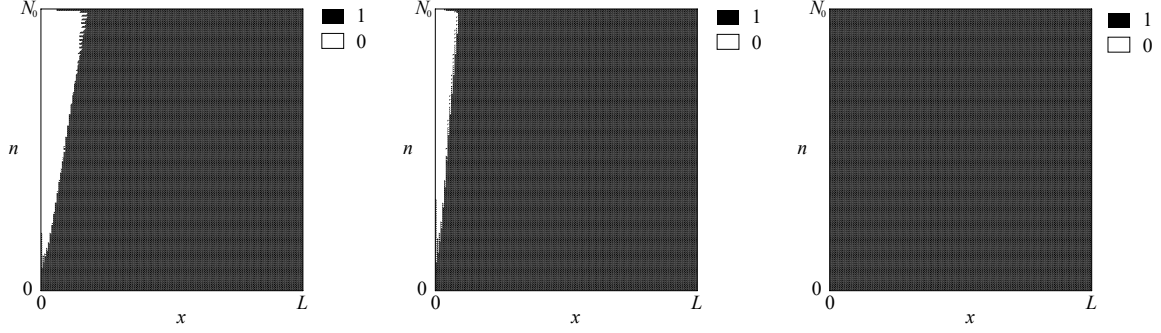


Fig. 3.2: Optimal control c^* at 90 (day) (left), 126 (day) (middle), and 162 (day) (right).

The indexes $J_{0,c}$, $J_{0,f}$ and $J_{0,v}$ for a variety of the values of the predation pressure k_0 and the noise intensity σ are computed in order to comprehend dependence of optimal management strategies against them. The parameter values examined are, $k_0 = (1.0 + 2.0i) \times 10^{-3}$ (1/day) and $\sigma = 0.05 + 0.02j$ (1/day^{1/2}) where $0 \leq i, j \leq 10$ are nonnegative integers. The value of $\omega = 1.0 \times 10^4$ (yen/day) is specified as in the previous case. Each panel of Figures.3.3 and 3.4 plots an index in the k_0 - σ plane where $c_M = 5.0 \times 10^{-3}$ (1/day) in Figure.3.3 and $c_M = 1.0 \times 10^{-2}$ (1/day) in Figure.3.4. Figures.3.3 and 3.4 show that the profit by selling the fish $J_{0,c}$ monotonically decreases as k_0 or σ increases and the total profit $J_{0,v}$ accordingly decreases. This result indicates that increase of k_0 or σ potentially leads to decrease of the total biomass of *P. altivelis*. The indexes $J_{0,c}$ and $J_{0,v}$ decrease in both k_0 and σ , and their dependence on k_0 is higher than that on σ . The gradient of $J_{0,c}$ and $J_{0,v}$ do not significantly depend on c_M . For $c_M = 5.0 \times 10^{-3}$ (1/day), $J_{0,v}$ is negative for every sets of parameters, while for $c_M = 1.0 \times 10^{-2}$ (1/day), $J_{0,v}$ is positive only for small k_0 and σ . The total cost for the extermination $J_{0,f}$ with $c_M = 1.0 \times 10^{-2}$ (1/day) is higher than that with $c_M = 5.0 \times 10^{-3}$ (1/day). This can be explained by the fact the profit by selling the fish effectively compensates the cost of taking countermeasures.

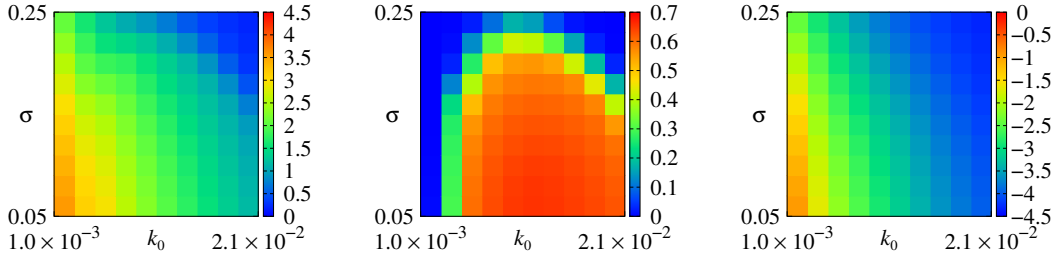


Fig. 3.3: Indices $J_{0,c}$ (left), $J_{0,f}$ (middle), and $J_{0,v}$ (right) for $c_M = 5.0 \times 10^{-3}$ (1/day) and $T_c = 90$ (day).

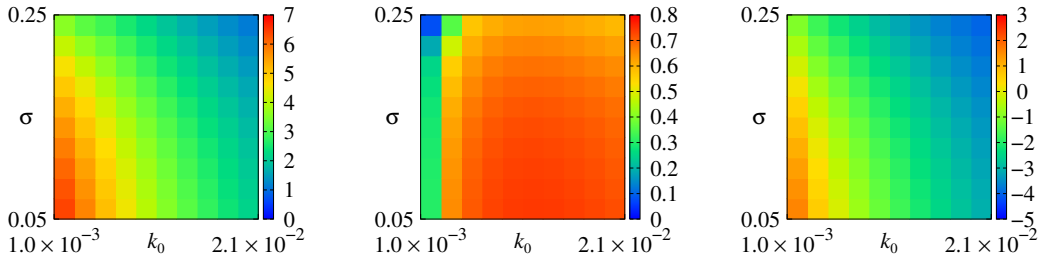


Fig. 3.4: Indices $J_{0,c}$ (left), $J_{0,f}$ (middle), and $J_{0,v}$ (right) for $c_M = 1.0 \times 10^{-2}$ (1/day) and $T_c = 90$ (day).

An additional set of numerical computation of the optimal management strategies is performed to assess impacts of the opening time T_c on the three statistical indexes. For that purpose, the indexes are computed with the earlier ($T_c = 72$ (day)) and later ($T_c = 108$ (day)) opening times with the fixed harvesting pressure $c_M = 5.0 \times 10^{-3}$ (1/day) and $c_M = 1.0 \times 10^{-2}$ (1/day). As in Figures.3.3 and 3.4, each panel of Figures.3.5 through 3.8 plots the indices in the k_0 - σ plane where $T_c = 72$ (day) in Figures.3.5 and 3.6 and $T_c = 108$ (day) in Figures.3.7 and 3.8. The maximum harvesting pressure c_M is set as $c_M = 5.0 \times 10^{-3}$ (1/day) in Figures.3.5 and 3.7 and $c_M = 1.0 \times 10^{-2}$ (1/day) in Figures.3.6 and 3.8. Comparisons among Figures. 3.3, 3.5, and 3.7 show that the dependence of the three indices is qualitatively the same for all the opening times examined for $c_M = 5.0 \times 10^{-3}$ (1/day). The same applies to the cases with the larger maximum harvesting pressure $c_M = 1.0 \times 10^{-2}$ (1/day) in Figures. 3.4, 3.6, and 3.8. These figures show that the indexes $J_{0,c}$ and $J_{0,v}$ increase as the opening time T_c becomes earlier for both smaller and larger maximum harvesting pressures. The index $J_{0,f}$

for the smaller maximum harvesting pressure $c_M = 5.0 \times 10^{-3}$ (1/day) seems to decrease as the opening time T_c is delayed. On the other hand, $J_{0,f}$ for the larger $c_M = 5.0 \times 10^{-3}$ (1/day) seems to increase as T_c is delayed. However, this difference of the index $J_{0,f}$ on the maximum harvesting pressure c_M is minor since it is one-order smaller than $J_{0,c}$ and $J_{0,v}$. The computational results suggest that the total profit, which is the index $J_{0,v}$, increases as the opening time T_c is set earlier with the specified parameter values.

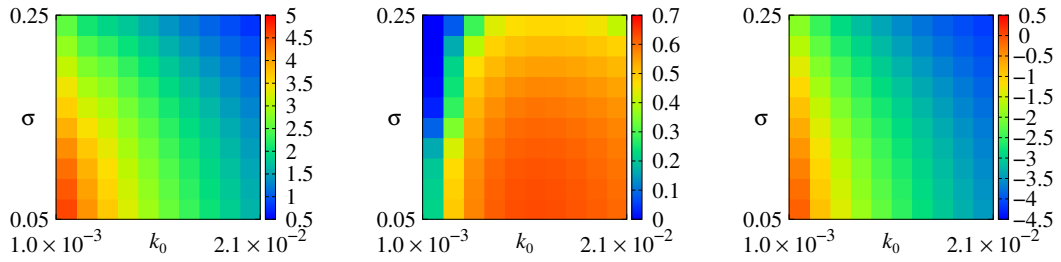


Fig. 3.5: Indices $J_{0,c}$ (left), $J_{0,f}$ (middle), and $J_{0,v}$ (right) for $c_M = 5.0 \times 10^{-3}$ (1/day) and $T_c = 72$ (day).

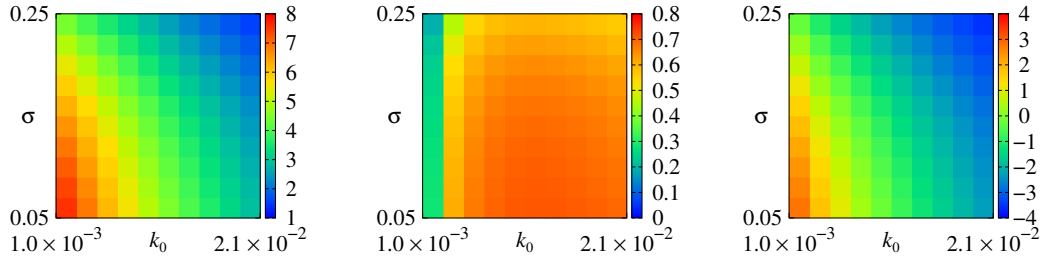


Fig. 3.6: Indices $J_{0,c}$ (left), $J_{0,f}$ (middle), and $J_{0,v}$ (right) for $c_M = 1.0 \times 10^{-2}$ (1/day) and $T_c = 72$ (day).

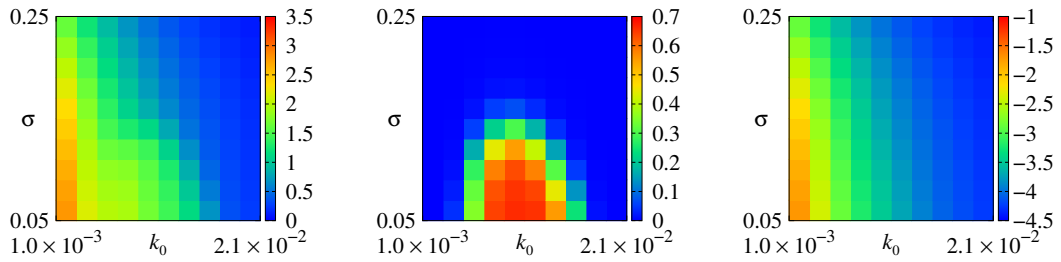


Fig. 3.7: Indices $J_{0,c}$ (left), $J_{0,f}$ (middle), and $J_{0,v}$ (right) for $c_M = 5.0 \times 10^{-3}$ (1/day) and $T_c = 108$ (day).

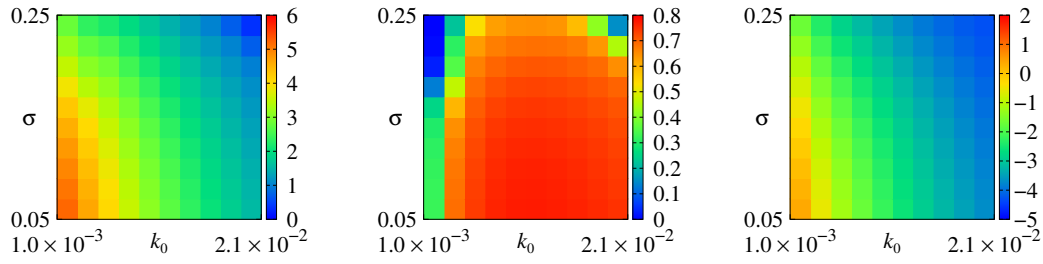


Fig. 3.8: Indices $J_{0,c}$ (left), $J_{0,f}$ (middle), and $J_{0,v}$ (right) for $c_M = 1.0 \times 10^{-2}$ (1/day) and $T_c = 108$ (day).

3.4 Conclusions

Starting from a pure death process, a stochastic control model for population dynamics of released *P. altivelis* in river environment under predation from *P. carbo* and harvesting by a human was formulated. Application of the present model to the Hii River with estimated parameter values could successfully compute the optimal management strategy for *P. altivelis*, which was more consistent with reality than the previous model. Numerical sensitivity analysis of the optimal management strategies using the statistical indexes revealed their parameter-dependence, which significantly facilitated the comprehension of the behavior of the model.

CHAPTER 4

A stochastic singular control model for management of *P. carbo*

4.1 Introduction

P. carbo is a colonial fish-eating bird with worldwide distribution [59]. Feeding damage from *P. carbo* has been especially severe on *P. altivelis*, one of the most economically and culturally important inland fishery resources in Japan [81]. The feeding damage at Lake Biwa, containing the largest colony of *P. carbo* in Japan, has been estimated as 1.0×10^6 kg of riverine fish per year [72]. The fish catch of *P. altivelis* in 2018 accounts for 7.9 % of the total catches of Japanese inland fisheries per year [62] and is the main source of their income. Without *P. altivelis*, inland fishery cooperatives would face significant management difficulties. Therefore, to maintain adequate populations of *P. altivelis*, inland fishery cooperatives have released farmed fish into rivers. Thus, establishing a sustainable management policy of the bird population that can effectively suppress its predation on riverine fish is needed. The sustainable management policy should also control the bird population without undue human interventions. However, such an approach has not been well established so far.

Mathematical modeling of the bird population dynamics would help to resolve the severe situation currently faced by inland fisheries in Japan. This chapter establishes and analyzes a tractable stochastic singular control model for a cost-effective, ecologically-sound, and

sustainable predator management policy. Specifically, a sustainable management policy of *P. carbo* populations by a new mathematical approach based on a stochastic singular control model [70] is found. Such a model includes a controlled SDE [66] with a performance index to be maximized or minimized by choosing an appropriate control. The stochastic singular control formulates a simple and tractable mathematical model with an exact solution, enabling a practical threshold-type optimal management policy. To the author's knowledge, stochastic singular control has rarely been applied to predator population management. Therefore, this chapter explores a predator management problem in a stochastic singular control model. The problem of finding a population management policy for the bird population (the predator) is reduced to an exactly-solvable HJBVI, from which a threshold-type, sustainable predator management policy is derived. The sensitivity of the management policy on the ecological, environmental, and social parameters is confirmed in mathematical analysis. The model is then applied to the management problem of *P. carbo* populations in Lake Biwa, Japan, using parameter values derived from public data. Next, the model verifies the population control threshold of the bird population currently used around Lake Biwa and investigates the conditions that optimize the current management policy by balancing the loss of riverine fish with the ecosystem services provided by the bird.

This chapter checks the validity of the current population-control threshold of *P. carbo* around Lake Biwa, Japan, from both scientific and quantitative viewpoints. Although the present model is simple, it directly computes the population-control threshold, which is a practical index in bird management plans. We are aware of few, if any, studies adopting this verification approach. The present model provides another useful quantity, the average reduction in bird population numbers per unit time. Moreover, the method developed in our paper is more mechanistic and systematic than statistical models [23] and other empirical models. Therefore, the present model better clarifies the physical meaning of the model parameters than empirical models. Although more refined mechanistic models with more degrees of freedom than ours have been proposed [18], they are less practical owing to a large

number of model parameters. Our model assures both practical applicability and biological and ecological feasibility.

The remainder of this chapter is structured as follows. Section 2 proposes the mathematical model of predator control, and Section 3 analytically derives the practical indices for assessing the management plans. Section 4 applies these indices to the management of the *P. carbo* population around Lake Biwa, Japan, where predation of riverine fish is the most severe in the country. Section 5 concludes this chapter.

4.2 Mathematical model

4.2.1 Stochastic differential equation

The population dynamics of *P. carbo* are described by a controlled SDE governing the temporal evolution of the bird population in a habitat (i.e., the bird colony around a river or a lake). The SDE is a linear stochastic population growth model driven by multiplicative noise; however, the population can decrease under human interventions. The linear growth assumption is crucial for establishing a tractable mathematical model and is justifiable when the number of same-species individuals in the habitat is smaller than the environmental capacity. In this chapter, the bird population is considered to be appreciably large, so it is treated as a continuous variable.

The bird population dynamics are assumed to follow Itô's SDE [66]

$$dX_t = X_t(\mu dt + \sigma dB_t) - d\eta_t, \quad X_{-0} = x \geq 0, \quad (4.1)$$

with the conventions $X_{-0} = \lim_{s \uparrow 0} X_s$ and in general $X_{t-0} = \lim_{s \uparrow t} X_s$ for $t > 0$. Here, X_t is the total number of birds at time t , $\mu > 0$ is the deterministic growth rate of the population, $\sigma > 0$ is the magnitude of the stochastic fluctuations in the population dynamics, and B_t is the one-dimensional (1-D) standard Brownian motion on the complete probability space [70], whose filtration is right-continuous and satisfies the usual conditions [39].

The model parameters of the SDE (4.1) [33] should satisfy

$$\mu > \frac{\sigma^2}{2}, \quad (4.2)$$

meaning that an uncontrolled bird population ($\eta_t = 0$ for all t) never becomes extinct. The variable η_t represents the population decrease through countermeasures such as gun shooting, a right-continuous adapted process [70] that directly reduces the bird population. Formally, the increment $d\eta_t$ is rewritten as

$$d\eta_t = u_t dt \quad (4.3)$$

where the measurable process $u_t \in [0, \infty)$ denotes the population reduction per unit time. The unboundedness of the range of u_t can be justified if the countermeasure occurs over a much shorter timescale than the population growth. Intensive gun shooting (the common countermeasure in Japan) fits this criterion. The control variable η_t is chosen such that the total population in the model remains positive ($X_t > 0$) for all t . We also assume $\eta_{-0} = 0$, meaning that η_t defines the total number of culled individuals during the time interval $(0, t)$. As in the conventional models [3], the control η_t is said to be admissible when SDE (4.1) admits a unique strong solution in which X_t is positive, adapted and right-continuous. In this chapter, a sustainable management policy is defined as a policy under which the population $X_t > 0$ remains bounded and positive for $t \geq 0$. Although the formulation of our singular stochastic control is somewhat idealized, it provides a tractable mathematical model with associated practical indices, as will be demonstrated later.

The SDE (4.1) potentially describes a minimal stochastic process model of the population dynamics, with the simplest form of the multiplicative noise term that reflects the inherent stochasticity in the population dynamics [45]. Owing to their mathematical tractability, similar SDEs have been applied in resource and population dynamics [17]. The mathematical properties of stochastic singular control models are analyzed in [70].

4.2.2 Performance index

The decision-maker of the present model is an organization that manages the *P. carbo* population, such as a local fishery cooperative or a local government. The decision-maker should maximize the performance index by choosing an optimal control $\eta_t = \eta_t^*$, namely, a

maximizer. The performance index for an admissible η_t is denoted as $v = v(x; \eta)$, and is set as

$$v(x; \eta) = E^x \left[- \int_0^\infty e^{-\delta s} (RX_s^M - rX_s^m) ds - \int_0^\infty e^{-\delta s} d\eta_s \right], \quad (4.4)$$

where $E^x[\cdot]$ represents the expectation (conditional on $X_{-0} = x \geq 0$), $\delta > 0$ is the discount rate of the profit, and R, r, m , and M are model parameters with $R, r \geq 0$ and $0 < M < 1 < m \leq 2$. This form of the performance index, namely, the expectation of the integrals with exponential discounting, is routinely used in long-term resource management [22].

The performance index v measures the expected net profit gained by the decision-maker. The discount rate δ represents the attitude of the decision-maker while managing the bird population; if δ is large, the decision-maker intends a long-term population control. Conversely, no sustainable management policy is possible when δ is small. For example, following Lemma 2 of Alvarez (2000) [4], the conditions $\mu > \delta$ and $R = 0$ lead to an immediate and utter extermination of the population at the initial time $t = 0$, which is not an ecologically-sound policy. Hereafter, the inequality

$$\delta > \mu m + \frac{\sigma^2}{2} m(m-1) \quad (4.5)$$

is assumed, meaning that the decision-maker manages the bird population from a sufficiently long-term, sustainable viewpoint. The terms RX_s^M and $-rX_s^m$ quantify the loss of riverine fish to *P. carbo* predation and the ecosystem services provided by the bird, respectively, per unit time [104]. The terms $RX_s^M - rX_s^m$ is unimodal and convex with respect to X_s , implying that no cost nor profit is possible when the bird population is zero ($X_s = 0$). The second term in the right-hand side of (4.4) represents the cost of taking countermeasures. When developing a population management policy of harvesting or population control, the costs and benefits must be carefully balanced to ensure the cost-effectiveness of the plan [76]. The first and second terms of the performance index v are weighted by the parameters R and r , respectively, depending on the attitude of the decision-maker.

The present model involves two types of parameters. The first type, the parameters of

the population dynamics μ and σ , are determined from the real bird population data. The second type, the performance parameters r , R and δ , are determined by the attitude of the decision-maker. Before implementing the model, the decision-maker specifies the specific parameter values of the performance index. Applying the model to the bird predation problem around Lake Biwa with possible ranges of the values of the performance parameters are also identified.

4.2.3 Hamilton-Jacobi-Bellman variational inequality

The value function V is defined as the maximum performance index v :

$$V(x) = \sup_{\eta} v(x; \eta) = v(x; \eta^*) \quad (4.6)$$

where η^* is the optimal control providing the best feed-back population management policy. A main goal of the present stochastic singular control problem is to find the maximizer η^* satisfying (4.6). According to the dynamic programming principle for stochastic control problems (Chapter 4.3 of [70]), the value function $V(x)$ is formally a solution to the HJBVI

$$\min \left(\delta V - \mu x \frac{dV}{dx} + \frac{1}{2} \sigma^2 x^2 \frac{d^2 V}{dx^2} - u x^M + dx^m, \frac{dV}{dx} + 1 \right) = 0 \quad (4.7)$$

and

$$V(0) = 0. \quad (4.8)$$

The boundary condition (4.8) implies that neither profit nor loss is possible in a zero population ($x = 0$). The left-hand and right-hand terms in the min operator correspond to the extreme situations of no and immediate countermeasures, respectively. As demonstrated in the next section, the HJBVI (4.7) admits a unique solution with a free boundary ($x = \bar{x}$ in the following). This solution corresponds to the threshold population-level above which the countermeasure should be carried out immediately.

4.3 Theoretical analysis

4.3.1 Exact solution

In general, solutions to degenerate elliptic problems like (4.7) must be understood in the context of viscosity solutions, which are appropriate weak solutions to this kind of problem [70]. However, such refined mathematical techniques are not applied in this chapter, because the variational inequality (4.7) has an exact solution in the classical point-wise sense, namely, the value function defined in (4.6).

Under Assumption (4.5), the viscosity solution to the present HJBVI is uniquely given by

$$V(x) = \begin{cases} ax^k + Ax^m + Bx^M & (0 < x \leq \bar{x}) \\ b - x & (x > \bar{x}) \end{cases} \quad (4.9)$$

with

$$k = \frac{1}{2} \left(1 - \frac{2\mu}{\sigma^2} + \sqrt{\left(\frac{2\mu}{\sigma^2} - 1 \right)^2 + \frac{8\delta}{\sigma^2}} \right) \quad (> m) \quad (4.10)$$

and

$$A = \frac{-r}{\delta - \mu m - \frac{\sigma^2}{2}m(m-1)} < 0, \quad B = \frac{R}{\delta - \mu M - \frac{\sigma^2}{2}M(M-1)} > 0, \quad (4.11)$$

following Chapter 4.5 of [70]. Here, a , b , and $\bar{x}$ are unknowns that solve the system of nonlinear equations

$$\begin{cases} a\bar{x}^k + A\bar{x}^m + B\bar{x}^M = b - \bar{x} \\ ka\bar{x}^{k-1} + mA\bar{x}^{m-1} + MB\bar{x}^{M-1} = -1 \\ k(k-1)a\bar{x}^{k-2} + m(m-1)A\bar{x}^{m-2} + M(M-1)B\bar{x}^{M-2} = 0 \end{cases}. \quad (4.12)$$

Here, $\bar{x}$ represents the population-control threshold; if the bird population is below the threshold, the countermeasure should not be implemented. On the other hand, if the population exceeds the threshold, then the decision-maker should activate the countermeasure immediately.

Combining the second and third equations of (5.34), the governing algebraic equation of $\bar{x}$ is obtained as

$$\bar{x}^{m-1} = \frac{-\left(MB\bar{x}^{M-1}(k-M) + k-1\right)}{mA(k-m)}. \quad (4.13)$$

According to the classical intermediate value theorem [5], (4.13) has a unique solution within $0 < \bar{x} < \infty$. The other unknowns a and b are found by substituting $\bar{x}$ into the first and second equations of (5.34). After a straightforward calculation, the viscosity solution (4.9) is found as a classical solution $V \in C^2(0, \infty) \cap C[0, \infty)$. Note that the exact solution (4.9) is smooth at the threshold $x = \bar{x}$. The above mathematical analysis shows that the qualitative profile of the value function V does not significantly change under the assumed conditions (4.5). Therefore, to investigate the optimal management policy, we need only analyze the dependence of the threshold $\bar{x}$ on the model parameters of the environment and the attitude of the decision-maker.

As indicated by Solution (4.9), the countermeasure should be performed when the bird population X_t reaches the threshold $\bar{x}$. The SDE (4.1) is then rewritten in terms of $\bar{x}$ as

$$dX_t = X_t(\mu dt + \sigma dB_t) - d\eta_t^*, \quad X_{-0} = x \geq 0, \quad (4.14)$$

with the optimal control policy expressed as [48]

$$\eta_t^* = \max(x - \bar{x}, 0) + L_t, \quad \eta_{-0}^* = 0. \quad (4.15)$$

In (6.7), L_t is the local time [8]. To ensure that process (4.14) is bounded within $x \in [0, \bar{x}]$, the local time is invoked only when the bird population is about to exceed the threshold $\bar{x}$. Around Lake Biwa, Japan, the threshold-type control intends to bound the population X_t within the range $[0, \bar{x}]$, with $\bar{x} = 4,000$ (-), as explained in the next section. The derived optimal management policy is therefore consistent with that employed in a real problem. In addition, whether the countermeasure should be taken or not is determined simply by observing the number of *P. carbo* individuals.

According to (6.7), if $0 \leq X_{-0} = x \leq \bar{x}$, the bird population should be monitored such that the threshold $\bar{x}$ is never exceeded. The SDE (4.14) then governs a geometric Brownian motion with a reflection at $x = \bar{x}$ [70]. If $X_{-0} > \bar{x}$, i.e., the population is above the threshold, the optimal strategy is immediately reducing the population to $X_0 = \bar{x}$, and thereafter ensuring that the population never exceeds the threshold $\bar{x}$. If $X_{-0} \leq \bar{x}$, then $X_t > \bar{x}$ for $t > 0$ does

not occur under the optimal management policy. Fig. 4.1 presents a computed sample path following SDE (4.14) under the optimal control (6.7). When the population exceeds the threshold (green lines in Fig. 4.1), the optimal policy (6.7) immediately reduces the bird population. The path is indeed reflected at the threshold $\bar{x}$ and its range is consequently bounded as $[0, \bar{x}]$. The properties of the SDE solution and their practical implications are discussed in the following sections.

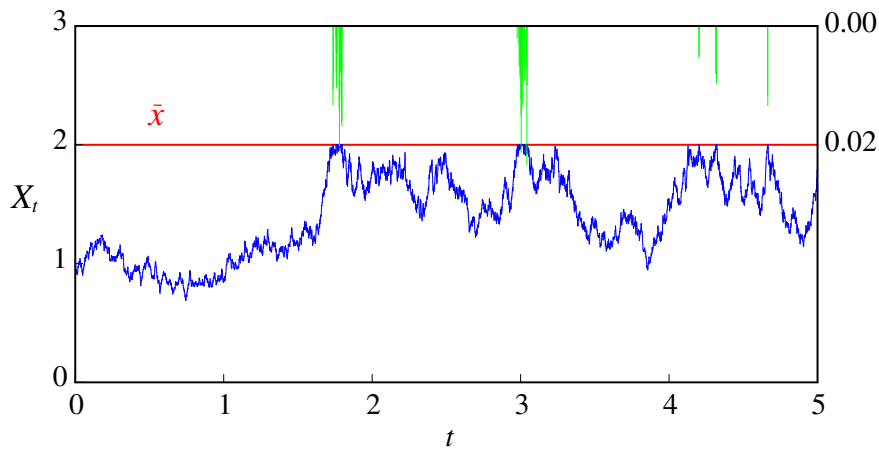


Fig. 4.1: Sample path of the optimized X_t with $\bar{x} = 2$ (red line), showing the reduced population when the population rises above the threshold (green line). The model parameters are $\mu = 0.17$ and $\sigma = 0.53$. The path is reflected at the level $\bar{x}$ by the countermeasure and satisfies $0 \leq X_t \leq \bar{x}$.

4.3.2 Probability density function

Having calculated the threshold $\bar{x}$ by solving the HJBVI (4.7), we can analytically calculate the probability density function (PDF) of the stochastic process X_t under the optimal management policy η^* , because the model is completely tractable. Hereafter, we assume $0 \leq x \leq \bar{x}$, namely, that the bird population is not above the threshold and that the decision-maker manages the population dynamics following the optimal management policy derived in the foregoing sub-section.

To evaluate the statistics of the PDF, we require the PDF of the stochastic process X_t with η^* . This PDF can be analytically solved in the present model because the first term of the SDE

(4.1) is linear. For this purpose, we can adopt the analytical formulas of the stationary PDF of the geometric Brownian motion with reflection [105]. The stationary PDF $p = p(x)$ for the process X_t (4.14) under the optimal control η^* is analytically derived from the Kolmogorov forward equation (KFE) [32]

$$\frac{d(\mu xp)}{dx} - \frac{d^2}{dx^2} \left(\frac{\sigma^2 x^2}{2} p \right) = 0, \quad 0 < x < \bar{x}, \quad (4.16)$$

subject to the boundary conditions $p(0) = 0$ and the zero-flux condition (reflection) at $x = \bar{x}$

$$\mu xp - \frac{d}{dx} \left(\frac{\sigma^2 x^2}{2} p \right) = 0 \quad (4.17)$$

under the usual normalization condition

$$\int_0^{\bar{x}} p(x) dx = 1. \quad (4.18)$$

At first, the condition $q = \frac{2\mu}{\sigma^2} - 1 > 0$ is assumed. Integrating (4.16) with respect to x yields

$$\mu xp - \frac{d}{dx} \left(\frac{\sigma^2 x^2}{2} p \right) + \alpha = 0. \quad (4.19)$$

The constant of integration α is 0 because the normalization condition (4.18) holds. By (4.19) with $\alpha = 0$, we have

$$\begin{aligned} \mu xp - \frac{d}{dx} \left(\frac{\sigma^2 x^2}{2} p \right) &= (\mu - \sigma^2) xp - \frac{\sigma^2 x^2}{2} \frac{dp}{dx}, \\ &= 0 \end{aligned} \quad (4.20)$$

and thus

$$\frac{2(\mu - \sigma^2)}{\sigma^2} \frac{dx}{x} = (q - 1) \frac{dx}{x} = \frac{dp}{p}. \quad (4.21)$$

Integrating both hand-sides of (4.21), we have

$$p(x) = \gamma x^{q-1}, \quad 0 \leq x \leq \bar{x}. \quad (4.22)$$

From the normalization condition (4.18), the coefficient γ is determined as $\gamma = q\bar{x}^{-q}$.

Consequently, the stationary PDF p is derived as

$$p(x) = q\bar{x}^{-q} x^{q-1}, \quad 0 \leq x \leq \bar{x}. \quad (4.23)$$

For the case with $q \leq 0$, we have

$$p(x) = \Delta(0) \quad (4.24)$$

where $\Delta(x)$ is the Dirac's delta function at the point x . Combining the results, we have the PDF as

$$p(x) = \begin{cases} \Delta(0) & (q \leq 0) \\ q\bar{x}^{-q}x^{q-1} & (q > 0) \end{cases}. \quad (4.25)$$

The PDF (4.25) implies that the non-dimensional parameter q critically affects the behavior of the stochastic process X_t . When the deterministic growth rate μ is sufficiently small ($q \leq 0$), the PDF (4.25) reduces to a Dirac's function at the origin $x = 0$. In this case, the population will become extinct without any population management, and intensive countermeasures to suppress the predation are not required. In the next section, we examine a real case of $q > 0$ that gives the non-trivial polynomial PDF (4.25).

4.3.3 Average reduction in bird population numbers per unit time

This subsection introduces the average reduction in the bird population per unit time under the optimal control η^* . This quantity is denoted as I . Using SDE (4.1), I can be calculated as the followings. By its definition, I can be rewritten as

$$\begin{aligned} I &= \lim_{T \rightarrow \infty} \left(\frac{1}{T} \int_0^T d\eta_s^* \right) \\ &= \lim_{T \rightarrow \infty} \left[\frac{1}{T} \int_0^T (-dX_s + X_s(\mu ds + \sigma dB_s)) \right] \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} (X_0 - X_T) + \lim_{T \rightarrow \infty} \left[\frac{1}{T} \int_0^T (X_s(\mu ds + \sigma dB_s)) \right] \end{aligned} \quad (4.26)$$

As the quantity $X_0 - X_t$ with $t \geq 0$ is bounded by the optimal control η^* (6.7), (4.26) reduces to

$$I = \lim_{T \rightarrow \infty} \left(\frac{1}{T} \int_0^T \mu X_s ds \right) + \lim_{T \rightarrow \infty} \left(\frac{1}{T} \int_0^T \sigma X_s dB_s \right). \quad (4.27)$$

The second term of (4.27) converges to 0 with probability 1, as described in Chapter 3 of [14].

Consequently, we have

$$I = \lim_{T \rightarrow \infty} \left(\frac{1}{T} \int_0^T \mu X_s ds \right). \quad (4.28)$$

By Chapter 5 of [75], (4.28) can be expressed in terms of the PDF as

$$I = \mu \int_0^{\bar{x}} xp(x)dx. \quad (4.29)$$

The right-hand side of (4.29) is calculated as

$$I = \mu \int_0^{\bar{x}} xp(x)dx = \left(\mu - \frac{\sigma^2}{2} \right) \bar{x} = \frac{\sigma^2 q}{2} \bar{x}. \quad (4.30)$$

This index is used in Applications, which applies the present model to a realistic bird population management problem. The dependence of I on the parameters (the growth rate μ and the stochasticity σ) will be numerically investigated in that section.

4.3.4 Sensitivity analysis

To qualitatively clarify the dependence of the population threshold on the model parameters, this sub-section conducts a theoretical sensitivity analysis of the threshold $\bar{x}$. The results of the sensitivity analysis are useful for comprehending the properties of the optimal management plans. The quantitative dependence of $\bar{x}$ on the parameters is explored through the model application in the next section.

For the sensitivity analysis, we partially differentiate both sides of (4.13) with respect to the parameters r , R , μ and σ . A straightforward calculation yields the inequalities

$$\frac{\partial \bar{x}}{\partial r} < 0, \quad \frac{\partial \bar{x}}{\partial R} > 0, \quad \frac{\partial \bar{x}}{\partial \delta} > 0, \quad \frac{\partial \bar{x}}{\partial \mu} < 0, \quad \frac{\partial \bar{x}}{\partial \sigma} > 0, \quad (4.31)$$

where the last two inequalities follow when δ is sufficiently large. According to the sensitivity analysis results (4.31), the countermeasure-activation threshold $\bar{x}$ decreases with increasing loss of riverine fish to predation r and/or increasing growth rate μ of the bird population. Therefore, the countermeasure should target a lower level of the fish population in situations with severer predation. On the other hand, the threshold increases with increasing ecosystem services R provided by the bird, the discount rate δ , and/or the uncertainty σ . Hence, the decision-maker should carefully decide whether to implement the countermeasure when the birds sufficiently benefit the lake ecology. A larger discount rate increases the timeframe

of the countermeasure, consistent with the sensitivity analysis result. Regarding the uncertainty σ , the sensitivity analysis implies a larger threshold when the decision-maker has less deterministic information. In a practical context, if a parameter change increases the bird population, the threshold ($\frac{\partial \bar{x}}{\partial \mu} < 0$, $\frac{\partial \bar{x}}{\partial \sigma} > 0$) should be decreased, but if the parameter change increases the profit generated by the bird, the threshold ($\frac{\partial \bar{x}}{\partial r} < 0$, $\frac{\partial \bar{x}}{\partial R} > 0$) should be increased. Note that increasing σ decreases the population by geometric Brownian motion.

4.4 Applications

The present stochastic singular control model was applied to the population management of *P. carbo* in Lake Biwa, Japan, where the predation of riverine fish by this bird is the most severe in the country. This section evaluates the behavior and sustainability of the optimal management plans using the identified parameter values.

4.4.1 Study area

The study site is Lake Biwa, the largest freshwater lake in the Kinki area, Japan. With a water surface area of 674 (km²), this lake is an important habitat for many aquatic species and waterfowls. The following review on *P. carbo* around the lake is based on [81]. *P. carbo* is an indigenous species around Lake Biwa [49]. It has established one of its largest populations around the lake, with huge colonies at the Isaki Peninsula and the Chikubu Island. The *P. carbo* population around Lake Biwa has increased very rapidly, from five nests at the Chikubu Island in 1982 to approximately 1.2×10^4 individuals in 1998. Most of the individuals leave the lake in the non-breeding season (winter), but approximately 1.2×10^3 to 2.7×10^3 individuals remain around the lake. Meanwhile, *P. altivelis* yields the largest catch among the fishes in Lake Biwa, in terms of both weight and price. The rapid increasing *P. carbo* population is regarded as a severe problem for the inland fisheries around the study area because this species majorly preys on *P. altivelis* in the spring and summer seasons. It also consumes a wide range of riverine fish around the study area. When interviewed, the members of Hirose Fisheries

Cooperatives of Lake Biwa mentioned that long-term countermeasures against predation by *P. carbo* are currently being enforced; otherwise, the increasing bird population could not be effectively suppressed. The members are also prioritizing the loss of riverine fish over the ecosystem services provided by the bird. The main countermeasure at Lake Biwa is shooting by licensed hunters. The same countermeasure is mainly employed by several other fisheries cooperatives in Japan.

4.4.2 Determination of parameter values

The two SDE parameters μ and σ are identified from the yearly public data of the *P. carbo* populations around Lake Biwa. These data are available at Shiga Prefecture [72] and presented in Fig 4.2. For parameter determination, we used the bird population data from 1994 to 2003, because Shiga Prefecture (2007) [72] reported successful *P. carbo* control (since 2004) by their population management plan. The population in year n , where $n = 1$ (1994) to $n = N = 10$ (2003), is denoted as $X^{(n)}$. The study period (1994–2003) was chosen because no countermeasures had been implemented in Lake Biwa until 2003. The uncontrolled population dynamics of *P. carbo* were required for identifying the parameter values in our model. Using a linear regression approach, the parameters μ and σ were determined as

$$\mu = \frac{\sum_{n=1}^{N-1} (X^{(n+1)} - X^{(n)})}{\sum_{n=1}^N X^{(n)}} = 1.7 \times 10^{-1} \quad (1/\text{year}) \quad (4.32)$$

and

$$\sigma^2 = \frac{\sum_{n=1}^{N-1} (X^{(n+1)} - X^{(n)})^2}{\sum_{n=1}^N (X^{(n)})^2} = 2.8 \times 10^{-1} \quad (1/\text{year}) \quad (4.33)$$

respectively. The identified μ and σ satisfy the condition $q > 0$ assumed in (4.2).

4.4.3 Sustainable bird population management plans

Based on the management plan of *P. carbo* in Shiga Prefecture [73], the population-control threshold was set to $\bar{x} = 4,000$ (-). The management policy was considered sustainable if the threshold satisfied $\bar{x} > 0$. Whereas μ and σ have ecological meanings, the parameters m , M ,

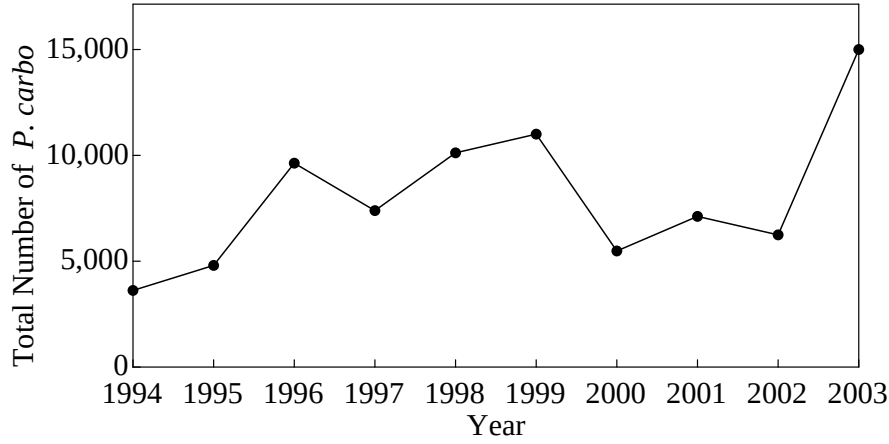


Fig. 4.2: Historical data of *P. carbo* population around Lake Biwa [72].

r , R , and δ depend on the attitude of the decision-maker, so their values should depend on the attitude of the decision-maker on the population management. Different attitudes thus result in different management policies. For example, when R is large, the decision-maker prioritizes the ecosystem services provided by *P. carbo*, whereas large r means that the decision-maker mostly avoids the riverine-fish loss to *P. carbo* predation. For a brief and simple analysis, we set $m = 2.0$ and $M = 0.5$. Other values of m and M satisfying $0 < M < 1 < m \leq 2$ were preliminarily found to give the same qualitative results.

Figures 4.3-4.7 show the threshold $\bar{x}$ in the R - r plane for different values of δ (1/year). The parameters r and R range from 10^{-4} (-) to 10^4 (-), and $\bar{x}$ for each (R, r) pair was numerically computed from (4.13) by a classical Picard iteration method. In all plots, the black line represents the current threshold set at Lake Biwa, namely, $\bar{x} = 4,000$ (-). In Figure 4.4 (the case of small δ), the threshold $\bar{x}$ in the white regions is 0 (-), and conditions (4.5) are not satisfied. Within this white region, the optimal management strategy is the immediate utter extermination of the bird population, which is not sustainable. Moreover, in the regions corresponding to $\bar{x} \approx 0$ (-) (green and blue regions), no virtually sustainable management policy is possible because X_t represents the total number of birds in the habitat. However, in the region corresponding to $\bar{x} \neq 0$ (-) (red region), a sustainable management policy can

be found. In these figures, some $\bar{x}$ exceed 10^5 (-). This result occurs because the governing equation of X_t is an SDE based on geometric Brownian motion, which (unlike the logistic models) imposes no upper bound on the population numbers [33]. As shown in Figures 4.3-4.7, $\bar{x}$ increases with increasing R and δ , and decreases with increasing r . The red region contracts as δ decreases, indicating that the solution space of sustainable management policies decreases as the decision-maker shortens the time frame of the countermeasure. The computational results are consistent with the sensitivity analysis results. In addition, $\bar{x}$ is more sensitive to r than R , implying that a management policy against the *P. carbo* population is more significantly influenced by the loss of riverine fish than by the ecosystem services provided by the bird. Therefore, to sustainably manage the bird population, decision-makers should soften their attitude toward the loss of riverine fish and should adopt a long-term perspective for the management plans. Meanwhile, when δ is small, we find that $\bar{x}$ depends on R (when R is sufficiently large). This result implies that if the bird population is managed from a short-term perspective, the ecosystem services are important for achieving sustainable management.

According to Figures 4.3-4.7, if the current threshold $\bar{x} = 4,000$ is optimal, the loss of riverine fish should be sufficiently small ($r \approx 10^{-4} \sim 10^{-2}$), and the r that optimizes the current threshold should increase with increasing discount rate δ . In addition, the ecosystem services should be sufficiently large ($R \approx 10^2 \sim 10^4$) if the discount rate δ is small. Based on the obtained analysis results, optimal solutions for the current management plans can be recommended. The loss of riverine fish should be considered in any case. However, if the decision-maker manages the bird population from a short-term perspective, the ecosystem services should be sufficiently large as well. In addition, the analysis results imply that the optimal threshold is smaller than the current value $\bar{x} = 4,000$ (-) inferred by interviewing the members of Hirose Fisheries Cooperatives. In their plans, the loss of riverine fish r is large and the ecosystem service R is relatively small.

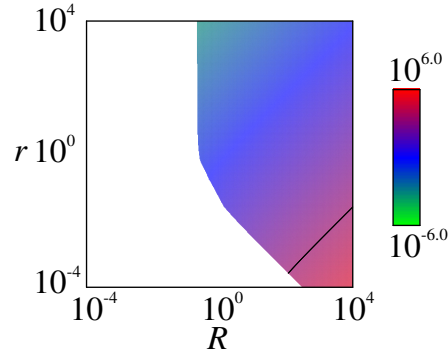


Fig. 4.3: Threshold $\bar{x}$ in (R, r) plane with $\delta = 1.0 \times 10^{-2}$ (1/year). No $\bar{x}$ exists in the white regions. Black lines represent the current threshold in Lake Biwa.

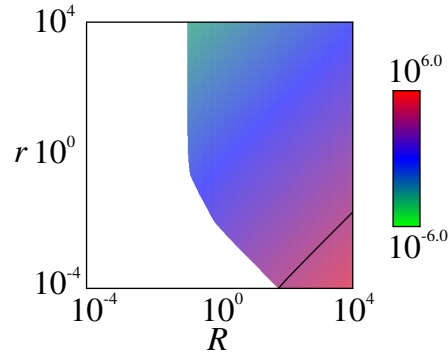


Fig. 4.4: Threshold $\bar{x}$ in (R, r) plane with $\delta = 1.0 \times 10^{-1}$ (1/year). No $\bar{x}$ exists in the white regions. Black lines represent the current threshold in Lake Biwa.

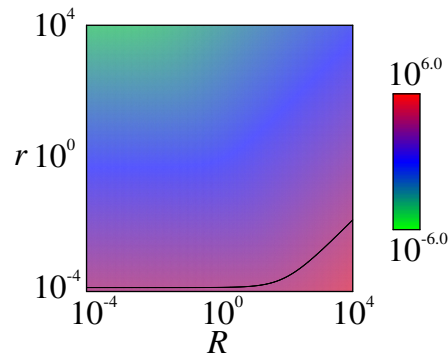


Fig. 4.5: Threshold $\bar{x}$ in (R, r) plane with $\delta = 1.0 \times 10^0$ (1/year). No $\bar{x}$ exists in the white regions. Black lines represent the current threshold in Lake Biwa.

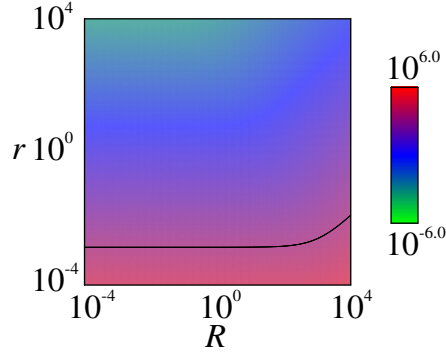


Fig. 4.6: Threshold $\bar{x}$ in (R, r) plane with $\delta = 1.0 \times 10^1$ (1/year). No $\bar{x}$ exists in the white regions. Black lines represent the current threshold in Lake Biwa.

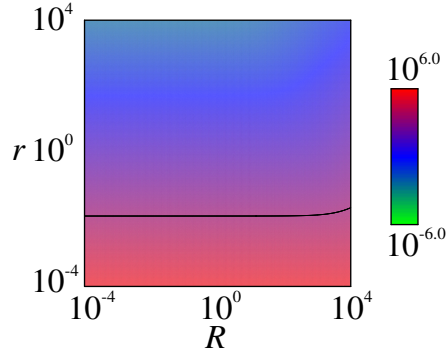


Fig. 4.7: Threshold $\bar{x}$ in (R, r) plane with $\delta = 1.0 \times 10^2$ (1/year). No $\bar{x}$ exists in the white regions. Black lines represent the current threshold in Lake Biwa.

4.4.4 Average reduction in bird population numbers per unit time around Lake Biwa

Finally, the parameter dependence of the average reduced bird population I per unit time, reported in (4.30), was investigated at Lake Biwa. Of particular interest is the dependence of I on the parameters μ and σ in the population dynamics (4.1). The examined ranges of μ and σ were varied as 0.05 (1/year) to 0.35 (1/year), and as 0.1 (1/year^{1/2}) to 0.8 (1/year^{1/2}), respectively. The average reduced bird population I was calculated for each (σ, μ) pair by (4.30). Figure 4.8 shows the calculated I in the (σ, μ) plane for the parameters $r = 1.0 \times 10^{-2}$ (-), $R = 1.0 \times 10^2$ (-), and $\delta = 1.0 \times 10^{-1}$ (1/year). As the profiles are fairly insensitive to choice of δ , we set $\delta = 1.0 \times 10^{-1}$ (1/year). Condition (4.2) is violated in the white region. The computational results indicate that I increases with respect to μ and decreases with respect to

σ .

Assuming that the current threshold in Lake Biwa, $\bar{x} = 4,000$ (-), is optimal, the average reduced bird population I per year is calculated as

$$I = \left(\mu - \frac{\sigma^2}{2} \right) \bar{x} = 1.2 \times 10^2 \quad (1/\text{year}). \quad (4.34)$$

According to the previous subsection, which discussed the optimality of the current management policy, this result may be an overestimate. Therefore, the truly optimal I might be smaller than that calculated in (4.34).

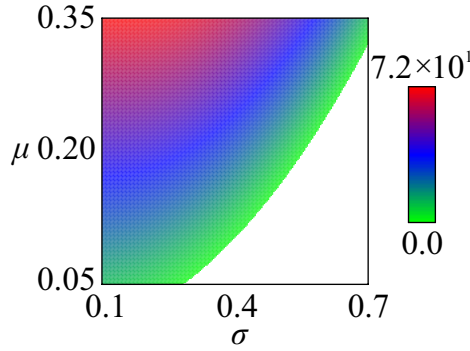


Fig. 4.8: Average reduction in bird population numbers per unit time I around Lake Biwa in the (σ, μ) plane. The parameters are $r = 1.0 \times 10^{-2}$, $R = 1.0 \times 10^2$, and $\delta = 1.0 \times 10^{-1}$ (1/year). No I exists in the white region.

4.5 Conclusions

This chapter proposed and analyzed an exactly-solvable stochastic singular control model that finds a sustainable, cost-effective, and ecologically-sound management policy for the population dynamics of *P. carbo*. The model and its determined parameter values were applied to the management of the *P. carbo* population in Lake Biwa, Japan. The current management plans around the lake were analyzed and verified from the viewpoint of the present mathematical model. A recommendation for optimizing the current management plans was made. The optimal policy should minimize the loss of riverine fish without compromising the ecosystem services provided by the birds.

CHAPTER 5

A stochastic impulse control model for management of *P. carbo*

5.1 Introduction

P. carbo is a colonial fish-eating bird distributed worldwide and causes feeding damage to *P. altivelis*. The establishment of a cost-effective and ecologically conscious management policy of the bird population is an urgent issue in Japan. In order to approach the above-mentioned issue, a stochastic control approach [70] based on a stochastic impulse control formalism [13, 63, 86] is employed. The stochastic impulse control problem aims at controlling a system driven by stochastic fluctuations, such as a diffusion process, through impulsive interventions so that a performance index is maximized. In a typical stochastic impulse control problem, the resulting optimal policy is of a threshold type taking an intervention only when the state variables reach a threshold. In the author's previous study [96], an exact solution of an HJBQVI was used because the HJBQVI was not so complicated. However, numerically solving HJBQVIs that are more complicated will be required when dealing with problems that are more realistic since such problems may not have exact solutions.

The objective of this chapter is the development of a stable numerical method for solving HJBQVIs associated with a stochastic impulse control model for *P. carbo* population management. The numerical methods are based on a finite element scheme with the penalty method and the policy iteration method and its full discretizations are presented in this chapter. The

performance of the schemes is examined against an exact solution, demonstrating satisfactory accuracies of the proposed schemes.

The remainder of this chapter is structured as follows. Section 2 presents the mathematical model and derives the HJBQVI. Section 3 proposes numerical schemes for the discretization of the HJBQVI, and Section 4 investigates the validity of the schemes based on an exact solution. Section 5 concludes the chapter.

5.2 Mathematical model

5.2.1 Population dynamics of *P. carbo*

An infinite-horizon management problem of the *P. carbo* population in a habitat is considered. We assume that the decision-maker, the manager of the *P. carbo* population, can reduce the *P. carbo* population through countermeasures, and also assume that they are carried out in a much shorter timescale than that of the *P. carbo* population dynamics, so that the impulse control formalism is justified. The population in the habitat at the time t is denoted as X_t , which is governed by an SDE with the fluctuation term defined in the Itô' s sense:

$$\begin{cases} dX_t = (\mu dt + \sigma dB_t), & \tau_i \leq t < \tau_{i+1} \\ X_{\tau} = X_{\tau_i-} - \zeta_i \end{cases} \quad (5.1)$$

with an initial condition $X_{0-} > 0$ where $\mu > 0$ is the intrinsic growth rate, $\sigma > 0$ is the magnitude of stochastic fluctuation involved in the population dynamics, B_t is the 1-D standard Brownian motion defined on a usual complete probability space [66], τ_i ($i = 0, 1, 2, \dots$, $\tau_0 = 0$) is the time when the countermeasure is performed to reduce the population and $\zeta_i > 0$ represents the magnitude of the countermeasure, the total population of the killed *P. carbo*, at the time τ_i . Whenever the countermeasure is performed, we assume the cost

$$K(\zeta_i) = k_1 \zeta_i + k_0 \quad (\zeta_i > 0) \quad (5.2)$$

where $k_1 > 0$ is the cost proportional to ζ and $k_0 > 0$ is the fixed cost incurred regardless of ζ_i . In this model, both the timing τ_i and the magnitude of the countermeasure ζ_i are the control variables to be optimized.

5.2.2 Performance index

The performance index J is the expected net profit of the decision-maker subject to a discount:

$$J(x, \eta) = E \left[\int_0^\infty e^{-\delta s} (uX_s^M - dX_s^m) ds - \sum_{i=0}^\infty e^{-\delta s} K(\zeta_i) \right] \quad (5.3)$$

with

$$0 < M < 1 < m \leq 2, \quad (5.4)$$

where $\delta > 0$ is the discount rate, u , d , M and m are the positive constants. In the right-hand side of (5.3), the term uX_s^M represents the ecological utility provided by the existence of the *P. carbo*, the term $-dX_s^m$ represents the disutility by the existence of the *P. carbo* and the last summation term is the cost of the countermeasure. The net profit $uX_s^M - dX_s^m$ is not monotone but concave with respect to X_s . The relationship (6.6) means that the increase rate of the disutility by the existence of the animal is higher than that of the utility by the existence of the animal, and the condition $0 < M < 1$ in (6.6) intrinsically means that the ecological services drastically change between the states where the population exists and where the population does not exist. This consideration is represented by the divergence of the derivative of uX_s^M with respect to X_s at the origin $X_s = 0$.

Here, η represents the management policy defined by the timing and magnitude of the countermeasure

$$\eta = (\tau_i, \zeta_i)_{i \geq 0} \quad (5.5)$$

and is called an admissible control if it satisfies the conditions stated in Definition 2.1 in [63], which are listed up in what follows:

$$0 \leq \tau_i < \tau_{i+1}, \text{ a.s. } i \geq 0, \quad (5.6)$$

$$\tau_i \text{ is an } \{\mathcal{F}_t\}_{t \geq 0} \text{ stopping time, } i \geq 0, \quad (5.7)$$

$$\zeta_i \text{ is } \mathcal{F}_{\tau_i} \text{ measurable, } i \geq 0, \quad (5.8)$$

$$P \left[\lim_{i \rightarrow \infty} \tau_i \leq T \right] = 0, \forall T \in [0, \infty), \quad (5.9)$$

and

$$X_t \geq 0, t \geq 0. \quad (5.10)$$

5.2.3 Hamilton-Jacobi-Bellman quasi-variational inequality

Let $\mathcal{A}$ be the set of the admissible controls. The objective of the decision-maker is to find the optimal control η^* such that

$$V(x) = \sup_{\eta \in \mathcal{A}} J(x; \eta) = J(x; \eta^*), \quad (5.11)$$

where $V = V(x)$ is the maximized performance index referred to as the value function. The dynamic programming principle [70] leads to a HJBQVI [13, 63, 86] as

$$\max \left(\mathcal{L}V + ux^M - dx^m, \mathcal{M}V - V \right) = 0, \quad x > 0 \quad (5.12)$$

with the differential operator $\mathcal{L}$ and the intervention operator $\mathcal{M}$ defined as

$$\mathcal{L}W = \frac{1}{2}\sigma^2 x^2 \frac{d^2 W}{dx^2} + \mu x \frac{dW}{dx} - \delta W \quad (5.13)$$

and

$$\mathcal{M}W = \sup_{\zeta} [-K(\zeta) + W(x - \zeta)], \quad (5.14)$$

respectively, for generic sufficiently smooth $W = W(x)$. The boundary condition for the HJBQVI is $V(0) = 0$, which means that no problem arises when there is no population. The left part in the max operator in the HJBQVI (6.13) corresponds to the situation where no countermeasure should be performed, while the right part corresponds to the situation where the countermeasure should be performed immediately.

5.2.4 Optimal control

Under the QVI controls (Definition 3.2 in [13])

$$(\tau_0, \zeta_0) = (0, 0), \quad (5.15)$$

$$\tau_i = \inf [t > \tau_{i-1} : V(X_t) = \mathcal{M}V(X_t)], \quad i \geq 0, \quad (5.16)$$

and

$$\zeta = \arg \sup_{\zeta} [-K(\omega) + V(X_{\tau_i} - \omega) : \omega \in \mathbb{R}, X_{\tau_i} - \omega \in (0, \infty)], \quad i \geq 0, \quad (5.17)$$

the following threshold-type management policy with the threshold values $\bar{x}$ and $\underline{x}$ ($\underline{x} < \bar{x}$) is found to be optimal (Verification theorem, Theorem 3.1 in [13]):

- (A) If $X_{t-} < \bar{x}$, then no intervention is performed. If $X_{t-} = \bar{x}$, the intervention is immediately performed and X_{t-} is reduced to $\underline{x}$ ($X_t = \underline{x}$).
- (B) If $X_{0-} > \bar{x}$, then X_{0-} is immediately reduced to $\underline{x}$ ($X_0 = \underline{x}$) by the intervention, and follows (A).

With the thresholds $\bar{x}$ and $\underline{x}$, the optimal amount of the intervention ζ^* is

$$\zeta^* = \begin{cases} x - \underline{x} & (x \leq \bar{x}) \\ 0 & (x > \bar{x}) \end{cases}. \quad (5.18)$$

A sample path of the process X_t following the above-mentioned policy is plotted in Figure.5.1. Note that Cadenillas (1999) [13] proved the verification theorem (Theorem 3.1 in the literature), which states that the solution to HJBQVI (6.13) under the QVI control (5.15) through (5.17) is in fact the value function (5.11) and the QVI control is the optimal impulse control. Thus, by solving the HJBQVI (6.13), the optimal impulse control can be obtained.

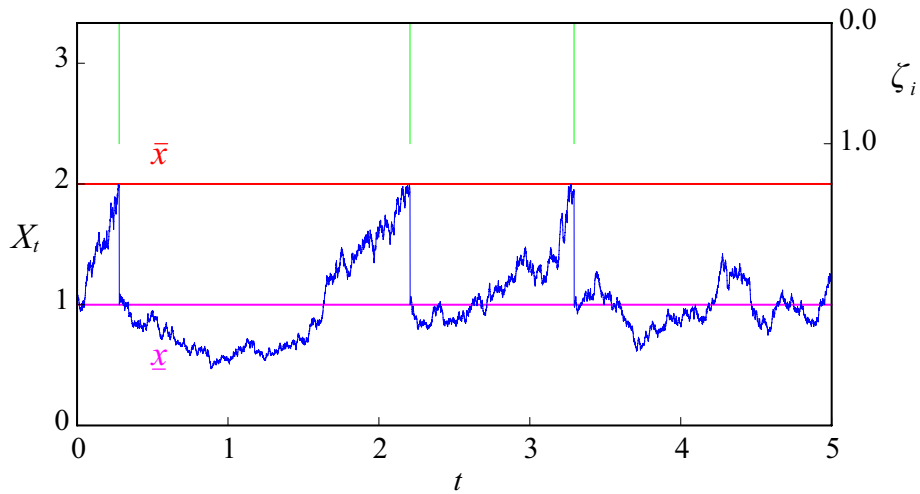


Fig. 5.1: The process X_t under the optimal management policy is blue line. Upper threshold is written in red line and lower threshold in pink line. The optimal magnitude of the killed bird ζ_i is shown as green line.

5.3 Exact solution

5.3.1 Exact solution form

In this section, we present a candidate of the solution to the HJBQVI (6.13), which later turns out to be a viscosity solution. We assume the following conditions to derive a non-trivial exact viscosity solution [96, 98]:

$$\mu > \frac{\sigma^2}{2}, \quad (5.19)$$

$$\delta - \mu m - \frac{1}{2}\sigma^2 m(m-1) > 0, \quad (5.20)$$

and

$$\beta = \frac{1}{2} \left(1 - \frac{2\mu}{\sigma^2} + \sqrt{\left(\frac{2\mu}{\sigma^2} - 1 \right)^2 + \frac{8\delta}{\sigma^2}} \right) > 2. \quad (5.21)$$

The inequality (5.19) means that the animal population does not become extinct without interventions. The relationship (5.20) and (5.84) means that the decision-maker manages the animal population from the sufficiently long-term perspectives.

Here, a formal exact solution to the HJBQVI (6.13) is presented, which is inspired from the impulse control models in the literature [63, 13, 65]. Let V be a candidate of the solution to the HJBQVI (6.13). At first, we consider the situation with $x < \bar{x}$, under which no countermeasure should be performed. In the HJBQVI (6.13), the equality [86]

$$\mathcal{L}V + ux^M - dx^m = 0 \quad (5.22)$$

has a solution

$$V(x) = a_1 x^{\beta_+} + a_2 x^{\beta_-} + A_1 x^M + A_2 x^m, \quad (5.23)$$

where

$$\beta_{\pm} = \frac{1}{2} \left(1 - \frac{2\mu}{\sigma^2} \pm \sqrt{\left(\frac{2\mu}{\sigma^2} - 1 \right)^2 + \frac{8\delta}{\sigma^2}} \right), \quad (5.24)$$

$$A_1 = \frac{u}{\delta - \mu M - \frac{\sigma^2}{2} M(M-1)} > 0, \quad (5.25)$$

and

$$A_2 = \frac{-d}{\delta - \mu m - \frac{\sigma^2}{2}m(m-1)} < 0. \quad (5.26)$$

In (5.23), $a_1 x^{\beta_+} + a_2 x^{\beta_-}$ is the general solution to $\mathcal{L}V = 0$ and $A_1 x^M + A_2 x^m$ is a particular solution to (5.22). Assume $a_2 \neq 0$, then we have $\lim_{x \rightarrow +0} a_2 x^{\beta_-} = \pm\infty$ since $\beta_- < 0$. However, this contradicts the boundary condition $V(0) = 0$. Hence, $a_2 = 0$ holds. At last, the solution (5.23) becomes

$$V(x) = ax^{\beta} + A_1 x^M + A_2 x^m \quad (5.27)$$

where the notations $a_1 = a$ and $\beta_+ = \beta$ are used.

For $x \geq \bar{x}$ where the countermeasure should be performed immediately, by the HJBQVI (6.13), the condition

$$\begin{aligned} V(x) &= -k_1(x - \underline{x}) - k_0 + V(\underline{x}) \\ &= -k_1(x - \underline{x}) - k_0 + ax^{\beta} + A_1 x^M + A_2 x^m, \end{aligned} \quad (5.28)$$

has to be satisfied. Consequently, a candidate of solutions to HJBQVI (6.13) is found as

$$V(x) = \begin{cases} ax^{\beta} + A_1 x^M + A_2 x^m & (x < \bar{x}) \\ -k_1(x - \underline{x}) - k_0 + ax^{\beta} + A_1 x^M + A_2 x^m & (x \geq \bar{x}) \end{cases} \quad (5.29)$$

where a , $\bar{x}$ and $\underline{x}$ are unknown at this stage.

Assume the following condition in the solution $V(x)$ to determine its unknown coefficients [63, 86]:

- (a) Continuity of $V(x)$ at $x = \bar{x}$ (Value matching),
- (b) Continuity of $V'(x)$ at $x = \bar{x}$ (Smooth pasting),
- (c) Optimality of the thresholds $\bar{x}$ and $\underline{x}$ in (6.15).

Firstly, from (a), we have

$$V(\bar{x}) = -k_1(\bar{x} - \underline{x}) - k_0 + V(\underline{x}). \quad (5.30)$$

From (b), we have

$$\begin{aligned} V'(\bar{x}) &= \lim_{x \downarrow \bar{x}} \frac{d}{dx} [-k_1(x - \underline{x}) - k_0 + V(\underline{x})] \\ &= -k_1 \end{aligned} \quad (5.31)$$

Finally, from (c), we have

$$V(\bar{x}) = \sup [-k_1(\bar{x} - y) - k_0 + V(y)], \quad (5.32)$$

namely, we have

$$\left. \frac{d}{dx} [-k_1(\bar{x} - y) - k_0 + V(y)] \right|_{y=\bar{x}} = 0. \quad (5.33)$$

Substituting (5.27) into (5.30), (5.31), and (5.33), the following system of nonlinear equations governing the coefficients is derived:

$$\begin{cases} a\bar{x}^\beta + A_1\bar{x}^M + A_2\bar{x}^m = -k_1(\bar{x} - \underline{x}) - k_0 + a\underline{x}^\beta + A_1\underline{x}^M + A_2\underline{x}^m \\ a\beta\bar{x}^{\beta-1} + A_1M\bar{x}^{M-1} + A_2m\bar{x}^{m-1} = -k_1 \\ a\beta\underline{x}^{\beta-1} + A_1M\underline{x}^{M-1} + A_2m\underline{x}^{m-1} = -k_1 \end{cases} \quad (5.34)$$

where a , $\bar{x}$ and $\underline{x}$ are unknown. According to the conditions (a) and (b), the solution (5.29) is $V \in C^1(0, \infty)$ and is twice differentiable except at $x = \bar{x}$; namely, we have $V \in C^2((0, \bar{x}) \cup (\bar{x}, \infty))$. Thus, the solution V is not a classical solution but a solution a.e.

Note that the exact solution (5.29) has an explicit upper- and lower-bounds. For obtaining these bounds, at first, an SDE without controls

$$dX_t = \mu dt + \sigma dB_t, \quad X_0 = x \quad (5.35)$$

is considered, and $E[e^{\gamma X_t}]$ is calculated as

$$\begin{aligned} E[e^{\gamma X_t}] &= \int_{-\infty}^{\infty} e^{\gamma z} dz \\ &= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{(z-(x+\mu t))^2}{2}} e^{\gamma z} dz \\ &= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{\gamma^2}{2}} e^{\gamma(\sigma y + x + \mu t)} \sigma dy \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(y-\sigma\gamma)^2} e^{\left\{\gamma(x+\mu t) + \frac{\sigma^2\gamma^2}{2}\right\}} dy \\ &= e^{\left\{\gamma(x+\mu t) + \frac{\sigma^2\gamma^2}{2}\right\}} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(y-\sigma\gamma)^2} dy \\ &= e^{\left\{\gamma(x+\mu t) + \frac{\sigma^2\gamma^2}{2}\right\}} \end{aligned} \quad (5.36)$$

because X follows $N(x + \mu t, \sigma^2 t)$. In (5.36), a transformation $y = \frac{z - (x + \mu t)}{\sigma}$ is performed, and $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(y - \sigma\gamma)^2} dy = 1$ holds true because it is an integration of a normal distribution with the mean $\sigma\gamma$ and the variance 1 from $-\infty$ to ∞ . Next, $E \left[\int_0^{\infty} e^{-\delta t} e^{\gamma X_t} dt \right]$ is calculated as

$$\begin{aligned}
 E \left[\int_0^{\infty} e^{\gamma X_t} dt \right] &= \int_0^{\infty} e^{-\delta t} E \left[e^{\gamma X_t} \right] dt \\
 &= \int_0^{\infty} e^{-\delta t} e^{\left\{ \gamma(x + \mu t) + \frac{1}{2} \sigma^2 t \gamma^2 \right\}} dt \\
 &= e^{\gamma x} \int_0^{\infty} e^{-(\delta - \mu\gamma - \frac{1}{2} \sigma^2 \gamma^2)t} dt \\
 &= - \frac{e^{\gamma x}}{\delta - \mu\gamma - \frac{1}{2} \sigma^2 \gamma^2} \left[e^{-(\delta - \mu\gamma - \frac{1}{2} \sigma^2 \gamma^2)t} \right]_0^{\infty} \\
 &= - \frac{e^{\gamma x}}{\delta - \mu\gamma - \frac{1}{2} \sigma^2 \gamma^2} (0 - 1) \\
 &= \frac{e^{\gamma x}}{\delta - \mu\gamma - \frac{1}{2} \sigma^2 \gamma^2}
 \end{aligned} \tag{5.37}$$

where (5.36) is used. Next, a process Y_t which follows

$$dY_t = \mu Y_t dt + \sigma Y_t dB_t, \quad Y_0 = y \tag{5.38}$$

is considered. A transformation

$$X_t = \ln Y_t = f(Y_t, t) \tag{5.39}$$

is introduced. Applying Itô's lemma [66] to (5.38) yields

$$\begin{aligned}
 dX_t = df &= \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial Y_t} dY_t + \frac{1}{2} \frac{\partial^2 f}{\partial Y_t^2} (dY_t)^2 \\
 &= \frac{1}{Y_t} (\mu Y_t dt + \sigma Y_t dB_t) - \frac{1}{2 Y_t^2} (\mu Y_t dt + \sigma Y_t dB_t)^2 \\
 &= \mu dt + \sigma dB_t - \frac{1}{2} \sigma^2 dt \\
 &= \left(\mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dB_t
 \end{aligned} \tag{5.40}$$

namely,

$$dX_t = \left(\mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dB_t. \tag{5.41}$$

From the transformation (5.39),

$$e^{\gamma X_t} = Y_t^\gamma \quad (5.42)$$

holds true. By applying (5.37) to the SDE (5.41),

$$\begin{aligned} \mathbb{E} \left[\int_0^\infty e^{-\delta t} Y_t dt \right] &= \mathbb{E} \left[\int_0^\infty e^{-\delta t} e^{\gamma X_t} dt \right] \\ &= \frac{e^{\gamma x}}{\delta - \left(\mu - \frac{1}{2} \sigma^2 \right) \gamma - \frac{1}{2} \sigma^2 \gamma^2} \\ &= \frac{y^\gamma}{\delta - \mu \gamma - \frac{1}{2} \sigma^2 \gamma (\gamma - 1)} \end{aligned} \quad (5.43)$$

holds true. Finally, the upper- and lower-bounds are calculated. By considering no control case, from the definition of the value function,

$$\mathbb{E} \left[\int_0^\infty e^{-\delta s} \left(u X_s^M - d X_s^m \right) ds \right] \leq V(x) \quad (5.44)$$

is obtained. From (5.43),

$$\begin{aligned} \mathbb{E} \left[\int_0^\infty e^{-\delta s} \left(u X_s^M - d X_s^m \right) ds \right] &= \frac{u x^M}{\delta - \mu M - \frac{1}{2} \sigma^2 M(M-1)} + \frac{-d x^m}{\delta - \mu m - \frac{1}{2} \sigma^2 m(m-1)} \\ &= A_1 x^M + A_2 x^m \end{aligned} \quad (5.45)$$

is obtained. Because for any X_t ,

$$u X_s^M - d X_s^m \leq u X_s^M, \quad (5.46)$$

the inequality

$$\int_0^\infty e^{-\delta s} \left(u X_s^M - d X_s^m \right) ds \leq \int_0^\infty e^{-\delta s} u X_s^M ds \quad (5.47)$$

holds true, and

$$\begin{aligned} \int_0^\infty e^{-\delta s} \left(u X_s^M - d X_s^m \right) ds - \sum_{i=1}^\infty e^{-\delta \tau_i} K(\zeta_i) &\leq \int_0^\infty e^{-\delta s} u X_s^M ds - \sum_{i=1}^\infty e^{-\delta \tau_i} K(\zeta_i) \\ &< \int_0^\infty e^{-\delta s} u X_s^M ds \end{aligned} \quad (5.48)$$

also holds true. Taking expectation on both hand side of (5.48) yields

$$\begin{aligned} J(x; \eta) &= E \left[\int_0^\infty e^{-\delta s} \left(uX_s^M - dX_s^m \right) - \sum_{i=1}^\infty e^{-\delta \tau_i} K(\zeta_i) \right] \\ &< E \left[\int_0^\infty e^{-\delta s} uX_s^M \right] \\ &= A_1 x^M \end{aligned} \quad (5.49)$$

Taking sup on the left hand side of (5.49) yields

$$\begin{aligned} V(x) &= \sup_{\eta \in \mathcal{A}} J(x; \eta) \\ &= \sup_{\eta \in \mathcal{A}} E \left[\int_0^\infty e^{-\delta s} \left(uX_s^M - dX_s^m \right) - \sum_{i=1}^\infty e^{-\delta \tau_i} K(\zeta_i) \right] \\ &= A_1 x^M \end{aligned} \quad (5.50)$$

Combining (5.45) and (5.50) yields the final result

$$A_1 x^M + A_2 x^m \leq V(x) \leq A_1 x^M. \quad (5.51)$$

5.3.2 Coefficients in exact solution form

Subtracting the middle equation in (5.34) from the below equation in (5.34) and solving for a yields

$$a = \frac{-A_1 M(\bar{x}^{M-1} - \underline{x}^{M-1}) - A_2 m(\bar{x}^{m-1} - \underline{x}^{m-1})}{\beta(\bar{x}^{\beta-1} - \underline{x}^{\beta-1})}. \quad (5.52)$$

Substituting (5.52) into the middle equation of (5.34) leads to

$$k_1 + \left[\frac{-A_1 M(\bar{x}^{M-1} - \underline{x}^{M-1}) - A_2 m(\bar{x}^{m-1} - \underline{x}^{m-1})}{\beta(\bar{x}^{\beta-1} - \underline{x}^{\beta-1})} \right] \beta \bar{x}^{\beta-1} + A_1 M \bar{x}^{M-1} + A_2 m \bar{x}^{m-1} = 0, \quad (5.53)$$

and substituting (5.52) into the above equation of (5.34) then yields

$$\begin{aligned} &-k_1(\bar{x} - \underline{x}) - k_0 + A_1(\underline{x}^M - \bar{x}^M) + A_2(\underline{x}^m - \bar{x}^m) \\ &+ \left[\frac{-A_1 M(\bar{x}^{M-1} - \underline{x}^{M-1}) - A_2 m(\bar{x}^{m-1} - \underline{x}^{m-1})}{\beta(\bar{x}^{\beta-1} - \underline{x}^{\beta-1})} \right] (\underline{x}^\beta - \bar{x}^\beta) = 0. \end{aligned} \quad (5.54)$$

By numerically solving the nonlinear equations (5.53) and (5.54), $\bar{x}$ and $\underline{x}$ would be found.

In this chapter, the nonlinear equations (5.53) and (5.54) with an artificial temporal term are

proposed as the system of ODEs, a dynamical system, as

$$\frac{d\mathbf{x}}{dt} = \mathbf{f} \text{ with } \mathbf{x} = \begin{pmatrix} \bar{x} \\ \underline{x} \end{pmatrix}, \quad (5.55)$$

where

$$\begin{aligned} \mathbf{f} &= \begin{pmatrix} f_1(\bar{x}, \underline{x}) \\ f_2(\bar{x}, \underline{x}) \end{pmatrix} \\ &= \begin{pmatrix} k_1(\bar{x} - \underline{x}) + k_0 - h(\underline{x}^\beta - \bar{x}^\beta) - A_1(\underline{x}^M - \bar{x}^M) - A_2(\underline{x}^m - \bar{x}^m) \\ k_1 + h\beta\bar{x}^{\beta-1} + A_1M\bar{x}^{M-1} + A_2m\bar{x}^{m-1} \end{pmatrix} \end{aligned} \quad (5.56)$$

and

$$h(\bar{x}, \underline{x}) = \frac{-A_1M(\bar{x}^{M-1} - \underline{x}^{M-1}) - A_2m(\bar{x}^{m-1} - \underline{x}^{m-1})}{\beta(\bar{x}^{\beta-1} - \underline{x}^{\beta-1})}. \quad (5.57)$$

Let $\bar{x} = \bar{x}^*$ and $\underline{x} = \underline{x}^*$ satisfy (5.34). Then, we have

$$h(\bar{x}, \underline{x}) = a. \quad (5.58)$$

The system (5.56) is numerically integrated until the quasi-steady state is attained with the forward Euler method. The coefficient a can be calculated with (5.57) using the obtained $\bar{x}^*$ and $\underline{x}^*$. Thus, $V(x)$ (5.29) can be obtained. Note that the system (5.55) can be proved to be asymptotically stable, meaning that the sequence of the system converges to $\bar{x}^*$ and $\underline{x}^*$ when an initial guess is sufficiently close to $\bar{x}^*$ and $\underline{x}^*$.

5.3.3 Numerical results

The system (5.56) is discretized with the conventional forward Euler method. The parameters are set as $\mu = 1.7 \times 10^{-1}$ (1/year), $\sigma = 5.3 \times 10^{-1}$ (1/year^{1/2}), $m = 1.8$ (-), $M = 0.5$ (-), $\delta = 1.0$ (1/year), $d = 5.0 \times 10^{-4}$ (-), $u = 1.0 \times 10^{-1}$ (-), $k_0 = 1.0 \times 10^1$ (-) and $k_1 = 1.0 \times 10^0$ (-) [97]. These parameters satisfy the assumptions (5.19) through (5.21). The increment for the time integration is set as $\Delta t = 0.01$ for stable temporal evolution of the system. The time integration marches until the error is under the error tolerance as

$$\text{error} = \max_x |x^{n+1} - x^n| < \epsilon \quad (5.59)$$

where n represents the time step and $\epsilon = 10^{-10}$ is the error tolerance. With the computed $\bar{x}$ and $\underline{x}$, a is calculated by (5.57). The initial estimations for the thresholds are set as $\bar{x} = 7500$ and

$\underline{x} = 5500$. The computed thresholds are $\bar{x}^* = 7352.93$ and $\underline{x}^* = 5757.82$. The computed a^* is $a^* = 4.39 \times 10^{-7}$. The same steady solution can be obtained for different initial estimations. Table 5.1 shows time evolution of the error, the residuals for each nonlinear equations (5.56) as

$$\text{Res 1} = \left| -k_1(\bar{x} - \underline{x}) - k_0 + a(\underline{x}^\beta - \bar{x}^\beta) + A_1(\underline{x}^M - \bar{x}^M) + A_2(\underline{x}^m - \bar{x}^m) \right|, \quad (5.60)$$

$$\text{Res 2} = \left| a\beta\bar{x}^{\beta-1} + A_1M\bar{x}^{M-1} + A_2m\bar{x}^{m-1} + k_1 \right|, \quad (5.61)$$

and

$$\text{Res 3} = \left| a\beta\underline{x}^{\beta-1} + A_1M\underline{x}^{M-1} + A_2m\underline{x}^{m-1} + k_1 \right|, \quad (5.62)$$

during the computational period, and the computed thresholds. According to Table 5.1, the errors and the residuals monotonically decreases as the time marches and the computation ends at $t = 5.5 \times 10^7$, indicating that the nonlinear equations (5.34) are correctly solved; each residual is less than 1.0×10^{-8} .

Table. 5.1: Time evolution of the error, the residuals and the computed thresholds.

$t (\times 10^7)$	Error	Res 1	Res 2	Res 3	$\bar{x}$	$\underline{x}$
0.0	1.6×10^{-1}	1.6×10^1	1.3×10^{-2}	1.3×10^{-2}	7500	5500
1.0	5.7×10^{-6}	5.7×10^{-4}	5.3×10^{-4}	5.3×10^{-4}	7376.14	5735.92
2.0	5.0×10^{-7}	5.0×10^{-5}	4.7×10^{-5}	4.7×10^{-5}	7354.98	5755.88
3.0	4.4×10^{-8}	4.4×10^{-6}	4.2×10^{-6}	4.2×10^{-6}	7353.11	5757.65
4.0	3.9×10^{-9}	3.9×10^{-7}	3.7×10^{-7}	3.7×10^{-7}	7352.95	5757.8
5.0	3.5×10^{-10}	3.5×10^{-8}	3.3×10^{-8}	3.3×10^{-8}	7352.93	5757.82
5.5	9.9×10^{-11}	1.0×10^{-10}	9.5×10^{-9}	9.5×10^{-9}	7352.93	5757.82

5.4 Numerical schemes for Hamilton-Jacobi-Bellman quasi-variational inequalities

In this section, two numerical schemes for HJBQVIs are presented. The first one employs the penalty method [26, 91] and the other the policy iteration method [7]. Both schemes employ the PGFEM [102].

The spatial domain of the HJBQVI is truncated as $x \in (0, L)$ with a sufficiently large constant $L > 0$ for the sake of numerical computation. The artificial boundary condition

$$V - \mathcal{M}V = 0 \quad (5.63)$$

is supplemented at $x = L$, which is based on the conjecture that the bird population should be immediately reduced if its population is excessively large.

5.4.1 Petrov-Galerkin finite element method

PGFEM [102] is employed for the spatial discretization. The trial and test functions of the PGFEM scheme are constructed based on the exact solutions to local two-point boundary value problems as in [19] to compute stable numerical solutions. This discretization enables us to compute nodally exact solutions for constant-coefficient problems, implying its possibility to accurately approximate solutions to the differential equations. The present scheme inherits this discretization technique.

The shape function ϕ and the weight function w are decided by the analytical solution of the following boundary value problem defined in the element $\Omega(i, j)$ constituting of the nodes i and j as

$$D_\Omega \frac{d^2 \phi}{dx^2} + a_\Omega \frac{d\phi}{dx} - \delta_\Omega \phi = 0, \quad 0 \leq x \leq L \quad (5.64)$$

and

$$D_\Omega \frac{d^2 w}{dx^2} - a_\Omega \frac{dw}{dx} - \delta_\Omega w = 0, \quad 0 \leq x \leq L \quad (5.65)$$

where x is the local one dimensional coordinate where $x = 0$ corresponds to the node i and $x = L$ to the node j , D_Ω is the diffusivity coefficient in the element $\Omega(i, j)$, a_Ω is the drift coefficient in the element $\Omega(i, j)$, and δ_Ω is decay coefficient in the element in the element $\Omega(i, j)$. The unknown variable V in the element $\Omega(i, j)$ is approximated as

$$\tilde{V} = V_i \phi_i + V_j \phi_j \quad (5.66)$$

where $\tilde{V}$ is approximated solution of V in the element $\Omega(i, j)$. Here, the shape functions ϕ_i

and ϕ_j are determined as

$$D_\Omega \frac{d^2 \phi_i}{dx^2} + a_\Omega \frac{d\phi_i}{dx} - \delta_\Omega \phi_i = 0, \quad 0 \leq x \leq L \text{ with } \phi_i(0) = 1 \text{ and } \phi_i(L) = 0 \quad (5.67)$$

and

$$D_\Omega \frac{d^2 \phi_j}{dx^2} + a_\Omega \frac{d\phi_j}{dx} - \delta_\Omega \phi_j = 0, \quad 0 \leq x \leq L \text{ with } \phi_j(0) = 0 \text{ and } \phi_j(L) = 1, \quad (5.68)$$

respectively. Solving (5.67) and (5.68) yields

$$\phi_i = \frac{e^{\nu_\Omega^+ L} e^{\nu_\Omega^- x} - e^{\nu_\Omega^- L} e^{\nu_\Omega^+ x}}{e^{\nu_\Omega^+ L} - e^{\nu_\Omega^- L}} \quad (5.69)$$

and

$$\phi_j = \frac{e^{\nu_\Omega^+ x} - e^{\nu_\Omega^- L}}{e^{\nu_\Omega^+ L} - e^{\nu_\Omega^- L}} \quad (5.70)$$

with

$$\nu_\Omega^\pm = \frac{1}{2} \left(-\text{Pe}_\Omega \pm \sqrt{\text{Pe}_\Omega^2 + 4\text{Da}_\Omega} \right), \quad (5.71)$$

the local Péclet number in the element $\Omega(i, j)$

$$\text{Pe}_\Omega = \frac{a_\Omega}{D_\Omega}, \quad (5.72)$$

and the local Damköhler number in the element $\Omega(i, j)$

$$\text{Da}_\Omega = \frac{\delta_\Omega}{D_\Omega}. \quad (5.73)$$

The weight functions associated with the nodes i and j are determined as

$$D_\Omega \frac{d^2 w_i}{dx^2} - a_\Omega \frac{dw_i}{dx} - \delta_\Omega w_i = 0, \quad 0 \leq x \leq L \text{ with } w_i(0) = 1 \text{ and } w_i(L) = 0 \quad (5.74)$$

and

$$D_\Omega \frac{d^2 w_j}{dx^2} - a_\Omega \frac{dw_j}{dx} - \delta_\Omega w_j = 0, \quad 0 \leq x \leq L \text{ with } w_j(0) = 0 \text{ and } w_j(L) = 1, \quad (5.75)$$

respectively. Solving (5.74) and (5.75) yields

$$w_i = \frac{e^{\mu_\Omega^+ L} e^{\mu_\Omega^- x} - e^{\mu_\Omega^- L} e^{\mu_\Omega^+ x}}{e^{\mu_\Omega^+ L} - e^{\mu_\Omega^- L}} \quad (5.76)$$

and

$$w_j = \frac{e^{\mu_{\Omega}^{+}x} - e^{\mu_{\Omega}^{-}x}}{e^{\mu_{\Omega}^{+}L} - e^{\mu_{\Omega}^{-}L}} \quad (5.77)$$

with

$$\mu_{\Omega}^{\pm} = \frac{1}{2} \left(\text{Pe}_{\Omega} \pm \sqrt{\text{Pe}_{\Omega}^2 + 4\text{Da}_{\Omega}} \right). \quad (5.78)$$

By using the above-explained shape functions and weight functions, a weak form can be obtained. Consider second order partial equation

$$\frac{\partial V}{\partial t} + a \frac{\partial V}{\partial x} - D \frac{\partial^2 V}{\partial x^2} + \delta V = S, \quad 0 < x < \Lambda \quad (5.79)$$

is discretized, where a is drift coefficient, D is diffusivity coefficient, δ is the decay coefficient, S is the source term, and Λ is the length of the computational domain. This can be interpreted as advection-diffusion-decay equation [102] with a source term. Multiplying the weighted function w by (5.79) yields

$$\int_0^{\Lambda} w \frac{\partial V}{\partial t} dx + \int_0^{\Lambda} wa \frac{\partial V}{\partial x} dx - \int_0^{\Lambda} wD \frac{\partial^2 V}{\partial x^2} dx + \int_0^{\Lambda} w\delta V dx = \int_0^{\Lambda} wS dx. \quad (5.80)$$

Applying integration by parts to second order partial derivative yields

$$\begin{aligned} \int_0^{\Lambda} w \frac{\partial V}{\partial t} dx + \int_0^{\Lambda} wa \frac{\partial V}{\partial x} dx - \int_0^{\Lambda} D \frac{\partial w}{\partial x} \frac{\partial V}{\partial x} dx + \int_0^{\Lambda} w\delta V dx \\ = \int_0^{\Lambda} wS dx + \left[wD \frac{\partial V}{\partial x} \right]_0^{\Lambda}. \end{aligned} \quad (5.81)$$

The computational domain is discretized into N elements with $N+1$ nodes as $0 = x_0 < x_1 < \dots < x_{N-1} < x_N = L$. The i th element is the interval (x_{i-1}, x_i) . The quantity discretized at x_i is represented with the subscript i as V_i . The length of the i th element is denoted as $\Delta x = x_i - x_{i-1}$. For the sake of brevity, uniform length is employed in this chapter. The coefficients are attributed to the elements.

Consider the discretization at an inner i th node ($1 \leq i \leq N$). For the first part in the HJBQVI (6.13), the scheme employs an exponentially fitted discretization based on the PGFE as a three-point scheme

$$\delta V - \mu x \frac{\partial V}{\partial x} - \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 V}{\partial x^2} \rightarrow \beta_i V_{i-1} + \alpha_i V_i + \gamma V_{i+1} \quad (5.82)$$

where the coefficients α_i , β_i , and γ_i are given as

$$\alpha_i = \frac{D_{i-1}}{\Delta x} \left(e^{\nu_{i-1}^+} - e^{\nu_{i-1}^-} \right) \left(\nu_{i-1}^- e^{\nu_{i-1}^+} - \nu_{i-1}^+ e^{\nu_{i-1}^-} \right) + \frac{D_i}{\Delta x} \left(e^{\mu_i^+} - e^{\mu_i^-} \right) \left(\mu_i^- e^{\mu_i^+} - \mu_i^+ e^{\mu_i^-} \right), \quad (5.83)$$

$$\beta_i = \frac{D_{i-1}}{\Delta x} \left(e^{\nu_{i-1}^+} - e^{\nu_{i-1}^-} \right) \left(\nu_{i-1}^+ - \nu_{i-1}^- \right) \quad (5.84)$$

and

$$\gamma_i = \frac{D_i}{\Delta x} \left(e^{\mu_i^+} - e^{\mu_i^-} \right) \left(\mu_i^+ - \mu_i^- \right) \quad (5.85)$$

with

$$\nu_i^\pm = \frac{1}{2} \left(-\text{Pe}_i \pm \sqrt{\text{Pe}_i^2 + 4\text{Da}_i} \right), \quad (5.86)$$

$$\mu_i^\pm = \frac{1}{2} \left(\text{Pe}_i \pm \sqrt{\text{Pe}_i^2 + 4\text{Da}_i} \right), \quad (5.87)$$

$$\text{Pe}_i = \frac{a_i \Delta x}{D_i}, \quad (5.88)$$

$$\text{Da}_i = \frac{\delta_i (\Delta x)^2}{D_i}, \quad (5.89)$$

$$a_i = \mu \left(\frac{x_i + x_{i+1}}{2} \right), \quad (5.90)$$

and

$$D_i = \frac{1}{2} \sigma^2 \left(\frac{x_i + x_{i+1}}{2} \right)^2. \quad (5.91)$$

The source term at the i th node is discretized as

$$-ux^M + dx^m \rightarrow (-ux_i^M + dx_i^m) \epsilon_i \quad (5.92)$$

with

$$\begin{aligned} \epsilon_i = & \frac{1}{e^{\nu_{i-1}^+ \Delta x} - e^{\nu_{i-1}^- \Delta x}} \left[\frac{1}{\nu_{i-1}^+} \left(e^{\nu_{i-1}^+ \Delta x} - 1 \right) - \frac{1}{\nu_{i-1}^-} \left(e^{\nu_{i-1}^- \Delta x} - 1 \right) \right] \\ & + \frac{1}{e^{\mu_i^+ \Delta x} - e^{\mu_i^- \Delta x}} \left[\frac{1}{\mu_i^+} \left(e^{\mu_i^+ \Delta x} - 1 \right) - \frac{1}{\mu_i^-} \left(e^{\mu_i^- \Delta x} - 1 \right) \right] \end{aligned} \quad (5.93)$$

where the source term upwinding is naturally performed.

The term $V - \mathcal{M}V$ is discretized at the i th node as

$$V - \mathcal{M}V \rightarrow V_i - (\mathcal{M}\bar{V})_i \quad (5.94)$$

where $\bar{V}$ is a continuous linear interpolation of V using the nodal values of the previous time step, which is a piecewise linear continuous function at each time step. Then, we set

$$(\mathcal{M}\bar{V})_i = -K(\omega_i^*) + \bar{V}(x_i - \omega_i^*) = (-k_1\omega_i^* - k_0)\chi_{\{\omega_i^* > 0\}} + \bar{V}(x_i - \omega_i^*) \quad (5.95)$$

where

$$\omega_i^* = \arg \max_{\omega_i} \left[(-k_1\omega_i^* - k_0)\chi_{\{\omega_i^* > 0\}} + \bar{V}(x_i - \omega_i) : \omega_i \in \mathbb{R}, x_i - \omega_i \geq 0 \right]. \quad (5.96)$$

The boundary conditions are set as

$$V_0 = 0 \quad (5.97)$$

at the 0 th node and

$$V_N - (\mathcal{M}\bar{V})_N = 0 \quad (5.98)$$

at the N th node.

5.4.2 Penalty method

A temporal term is added to the HJBQVI as

$$\max \left(\frac{\partial V}{\partial t} + \mathcal{L}V + ux^M - dx^m, \mathcal{M}V - V \right) = 0, x > 0 \quad (5.99)$$

with a terminal condition $V(x) = 0$. Note that the derivatives are reinterpreted as partial derivatives. Steady solutions are calculated by solving the unsteady problem with a sufficiently large computational period. A penalized technique [26] is applied to the unsteady HJBQVI (5.99) as

$$-\frac{\partial V}{\partial s} + \mathcal{L}V + ux^M - dx^m + \lambda \min(V - \mathcal{M}V, 0) = 0, \quad 0 < x < L \quad (5.100)$$

where $s = -t$ and λ is the penalty parameter to formally recover the unsteady HJBQVI (5.99) under the limit $\lambda \rightarrow \infty$. For proving the validity of the penalty method, the following two conditions are considered; $V - \mathcal{M}V \geq 0$ and $V - \mathcal{M}V < 0$. For the case of $V - \mathcal{M}V \geq 0$,

$$\lambda \min(V - \mathcal{M}V, 0) = 0 \quad (5.101)$$

holds true, and (5.100) becomes

$$-\frac{\partial V}{\partial s} + \mathcal{L}V + ux^M - dx^m = 0 \quad (5.102)$$

and thus the HJBQVI (6.13) is satisfied in this case. For the case of $V - \mathcal{M}V < 0$,

$$\lambda \min(V - \mathcal{M}V, 0) = \lambda(V - \mathcal{M}V) \quad (5.103)$$

holds true, and (5.100) becomes

$$-\frac{\partial V}{\partial s} + \mathcal{L}V + ux^M - dx^m + \lambda(V - \mathcal{M}V) = 0. \quad (5.104)$$

Dividing (5.104) by λ and putting $(V - \mathcal{M}V)$ into the right hand side yields

$$\frac{1}{\lambda} \left[-\frac{\partial V}{\partial s} + \mathcal{L}V + ux^M - dx^m \right] = -(V - \mathcal{M}V) > 0, \quad (5.105)$$

which means

$$-\frac{\partial V}{\partial s} + \mathcal{L}V + ux^M - dx^m > 0. \quad (5.106)$$

In addition, when the limit $\lambda \rightarrow \infty$ is taken,

$$-(V - \mathcal{M}V) \approx 0 \quad (5.107)$$

formally holds true because the left hand side of (5.105) approaches 0 as $\lambda \rightarrow \infty$. Thus, the HJBQVI (6.13) is satisfied also in this case.

5.4.3 Policy iteration method

The policy iteration method [7] is applied to the steady HJBQVI. At first, the HJBQVI is equivalently rewritten as

$$\min_{u \in \{0,1\}} \left[(1-u)\{\mathcal{L}V - ux^M + dx^m\} + u\{V - \mathcal{M}V\} \right] = 0 \quad (5.108)$$

where $u \in \{0,1\}$ is an auxiliary control variable [7]. The discretized version of the auxiliary control problem (5.108) at the inner node i is written as

$$\min_{u_i \in \{0,1\}} \left[(1-u_i)\{\beta_i V_{i-1} + \alpha_i + V_i \gamma V_{i+1} + (-ux_i^M + dx_i^m)\epsilon_i\} + u_i \{V_i - (\mathcal{M}\bar{V})_i\} \right] = 0. \quad (5.109)$$

Let $n \geq 0$ be the step number in the policy iteration. In the policy iteration algorithm, the discretized problem (5.109) is solved recursively as

$$\min_{u_i \in \{0,1\}} \left[(1-u_i) \left\{ \beta_i V_{i-1}^n + \alpha_i + V_i^n + \gamma V_{i+1}^n \right\} + u_i \left\{ V_i^n - (\mathcal{M}\bar{V})_i^{n-1} \right\} \right] = 0, \quad n \geq 1 \quad (5.110)$$

starting from an initial guess $\mathbf{V}^0 = \mathbf{0}$. Here, $\mathbf{V}^n$ represents all the values at the step n . In (5.110), the piecewise linear interpolation $\bar{V}$ in the term $(\mathcal{M}\bar{V})_i^{n-1}$ is constructed with $\mathbf{V}^{n-1}$ to avoid solving non-linear system at each step. The control and the value function are thus updated in a staggered manner. The boundary condition (5.97) is evaluated at the time step n , and the boundary condition (5.98) is used to find V_N^n as

$$V_N^n = (\mathcal{M}\bar{V})_N^{n-1}. \quad (5.111)$$

Assembling the discretized equations (5.110) for all i , we obtain the system of linear equations

$$\mathbf{A}^{n-1} \mathbf{V}^n = \mathbf{B}^{n-1}, \quad (5.112)$$

where $\mathbf{A}$ is $(N+1) \times (N+1)$ dimensional coefficient matrix and $\mathbf{B}$ and $\mathbf{V}$ are $N+1$ dimensional vectors. $\mathbf{A}$ and $\mathbf{B}$ are independent of $\mathbf{V}^n$ and dependent only of $\mathbf{V}^{n-1}$. The iteration step in the policy iteration continues until the condition

$$\max_i |V_i^{n+1} - V_i^n| < 10^{-14} \quad (5.113)$$

is satisfied. The policy iteration approach has some advantage over other methods. The policy iteration approach does not need artificial parameters such as the time increment in the pseudo time-stepping methods [42] and the penalty parameter employed in the penalty method [26, 91].

5.5 Numerical results for Hamilton-Jacobi-Bellman quasi-variational inequality

In this section, the solutions obtained by the presented numerical schemes and an exact solution are compared.

5.5.1 Numerical results with policy iteration method

The model parameters are set as $\mu = 0.7$, $\sigma = 0.1$, $d = 5.0 \times 10^{-3}$, $u = 1.0 \times 10^{-4}$, $m = 2$, $M = 0.5$, $\delta = 2.0$, $k_0 = 0.01$ and $k_1 = 0.3$ [96, 98]. In this chapter, L is set as 100, which has been checked to be a sufficiently large value by preliminary numerical experiments. Both

the accuracy of numerically calculated value function $V(x)$ and that of the thresholds $\bar{x}$ and $\underline{x}$ are investigated. Especially, the accuracy of the two threshold values is important because it is necessary for calculating the optimal policy. In previous studies about numerical methods for HJBQVIs, computational accuracy of numerically calculated thresholds (free boundaries) was not investigated in detail [27, 7] despite it is the most important.

At first, the accuracy of the numerically calculated value function is investigated. Table 5.2 shows l^∞ error norms and l^2 error norms with changing the number of elements. According to Table 5.2, both l^∞ error norms and l^2 error norms monotonically decrease as the number of elements increases. Table 5.3 shows convergence rates (CRs) with l^∞ error norms and l^2 error norms. Table 5.3 indicates that the presented numerical method is more than first-order in the sense of l^∞ error norm and l^2 error norm. Figure. 5.2 shows the numerically computed value function (red circles) and the exact solution (black line) (5.29) with 100 elements. In addition, the blue and green curves are the lower bound and the upper bound (5.51), respectively. It is confirmed that the numerically calculated value functions satisfy the exact lower and upper bounds (5.51). This fact supports the validity of the proposed scheme.

Second, the accuracy of the computed threshold values is investigated. Under the above-presented model parameters, the exact threshold values are $\bar{x} = 43.0583$ and $\underline{x} = 36.07$. Table 5.4 shows the computed threshold values against the different resolutions. According to Table 5.4, the computed threshold values approach the exact values. Table 5.5 shows absolute differences between the numerical and exact solutions. Table 5.5 indicates that the absolute difference between the numerically calculated thresholds and that of the exact solution decreases. According to Tables 5.4 and 5.5, computed $\underline{x}$ with 100, 200 and 400 elements do not change and thus the absolute differences do not change. The reason for this phenomenon can be explained as follows. The resolution of $\underline{x}$ with 100 and 200 elements is relatively high: the absolute differences are both 0.07, which is smaller than the size of the elements 1 and 0.5. In each computation, the absolute difference is smaller than the size of the element Δx , indicating the satisfactory ability of the presented numerical method to calculate

the thresholds and thus the optimal plans.

Table. 5.2: l^∞ error norm and l^2 error norm for the value function $V(x)$ calculated with the policy iteration method.

Number of elements	l^∞ error norm	l^2 error norm
100	4.7×10^{-2}	4.3×10^{-1}
200	1.3×10^{-2}	1.3×10^{-1}
400	3.5×10^{-3}	3.4×10^{-2}
800	9.0×10^{-4}	8.8×10^{-3}
1600	2.4×10^{-4}	2.1×10^{-3}

Table. 5.3: Convergence rates (CRs) with l^∞ error norm and l^2 error norm for the value function $V(x)$ calculated with the policy iteration method.

Compared elements	CR with l^∞ error norm	CR with l^2 error norm
100/200	1.81	1.77
200/400	1.94	1.91
400/800	1.96	1.95
800/1600	1.91	2.06

Table. 5.4: Thresholds calculated with the policy iteration method.

Number of elements	$\bar{x}$	$\underline{x}$
100	42.5	36.0
200	42.75	36.0
400	42.875	36.0
800	42.9375	36.125
1600	43.03125	36.0625

Table. 5.5: Absolute difference between numerically calculated thresholds with the policy iteration method and exact solutions.

Size of elements	Absolute difference in $\bar{x}$	Absolute difference in $\underline{x}$
1	0.5583	0.07
0.5	0.3083	0.07
400	0.1833	0.07
800	0.1208	0.055
1600	0.02705	0.0075

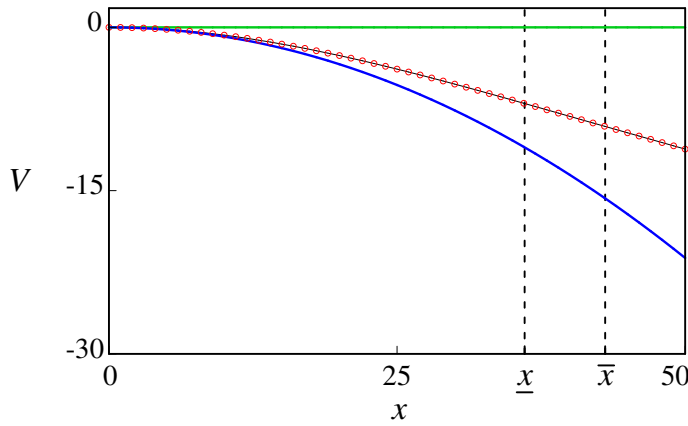


Fig. 5.2: The exact solution (black line), the exact thresholds (dashed black lines), and the numerically computed value function (red circles) with 100 elements with the policy iteration method. Only the domain $[0, 50]$ is shown here. The blue and green curves are the lower bound and the upper bound, respectively.

5.5.2 Numerical results with penalty method

The model parameters are the same as the policy iteration method case. The penalty parameter $\lambda = 10^5$ is employed. Both the accuracy of numerically calculated value function $V(x)$ and that of the thresholds $\bar{x}$ and $\underline{x}$ are investigated.

At first, the accuracy of the numerically calculated value function is investigated. Table 5.6 shows l^∞ error norms and l^2 error norms with changing the number of elements. According to Table 5.6, both l^∞ error norms and l^2 error norms monotonically decrease as the number of elements increases. Table 5.7 shows convergence rates (CRs) with l^∞ error norms and l^2 error

norms. Table 5.7 indicates that the presented numerical method is more than first-order in the sense of l^∞ error norm and l^2 error norm. Figure. 5.3 shows the numerically computed value function (red circles) and the exact solution (black line) (5.29) with 100 elements. In addition, the blue and green curves are the lower bound and the upper bound (5.51), respectively. It is confirmed that the numerically calculated value functions satisfy the exact lower and upper bounds (5.51). This fact supports the validity of the proposed scheme.

Second, the accuracy of the computed threshold values is investigated. Under the above-presented model parameters, the exact threshold values are $\bar{x} = 43.0583$ and $\underline{x} = 36.0700$. Table 5.8 shows the computed threshold values against the different resolutions. According to Table 5.8, the computed threshold values approach the exact values. Table 5.9 shows absolute differences between the numerical and exact solutions. Table 5.9 indicates that the absolute difference between the numerically calculated thresholds and that of the exact solution decreases. According to Tables 5.8 and 5.9, computed $\underline{x}$ with 100, 200 and 400 elements do not change and thus the absolute differences do not change. The reason for this phenomenon can be explained as follows. The resolution of $\underline{x}$ with 100 and 200 elements is relatively high: the absolute differences are both 0.07, which is smaller than the size of the elements 1 and 0.5. In each computation, the absolute difference is smaller than the size of the element Δx , indicating the satisfactory ability of the presented numerical method to calculate the thresholds and thus the optimal plans.

Table. 5.6: l^∞ error norm and l^2 error norm for the value function $V(x)$ calculated with the penalty method.

Number of elements	l^∞ error norm	l^2 error norm
100	4.7×10^{-2}	4.3×10^{-1}
200	1.4×10^{-2}	1.3×10^{-1}
400	3.5×10^{-3}	3.4×10^{-2}
800	9.0×10^{-4}	8.8×10^{-3}
1600	2.5×10^{-4}	2.1×10^{-3}

Table. 5.7: Convergence rates (CRs) with l^∞ error norm and l^2 error norm for the value function $V(x)$ calculated with the penalty method.

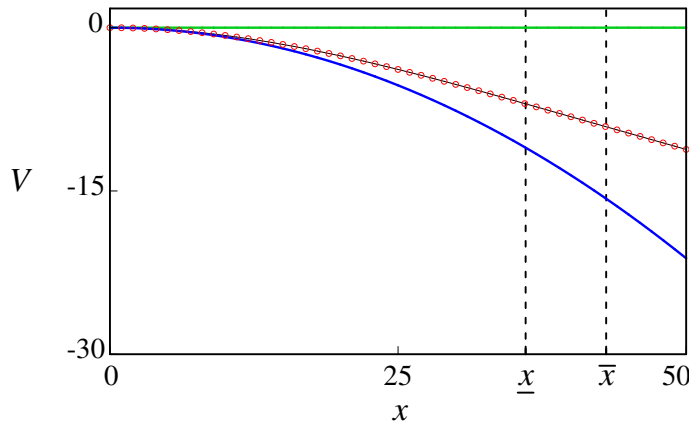
Compared elements	CR with l^∞ error norm	CR with l^2 error norm
100/200	1.81	1.77
200/400	1.94	1.91
400/800	1.96	1.95
800/1600	1.85	2.05

Table. 5.8: Thresholds calculated with the penalty method.

Number of elements	$\bar{x}$	$\underline{x}$
100	42.5	36.0
200	42.75	36.0
400	42.875	36.0
800	42.9375	36.125
1600	43.03125	36.0625

Table. 5.9: Absolute difference between numerically calculated thresholds with the penalty method and exact solutions.

Size of elements	Absolute difference in $\bar{x}$	Absolute difference in $\underline{x}$
1	0.5583	0.07
0.5	0.3083	0.07
400	0.1833	0.07
800	0.1208	0.055
1600	0.02705	0.0075

Fig. 5.3: The exact solution (black line), the exact thresholds (dashed black lines), and the numerically computed value function (red circles) with 100 elements with the penalty method. Only the domain $[0, 50]$ is shown here. The blue and green curves are the lower bound and the upper bound, respectively.

5.5.3 Comparison between policy iteration method and penalty method

Both schemes have almost the same l^∞ error norms and l^2 error norms. The CRs are thus almost the same. Accuracy for the thresholds is the same between the two schemes, indicating that both schemes can be utilized for solving HJBQVIs. Note that the time for computation with the policy iteration method is shorter than that with the penalty method.

5.6 Conclusions

For finding the optimal management plans of *P. carbo*, a stochastic impulse control model was introduced. Numerical methods for time-independent HJBQVIs based on the penalty method and the policy iteration approach were proposed and their accuracies in the solution itself and the thresholds were investigated by comparing with an exact solution. The result indicated that the proposed numerical schemes can successfully compute the solution of the HJBQVI at least with first-order and the thresholds for population control can be obtained within the length of the elements.

CHAPTER 6

A coupling model for management of *P. altivelis* with *P. carbo*

6.1 Introduction

Feeding damage from predators, such as waterfowls, to fishery resources has recently been increasing, which is an urgent and serious problem of inland fisheries. An example is *P. altivelis*, an economically important fish for Japanese inland fisheries and the waterfowl *P. carbo* [81]. Fishery Cooperatives and local governments have been trying to suppress the feeding damage by performing various countermeasures. Currently, a more cost-effective as well as ecologically sound strategy to suppress the predation is necessary for the sustainable management of both the predators and the fishery resources since *P. carbo* is not an alien species and provides valuable ecosystem services [74].

The above-mentioned issue is resolved with a population dynamics model [70]. The model tracks the population dynamics of the predator and the fishery resource where the countermeasure to suppress the predation is an impulse control. A HJBQVI governs the optimal strategy of predator suppression [68, 86], which is solved with PGFEM [102] with the conventional penalty and operator-splitting methods [30]. The constructed model is applied to the management problem of *P. altivelis* under the predation pressure from *P. carbo*.

6.2 Mathematical model

In this section, the mathematical model is presented. In the model, predator-prey dynamics between *P. carbo* and *P. altivelis* during a year is considered. During the year the former population is renewable while the latter is not renewable [95]. The time is denoted as t with the terminal time T .

6.2.1 Population dynamics of *P. carbo*

The total number of *P. carbo* at the time t is denoted as X_t , and is governed by the following Itô's SDE of geometric Brownian motion type [66]:

$$\begin{cases} dX_t = X_t(\mu(t)dt + \sigma(t)dB_t), & \tau_i \leq t < \tau_{i+1} \\ X_{\tau_i} = X_{\tau_i-} - \zeta_i \end{cases} \quad (6.1)$$

with the initial condition $X_{0-} = x \geq 0$ where $\mu(t)$ is the time-dependent drift as the deterministic growth rate, $\sigma(t)$ with $\mu > \sigma^2$ is the time-dependent volatility, B_t is the 1-D standard Brownian motion defined on a usual complete probability space [66], τ_i ($i = 0, 1, 2, \dots, P, \tau_0 = 0$) is the stopping time at which the countermeasure is taken, ζ_i represents the killed *P. carbo* population at the time τ_i , X_{0-} represents the left limit, and P is the maximum time of taking countermeasure during the period $[0, T]$.

6.2.2 Population dynamics of *P. altivelis*

The total number and the weight of *P. altivelis* are denoted as N_t and W_t at the time t . The governing equations of N_t and W_t are set as

$$dN_t = -(D + aX_t + \chi_{\{\tau \leq t\}}c)N_t dt \quad (6.2)$$

and

$$dW_t = r \left(1 - \frac{W_t}{C}\right) W_t dt \quad (6.3)$$

with the initial conditions $N_{0-} = n \geq 0$ and W_0 where D is the mortality rate of *P. altivelis*, a is a positive constant that modulates the predation pressure from *P. carbo*, τ is the opening

time of harvesting *P. altivelis*, $\chi_{\{\tau \leq t\}}$ is the indicator function such that $\chi_{\{\tau \leq t\}} = 1$ for $\tau \leq t$ and $\chi_{\{\tau \leq t\}} = 0$ otherwise, c is the harvesting pressure, r is the intrinsic growth rate, and C is the maximum body weight. Equation (6.2) is a Malthus growth type not-increasing model and (6.3) is a logistic growth model [95]. Whenever the countermeasure is taken, the cost

$$K(\zeta_i) = k_1 \zeta_i + k_0 \quad (\zeta_i > 0) \quad (6.4)$$

is incurred where $k_1 > 0$ is the cost proportional to ζ_i and $k_0 > 0$ is the fixed cost incurred regardless of ζ_i . In this model, both the timing τ_i and the magnitude of the intervention ζ_i are the control variables to be optimized.

6.2.3 Performance index

The performance index $J(x, n, t; \eta)$ to be maximized by the decision-maker, the manager of *P. altivelis* is set as

$$J(x, n, t; \eta) = E^{x, n, t} \left[\begin{aligned} &\alpha \int_t^T e^{-\delta(s-t)} \left(u X_s^M - d X_s^m \right) N_s ds \\ &+ \beta \int_t^T e^{-\delta(s-t)} \chi_{\{\tau \leq s\}} c N_t W_t ds - \gamma \sum_{i=0}^P e^{-\delta(s-t)} K(\zeta_i) \end{aligned} \right] \quad (6.5)$$

with

$$0 < M < 1 < m \leq 2, \quad (6.6)$$

where $E[\cdot]$ is the expectation conditioned on X_{0-} and N_{0-} , $\delta > 0$ is the discount rate, u and d are the parameters such that $0 < u < d$, m and M are the parameters such that and $\alpha, \beta, \gamma > 0$ are weight constants. Without loss of generality, $\gamma = 1$ is assumed. The discount rate δ represents the attitude of the decision-maker; larger δ means that he/she performs the countermeasure from a longer-term viewpoint. The performance index (6.5) represents the net profit of the decision-maker. The term $u X_s^M$ considers the ecosystem services that the bird provides [74] and the term $-d X_s^m$ quantifies the loss of the fish by the bird per unit time. The first term is multiplied by N_s because the more *P. altivelis* exists, the much feeding damage occur and the more nutrient salts are transported by the bird. The second term in the right-hand side of (6.5) is the cost of taking the countermeasure.

Here, η represents the management policy defined by the timing and magnitude of the countermeasure

$$\eta = (\tau_i, \zeta_i)_{i \geq 0} \quad (6.7)$$

and is called an admissible control if it satisfies the conditions stated in Definition 2.1 in [63], which are listed up in what follows:

$$0 \leq \tau_i < \tau_{i+1}, \text{ a.s. } 0 \leq i \leq P-1, \quad (6.8)$$

$$\tau_i \text{ is an } \{\mathcal{F}_t\}_{t \geq 0} \text{ stopping time, } 0 \leq i \leq P, \quad (6.9)$$

$$\zeta_i \text{ is } \mathcal{F}_{\tau_i} \text{ measurable, } 0 \leq i \leq P, \quad (6.10)$$

and

$$0 \leq X_t, 0 \leq t \leq T. \quad (6.11)$$

6.2.4 Hamilton-Jacobi-Bellman quasi-variational inequality

Let $\mathcal{A}$ be the set of the admissible controls. The value function, or the maximized net profit of the decision-maker $V(x, n, t)$ is defined as

$$V(x, n, t) = \sup_{\eta \in \mathcal{A}} \mathbb{E}[v(x, n, t; \eta)] = \mathbb{E}[v(x, n, t; \eta^*)] \quad (6.12)$$

where η^* is the optimal control. The spatial domain Ω of (x, n) is defined as $(0, \infty) \times (0, \infty)$. Applying the dynamic programming principle [70] to (6.12) leads to the governing HJBQVI [86]

$$\max \left[\frac{\partial V}{\partial t} + \mathcal{L}V + \alpha \left(ux^M - dx^m \right) n + \beta \chi_{\{\tau \leq t\}} cnw(t), \mathcal{M}V - V \right] = 0, \text{ in } [0, T) \times \Omega \quad (6.13)$$

with the differential operator $\mathcal{L}$ and the intervention operator $\mathcal{M}$ defined as

$$\mathcal{L}Q = \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 Q}{\partial x^2} + \mu x \frac{\partial Q}{\partial x} - \delta Q - (D + ax + \chi_{\{\tau \leq t\}} c) n \frac{\partial Q}{\partial n} \quad (6.14)$$

and

$$\mathcal{M}Q = \sup_{\zeta} [-K(\zeta) + Q(x - \zeta, n, t)], \quad (6.15)$$

respectively, for generic sufficiently smooth $Q = Q(x, n, t)$. Here, the notation $w(t) = W_t$ is used. The left part in max operator in the HJBQVI (6.13) corresponds to the situation where

no countermeasure should be performed, while the right part corresponds to the situation where the countermeasure should be performed immediately. The terminal condition

$$V(x, n, T) = f(x, n) \quad (6.16)$$

is prescribed where $f(x, n)$ is an appropriate function. The boundary conditions are prescribed as

$$V(0, n, t) = 0 \quad (6.17)$$

and

$$V(x, 0, t) = 0. \quad (6.18)$$

The former condition means that there is no loss when there is no *P. carbo*. The latter condition means that there is no profit when there is no *P. altivelis*. Note that a boundary condition is not necessary along $n = N_0$ considering the direction of the characteristic curve.

6.3 Numerical scheme

The 1-D domains $x \in (0, X_{\max})$ and $n \in (0, N_0)$ with a large $X_{\max}$ is considered. x axis is truncated for the sake of numerical computation. The computational domain for (x, n) is denoted as $\Omega_{\text{comp}} \in (0, X_{\max}) \times (0, N_0)$. The artificial boundary condition

$$V - \mathcal{M}V = 0 \quad (6.19)$$

is supplemented at $x = X_{\max}$, which is based on the conjecture that the bird population should be immediately reduced if its population is excessively large. The HJBQVI (6.13) is approximated with the penalty method explained in Chapter 5, and is discretized in time based on the three-stage operator-splitting technique [30] that describes the equation to be solved at each discrete time step as

$$-\frac{\partial V}{\partial s} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 V}{\partial x^2} + \mu x \frac{\partial V}{\partial x} - \delta V + \alpha \left(u x^M - d x^m \right) n + \lambda \min(V - \mathcal{M}V, 0) = 0 \text{ in } (0, X_{\max}) \quad (6.20)$$

and

$$-\frac{\partial V}{\partial s} - (D + ax + \chi_{\{\tau \leq t\}}c)n \frac{\partial V}{\partial n} + \beta \chi_{\{\tau \leq t\}}cnw(t) = 0 \text{ in } (0, N_0) \quad (6.21)$$

where λ is the penalty parameter, which formally recovers the HJBQVI (6.13) when $\lambda \rightarrow \infty$, and $s = T - t$. At each time step, (6.21) is integrated with the half increment $\frac{\Delta s}{2}$, then (6.20) is integrated with the increment Δs , and finally (6.21) is again integrated with the half increment $\frac{\Delta t}{2}$. A fully implicit discretization is employed at each stage.

The computational domain Ω_{comp} is discretized into $NX \times NY$ elements with $(NX + 1) \times (NY + 1)$ nodes as structured grid. The Petrov-Galerkin finite element scheme explained in Chapter 5 is employed for spatial discretization of (6.20) except for the penalty term. The element i, j is the rectangular $(x_i, x_{i+1}) \times (n_j, n_{j+1})$. The quantity discretized at i, j at the time step n is represented with the subscript i and j and superscript n as $V_{i,j}^n$. The length for x of the element i, j is denoted as $\Delta x = x_{i+1} - x_i$ and the length for n as $\Delta n = n_{j+1} - n_j$. For the sake of brevity, uniform length is employed in this chapter. The coefficients are attributed to the elements. The term $V - \mathcal{M}V$ is discretized at the i, j th node as

$$V - \mathcal{M}V \rightarrow V_{i,j} - (\mathcal{M}\bar{V})_{i,j} \quad (6.22)$$

where $\bar{V}$ is a continuous linear interpolation of V using the nodal values of the previous time step, which is a piecewise linear continuous function at each time step. Then, we set

$$(\mathcal{M}\bar{V})_{i,j} = -K(\omega_i^*) + \bar{V}(x_i - \omega_i^*, n_j, t) = (-k_1\omega_i^* - k_0)\chi_{\{\omega_i^* > 0\}} + \bar{V}(x_i - \omega_i^*, n_j, t) \quad (6.23)$$

where

$$\omega_i^* = \arg \max_{\omega_i} \left[(-k_1\omega_i^* - k_0)\chi_{\{\omega_i^* > 0\}} + \bar{V}(x_i - \omega_i, n_j, t) : \omega_i \in \mathbb{R}, x_i - \omega_i \geq 0 \right]. \quad (6.24)$$

The Dirichlet boundary conditions are directly imposed as

$$V_{0,j}^n = 0 \quad (6.25)$$

and

$$V_{i,0}^n = 0 \quad (6.26)$$

for all time step n . The conventional first order upwind difference method is used for the spatial discretization of (6.21) as

$$(D + ax + \chi_{\{\tau \leq t\}}c)n \frac{\partial V}{\partial n} \rightarrow (D + ax + \chi_{\{\tau \leq t\}}c)n_j \frac{V_{i,j} - V_{i,j-1}}{\Delta n} \quad (6.27)$$

because the drift $(D + ax + \chi_{\{\tau \leq t\}}c)n$ is always positive. Finally, the discretized equation for (6.21) is represented as

$$V_{i,j}^{n+1} = \frac{\left(V_{i,j}^n + \frac{\Delta s/2}{\Delta n} V_{i,j-1}^{n+1} (D + ax_i + \chi_{\{\tau \leq n\Delta s\}}c)n_j + \beta \chi_{\{\tau \leq n\Delta s\}}cn_j w(n\Delta s)\Delta s/2 \right)}{\left(1 + \frac{\Delta s/2}{\Delta n} (D + ax_i + \chi_{\{\tau \leq n\Delta s\}}c)n_j \right)}. \quad (6.28)$$

Note that the above-equation can be successively solved from $j = 1$ by using the boundary condition $V_{i,0}^{n+1} = 0$.

6.4 Application

The model parameters are set according to the previous studies [94, 95, 97, 98, 96] and are in Table 6.1. The numbers of elements for x direction are 300 and for n direction are 300. The time increment is set as $\Delta t = 0.01$. The truncation parameter $X_{\max}$ is given as 3.0×10^5 , which is sufficiently large value and was confirmed that the computational results did not change if larger value was prescribed. The terminal condition is prescribed as

$$f(x, n) = 0. \quad (6.29)$$

In this section, a case with seasonality for growth of *P. carbo* is considered: the growth rate μ is time-dependent. For the sake of brevity, the magnitude of fluctuation σ is time-independent. The growth rate of *P. carbo* is described as a time-depending function

$$\mu(s) = \bar{\mu} + \gamma \sin\left(\frac{\pi s}{T}\right), \quad (6.30)$$

which expresses seasonality for growth of *P. carbo*, where the coefficient γ is a coefficient and is set as $\gamma = 9.4 \times 10^{-4}$, assuming that the situation is more severe (the peak is three times larger than $\bar{\mu}$). During the period $[0, T]$, the growth rate increases at first, takes a peak, and then decreases.

6.4.1 Model parameters

The model parameters used in this paper are summarized in Table 6.1. The terminal time $T = 180$ (day) is specified considering that the juveniles of *P. altivelis* move from the lake

to rivers and *P. altivelis* spawn during November. The opening time of harvesting *P. altivelis* is set as $T_c = 60$ (day) in summer according to the rule in the lake. The natural mortality rate of *P. altivelis* R from May to November is 50% [56], leading to 3.9×10^{-3} (1/day). The growth rate $\bar{\mu}$ and the magnitude of stochastic fluctuation σ of *P. carbo* are determined as 4.7×10^{-4} (1/day) and 4.0×10^{-4} (1/day^{1/2}), and the predation pressure from *P. carbo* to *P. altivelis* a as 2.0×10^{-1} (1/day) according to the time-series data of the number of *P. carbo* in Chapter 4. The discount rate δ is set as 0 (1/day) because finite period problems are considered. The parameters associated with the attitude of the decision-maker u and d are determined as 1.0×10^{-4} (-) and 1.0×10^{-3} (-) and M and m as 0.5 (-) and 2.0 (-) according to the results in Chapter 4. The weighting parameters α and β are specified as 1.0×10^0 (-) and 1.0×10^5 (-) assuming that the decision-maker puts on emphasis on *P. altivelis* according to the results in Chapter 4. The following parameters are estimated according to the data obtained through personal communications with the Hirose River Fishery Cooperatives. From the growth curve of *P. altivelis*, the intrinsic growth rate of *P. altivelis* is specified as 2.8×10^{-2} (1/day) and the maximum capacity of *P. altivelis* $C = 6.5 \times 10^{-2}$ (kg). The initial bodyweight of *P. altivelis* W_0 is specified as 4.0×10^{-3} (kg), the initial number of *P. altivelis* as 1.0×10^6 (-) and the fishing pressure rate c as 1.0×10^{-2} (1/day). The proportional cost for a countermeasure k_1 and the fixed cost for a countermeasure k_0 are specified as 1.0×10^2 (-) and 1.0×10^6 (-).

6.4.2 Computational results

Figures 6.1 to 6.6 represent evolution of the value function $V(t, x, n)$ (contour) and two thresholds $\bar{x}$ and $\underline{x}$ (black curves) at $s = 1$ (day) (6.1), $s = 70$ (day) (6.2), $s = 80$ (day) (6.3), $s = 120$ (day) (6.4), $s = 150$ (day) (6.5) and $s = 180$ (day) (6.6). In each Figure, left black curves are the lower threshold $\underline{x}$ and right black curves are the upper threshold $\bar{x}$. We can see that both the upper threshold and lower threshold are functions of n and s and can be written as $\bar{x}(n)$ and $\underline{x}(n)$ at each time. This fact means that the thresholds can be determined by the current number of *P. altivelis* at each time. For the sake of visibility, x axis is shown only

Table. 6.1: Values of model parameters.

Parameter	Name	Unit	Value
T	Terminal time	day	180
$\bar{\mu}$	Growth rate of <i>P. carbo</i>	1/day	4.7×10^{-4}
σ	Magnitude of fluctuation in population of <i>P. carbo</i>	1/day ^{1/2}	4.0×10^{-4}
D	Mortality rate of <i>P. altivelis</i>	1/day	3.9×10^{-3}
c	Fishing pressure rate	1/day	1.0×10^{-2}
r	Growth rate of <i>P. altivelis</i>	1/day	2.8×10^{-2}
N_0	Initial number of <i>P. altivelis</i>	–	1.0×10^6
W_0	Initial weight of <i>P. altivelis</i>	kg	4.0×10^{-3}
C	Carrying capacity	kg	6.5×10^{-2}
k_1	Proportional cost for countermeasure	–	1.0×10^2
k_0	Fixed cost for countermeasure	–	1.0×10^6
δ	Discount rate	1/day	0.0
τ	Opening time for fishing	day	60
α	Weighting parameter	–	1.0×10^0
β	Weighting parameter	–	2.0×10^4
a	Predation pressure from <i>P. carbo</i> to <i>P. altivelis</i>	1/day	2.0×10^{-1}
u	Coefficient for utility of <i>P. carbo</i>	–	1.0×10^{-4}
d	Coefficient for disutility of <i>P. carbo</i>	–	1.0×10^{-3}
M	Exponent for utility of <i>P. carbo</i>	–	0.5
m	Exponent for disutility of <i>P. carbo</i>	–	2.0
$X_{\max}$	Truncation parameter for x	–	3.0×10^4

from 0 to 2.0×10^4 . Each figure shows $V(x, n)$ at each time. According to these graphs, the decision-maker can determine the management plan, or whether he takes countermeasures or not. At first, the time is fixed. For each fixed time, we have the graph of two thresholds $\bar{x}(n)$ and $\underline{x}(n)$. Next, we have the current number of *P. carbo* x' and that of *P. altivelis* n' . Then, we can plot a corresponding point (x', n') in the graph. If the plotted point is equal to or larger than the upper threshold $\bar{x}(n')$, the decision-maker should take countermeasures and reduce the number of *P. carbo* to $\underline{x}(n')$. Otherwise, the decision-maker does not have take countermeasures. In short, the decision-maker can find the information necessary for optimal management plans from these graphs.

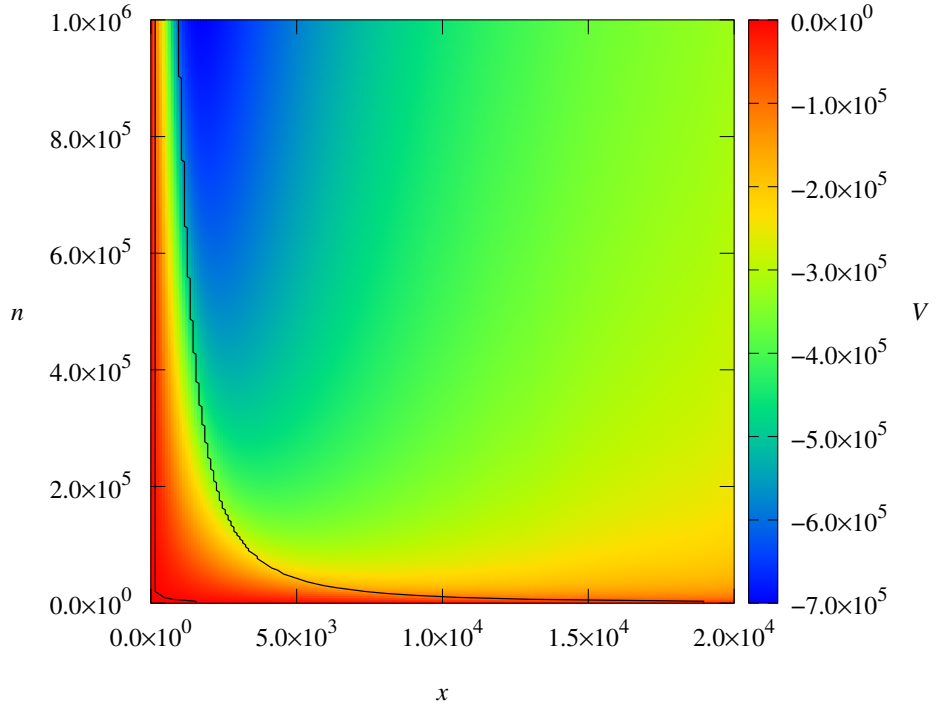


Fig. 6.1: The value function at $s = 1$ (day) (contour) and upper and lower thresholds (black curves).

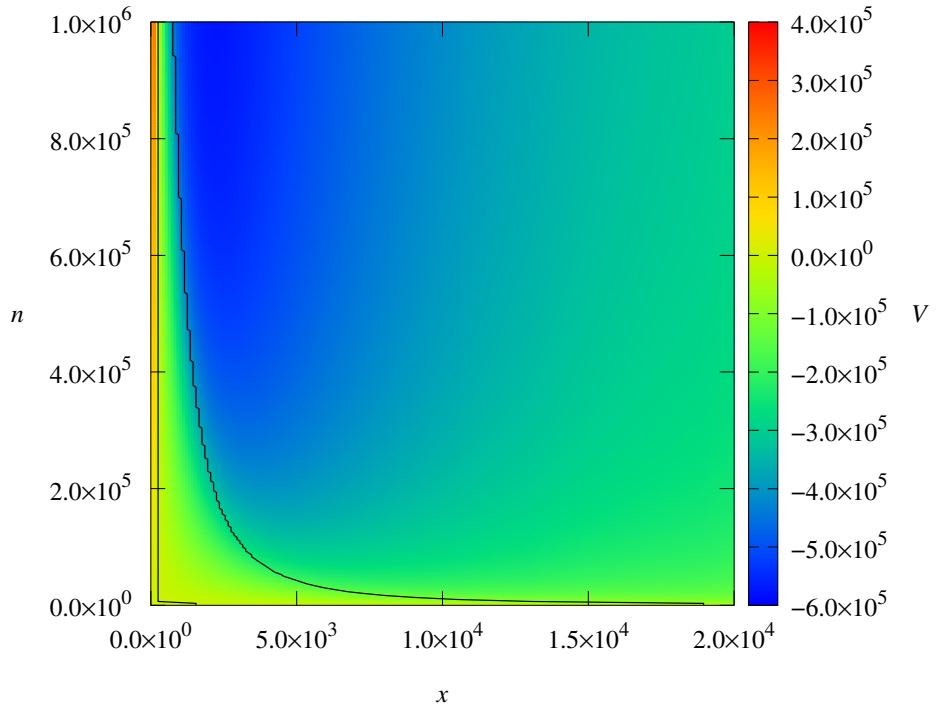


Fig. 6.2: The value function at $s = 70$ (day) (contour) and upper and lower thresholds (black curves).

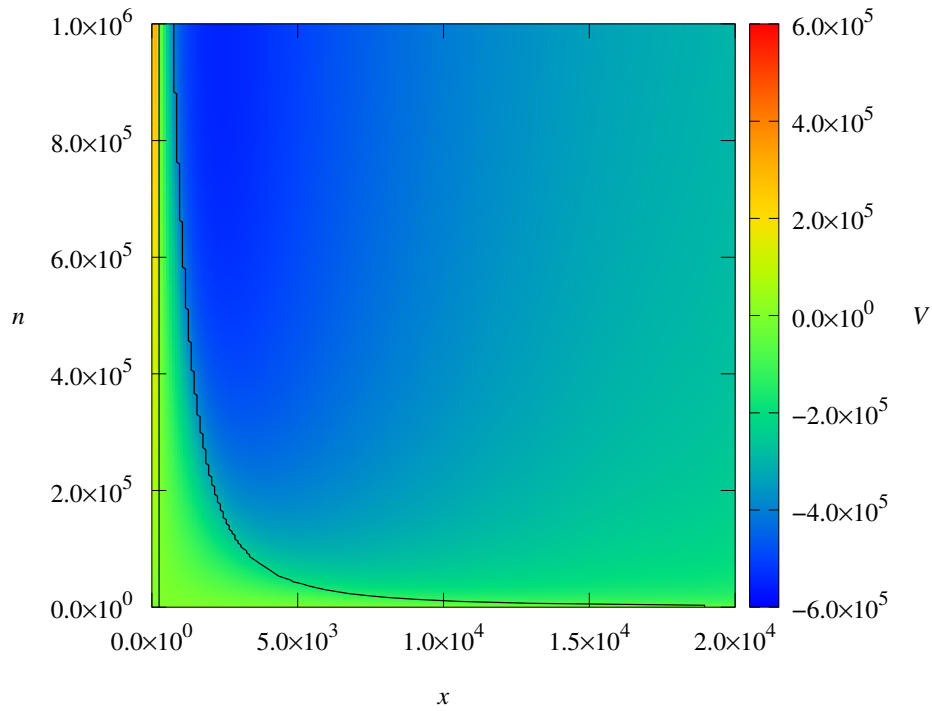


Fig. 6.3: The value function at $s = 80$ (day) (contour) and upper and lower thresholds (black curves).

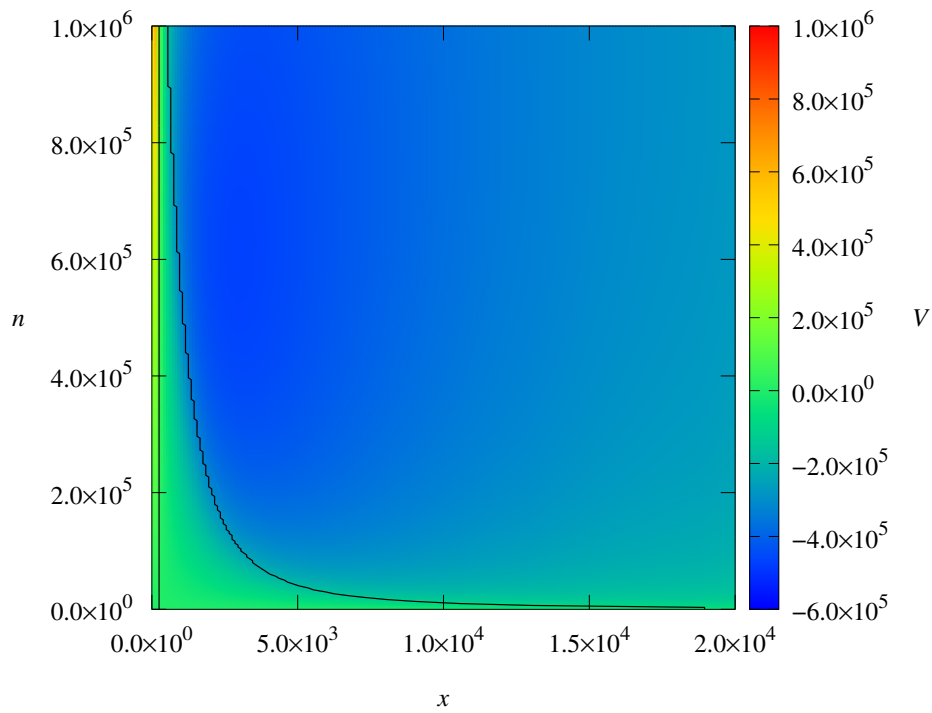


Fig. 6.4: The value function at $s = 120$ (day) (contour) and upper and lower thresholds (black curves).

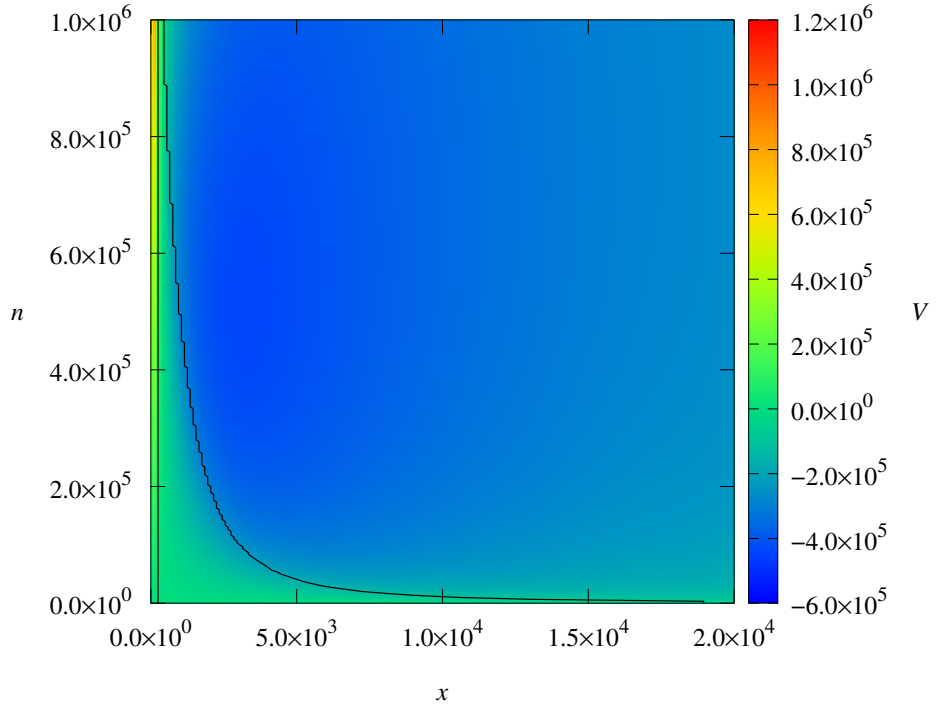


Fig. 6.5: The value function at $s = 150$ (day) (contour) and upper and lower thresholds (black curves).

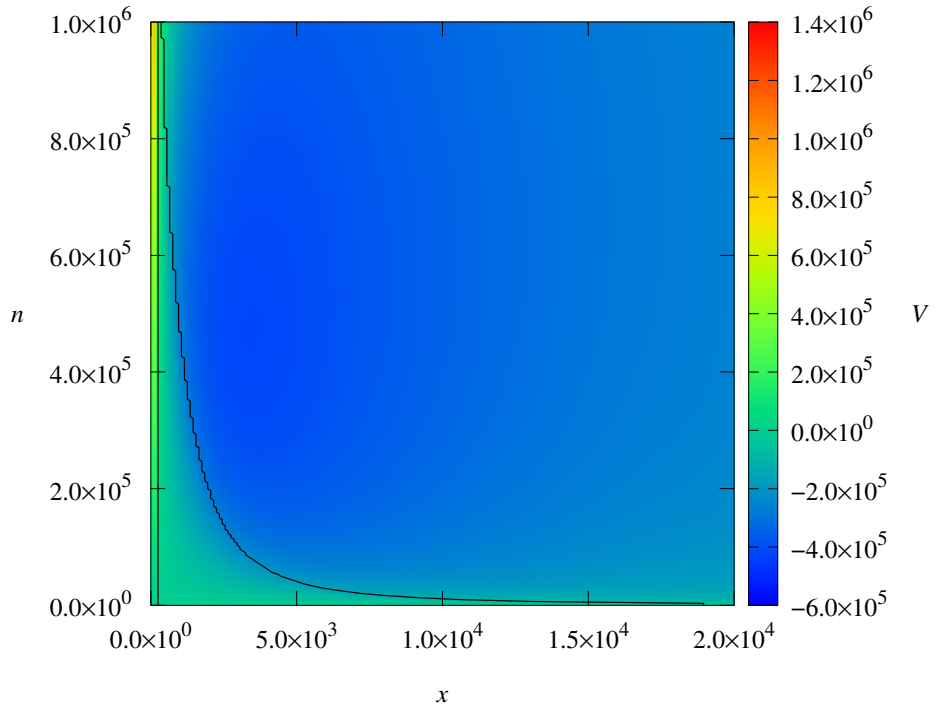


Fig. 6.6: The value function at $s = 180$ (day) (contour) and upper and lower thresholds (black curves).

According to the computational results, the value function V is not monotone with respect to x , however, is increasing with respect to n and decreasing with respect to s . The upper threshold is decreasing with respect to n at each time. It means that the decision-maker should set the upper threshold as small when there remains a large number of *P. altivelis* and as large when there remains a small number of *P. altivelis*. The latter fact implies that the decision-maker virtually does not have to take countermeasures when there remains a small number of *P. altivelis*. For such cases, the cost of taking countermeasures would surpass the profit obtained from catching *P. altivelis*. The lower threshold is constant with respect to n except for the neighborhood of $n = 0$ and is a small value at each time. This fact implies that the growth rate of *P. carbo* is sufficiently large and we do not have to consider its extinction in this parameter set. Thus, the decision-maker just reduces the number of *P. carbo* when the population size attains the upper threshold. This management plan is easy to implement in real situations. As the time elapses, the profiles of the value function and the thresholds do not qualitatively change. However, the profiles of the value function and the thresholds change quantitatively. Especially, between the no-fishing period $s < 60$ (day) and the fishing period $s \geq 60$ (day), the profiles change remarkably. In the no-fishing period, the upper and lower threshold do not change ($\underline{x} = 150$). At the beginning of the fishing period, the profiles remarkably change; the value function decreases in the whole domain, the upper threshold moves leftward and the lower threshold moves to rightward ($\underline{x} = 250$). In the fishing period, the upper threshold moves leftward as the time elapses and the lower threshold becomes constant; the increase around $n = 0$ disappears. This fact means that the decision-maker should more intensely protect *P. altivelis* after the beginning of the fishing period. Time-independent thresholds (management plans) are not appropriate when the dynamics of *P. carbo* and *P. altivelis* are considered simultaneously. In summary, the optimal management plan can be indicated as follows. In the no-fishing period, the decision-maker does not have to change the thresholds. At the beginning of the fishing period, the decision-maker should set the upper threshold smaller and the lower threshold larger. In the fishing period, the decision-maker should set

the upper threshold smaller as the time elapses while the lower threshold constant.

When other model parameters are employed for the computation, the profiles of the value function and thresholds do not qualitatively change, implying that similar management plans would appear in other study areas. Note that the upper threshold and lower threshold sometimes coincides when the weighting parameter β is large. This situation corresponds to the singular control explained in Chapter 4.

6.5 Conclusions

This chapter proposed a coupling stochastic impulse control model for deriving management plans of *P. altivelis* under the predation pressure from *P. carbo*. The maximized net profit or the value function is governed by an HJBQVI. The HJBQVI was numerically solved by the Petrov-Galerkin finite element scheme with the penalty method and the operator splitting. The management plan was computed and can be found from the computed graphs. The computational results suggested that the decision-maker should more intensely protect *P. altivelis* after the beginning of the fishing period. This methodology would contribute to the establishment of cost-effective as well as ecologically-sound management plans.

CHAPTER 7

Summary and future perspectives

7.1 Summary

This thesis addressed stochastic optimal control models for the management of *P. altivelis* under predation pressure from *P. carbo*. The obtained results in this thesis can be summarized as follows.

1. The stochastic regular control model for population management of released *P. altivelis* in river environment under predation from *P. carbo* and harvesting by a human was formulated. Numerical sensitivity analysis of the optimal management plans using the statistical indexes revealed their parameter-dependence.
2. The stochastic singular control model that finds a sustainable, cost-effective, and ecologically-sound management plans for the population dynamics of *P. carbo* was proposed and analyzed. The model and its determined parameter values were applied to the management plans of the *P. carbo* population in Lake Biwa, Japan. The current management plans around the lake were analyzed and verified from the viewpoint of the present mathematical model. A recommendation for optimizing the current management plans was made. The optimal policy should minimize the loss of riverine fish without compromising the ecosystem services provided by the birds.
3. The stochastic impulse control model for population management of *P. carbo* was introduced. The numerical schemes for time-independent HJBQVIs based on the

policy iteration approach and the penalty method were proposed. The accuracy of the schemes in the solution itself and the upper and lower thresholds were investigated by comparing them with an exact solution. The result indicated that the proposed numerical schemes can successfully compute the solution of the HJBQVIs at least with first-order and the thresholds for population control can be obtained within the length of the elements.

4. The coupling stochastic impulse control model for deriving management plans of *P. altivelis* under the predation pressure from *P. carbo* was proposed. The numerical scheme for solving HJBQVI, Petrov-Galerkin finite element scheme with the penalty method and the operator splitting was proposed. The computed management plans suggested that the decision-maker should change the management strategy before and after the opening time for fishing period.

7.2 Future perspectives

This thesis developed stochastic optimal control models for the management of *P. altivelis* under predation pressure from *P. carbo*. However, there exist several unresolved issues, which have to be addressed in future studies. At least, the following issues remain to be resolved.

1. The model parameters should be identified more in detail, especially the parameters that are relevant to the attitude of decision-makers. Such parameters are difficult to be identified.
2. Regularity and profile of the upper and lower thresholds in the coupling model should mathematically be investigated because only numerical simulations were performed in this thesis.
3. Probability density function associated with the coupling model should be investigated, which would lead to a deeper understanding of the dynamics under the optimal controls.
4. In this thesis, the dynamics are ordinary differential equations because spatial depen-

dence was ignored. However, there exist many kinds of real problems which have spatial dependence. In such cases, the dynamics are described by partial differential equations, and thus optimal control theory of partial differential equations is necessary. By using the optimal control theory of partial differential equations, the coupling model can be extended to a model with spatial dependence.

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