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Evolutionary Explanation”

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Favorite-Longshot Bias in Parimutuel Betting: an Evolutionary Explanation*

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Abstract

We offer an evolutionary explanation for the favorite-longshot bias in pari-mutuel betting, in a simple evolutionary market model. Because of a positive track take, the expected returns of any strategy stay negative and so any agent must vanish in the long run. Those who bet on favorites lose steadily whereas those who favor long shots have some chances of getting ahead with rare but large gains to survive longer. This relative advantage results in overvaluation of long shots.

1 Introduction

The explanation of the favorite - longshot bias (FLB) is perceived as one of the most crucial questions in research on betting markets. Griffith (1949) first reported that the realized average rates of returns from betting on favorite horses tend to be robustly and significantly greater than those from betting on long shot horses in American horse races. A number of empirical studies¹

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¹See Thaler and Ziemba (1988) and Hausch and Ziemba (1995) for surveys. Snowberg and Wolfers (2010) finds FLB in a very large date set of American horse races.

have documented that FLB emerges in horse race tracks among different countries, and in other kinds of gambling markets as well.

Intrigued and inspired by these findings, several theoretical explanations have been proposed.² FLB is deemed as a consequence of mis-perception of probabilities or irrationality of agents in early studies, such as Griffith (1949). Subsequent studies explain FLB through special features of preferences of agents: for instance, Weitzman (1965) considers preference for risk taking, and Ali (1977) relies on some heterogeneity of beliefs.³ Recent studies, such as Shin (1991), Hurley and McDonough (1995), and Ottaviani and Sørensen (2010), focus on strategic behavior among bettors as well as book makers under asymmetric information.

American horse race tracks adopt a pari-mutuel betting system. It has been pointed out that the odds determined in such a pari-mutuel betting system can be regarded as competitive market prices.⁴ FLB therefore offers a challenge to the efficient market hypothesis (EMH). In its strong form, the EMH asserts that the prices would be adjusted to equate the expected returns of various investment opportunities available in the markets. In contrast, FLB occurs indeed when the expected returns of favorites and long shots are not the same.

Although the aforementioned theoretical works can also be regarded as explanations of EMH failure, they are not very attractive in this context in our view. To elaborate, let us first ask why the EMH is expected to hold to begin with. A common and plausible answer is the Friedman hypothesis (Friedman, 1953), which roughly argues as follows: suppose that the expected returns are not equated in the markets. Then some agents must be holding assets with inferior expected returns, and these agents' demands must be significant enough to meet the supplies. But as time passes by such agents' wealth will diminish to a non-significant level relative to the size of the markets because of inferior returns. So if such inequality persists, their demands can meet only insignificant parts of the supplies, contrary to the assertion above. Consequently, the expected returns must be equated in the long run. Only those who are willing to support the equality will survive, and the rest will become insignificant and vanish ultimately.

This market selection process fueled by growth of wealth was formally

²Ottaviani and Sørensen (2008) is an excellent review of theoretical attempts.

³See also Snowberg and Wolfers (2010).

⁴See Snyder (1978).

modeled by Blume and Easley (1992) and then examined further by several works to be followed. If the same logic is applied to pari-mutuel betting markets, those who happen to favor lower than average returns, irrespective of the underlying reasons, e.g., mis-perception, risk preferences, beliefs, or information, will be driven out of the markets, and thus FLB would not arise in the long run. Therefore, the previous works do not explain why such special characteristics are relevant in the growth dynamics.

The motivation of this paper is to ask why Friedman’s compelling idea does not work when one views FLB as failure of the EMH. That is, we want to reconcile the logic of wealth dynamics with FLB to explain why agents who choose low expected returns tend to matter in the long run. For this purpose, we establish a simple evolutionary model of pari-mutuel betting markets for horse races. There are two horses in every race, one has a higher probability of winning than the other. There are a continuum of price taking agents who know the probability of winning for each horse. Agents’ preferences belong to one of two types: one type bets to maximize the expected returns, whereas the other type is interested in the variance of returns. Each agent bets one unit, and after a track take is subtracted, the pool of bets is paid out for the winning bets. The resulting odds can be seen as market clearing prices, and in a competitive equilibrium, all the expected return maximizers bet on the horse with high winning probability if FLB is exhibited.

Thus in our model, the expected return maximizers tend to gain more than the variance maximizers on average because they never bet on a over-valued horse. But notice that because of the track take, the wealth of neither types grows if the size of bias is moderate. Everybody loses in gambling in the long run. Hence the long run growth of wealth does not constitute a reasonable criterion for survival, unlike in Friedman’s world. This is the crucial point of departure.

We therefore postulate the following criterion: an agent is forced to exit the market with some probability if he does not win more than some specified amount. For the sake of exposition, let’s say it is the break even line. After betting on some races at the end of one day, if an agent is ahead within the day, he will return to the race track next day with a fresh pocket. But if not, no matter what the past experiences might be, the agent will think about quitting and with some probability he actually does. Then there will be new agents from some potential pool of agents who replace those who quit, and the next day of races begins.

We show that in this simple environment, FLB is exhibited in the long

run, under reasonable assumptions. The basic idea is very simple. Suppose FLB does not occur so that the expected net returns from the favorite and the long shot are the same. But this equality implies that the common return is negative because of the track take. Then the probability of a positive gain is larger for the long shot bet because of the higher variance. For instance, in an environment where a one dollar bet results in 80 cents on average, if one keeps betting on a horse who wins with a very high probability, the realized average returns per race will be concentrated around 80 cents. Therefore, the chance of making more than one dollar on average is very small. On the other hand, if one keeps betting on the unlikely winner, the realized returns will still be 80 cents on average but they are more spread out because of occasional large gains. Therefore, the chance of making more than one dollar per race is larger. Hence without FLB, the variance seekers are a better fit to the environment. FLB must occur to reduce the market fitness of variance seekers so that both types are equally fit in a long run steady state.

We do not regard the variance seekers, exhibiting extreme risk loving, as irrational. In an environment where the objective of an decision maker is to clear some difficult target value within a specified time period, it is often optimal to select a strategy to maximize the variance of random outcomes even for an expected value maximizers, as is explained in Dubins and Savage (1965). When the target is too high to reach by playing safely, it is better to adopt a risky strategy which gives one some chance of making it. So in fact our variance seekers are optimizing when no FLB exists, and in this sense, it is not irrationality of agents which creates the bias in our model.

We are also interested in how the size of the long run bias changes as the size of track take increases. We show that when it is small enough, FLB gets magnified as the track take increases. But this relation is ambiguous if the track take is large. The size of bias might decrease as the track take increases. We speculate that this might explain why FLB is not clearly observed in the Japanese horse race tracks where the track take is larger than in the other comparable countries.⁵ Theoretically, this reversal occurs since for a large track take, the market forces a large proportion of agents to exit, and hence the property of the long run steady state population is primarily determined by the property of the potential pool of agents replacing the losers.

This paper is organized as follows. The basic pari-mutuel market model is set up in Section 2, where we discuss how it can be viewed as a com-

⁵See Snowberg and Wolfers (2010) and its reference.

petitive equilibrium in detail. Section 3 summarizes the properties of the evolutionary dynamics we consider, taking the exit rule and the replacement rule as exogenously given. The exit criterion we proposed above is formally described in Section 4, and we show that FLB occurs as a unique and stable long run equilibrium. Section 5 is devoted for a comparative statics exercise, and we conclude with discussion on the generality of our idea beyond race tracks in section 6.

2 Simple Pari-mutuel Track Races Model

We shall begin with a very simple static model of race tracks with a pari-mutuel system. There are two horses, Favorite (F) and Longshot (L). Horse F wins with probability p and horse L wins with probability $1 - p$, where $\frac{1}{2} < p < 1$.

There is a continuum of agents of total size one, who are price takers. Each agent is either an expected payoff maximizer (type E) or a variance lover (type V), and bets a fixed amount β . A type E agent bets on the horse with the higher expected returns, while a type V agent bets on the horse with the higher variance of returns. Since we have normalized the total size of agents to be one, the total amount of bets is β .

There is a track take τ per unit of bet, $0 \leq \tau < 1$. That is, $\beta(1 - \tau)$ is paid out to the winning bet. So let B be the total number of agents who bet on F . Then the gross return on a unit of bet on F is $\beta(1 - \tau)/\beta B = (1 - \tau)/B$ if F wins. Similarly, the gross return on a unit of bet on L is $(1 - \tau)/(1 - B)$. Thus the expected returns as well as the variances of these binomial random variables can be readily found, which are reported in the following table:

	Expected returns	Variances
horse F	$p(1 - \tau)/B$	$p(1 - p)(1 - \tau)^2/B^2$
horse L	$(1 - p)(1 - \tau)/(1 - B)$	$p(1 - p)(1 - \tau)^2/(1 - B)^2$
indifference	$p = B$	$B = \frac{1}{2}$

where the last row indicates when the expected returns and the variances are equated for the two horses, respectively. Notice that for horse F , both terms are decreasing in B , and for horse L , they are increasing in B .

The price taking assumption implies that each agent in effect takes B as given because B completely determines the odds. Since a type E chooses a horse giving a higher expected revenue, a type E agent prefers F if $p > B$,

is indifferent between F and L if $B = p$, and strictly prefers L if $p < B$. On the other hand, type V agent bets on a horse with the larger variance, so type V bets on L if $B > 1/2$, is indifferent between F and L if $B = 1/2$, and strictly prefers F if $B < 1/2$.

Let y , $0 \leq y \leq 1$, be the number of type E agents in the race track. By definition a competitive equilibrium occurs at B where the total amount of bets on F given B coincides with B itself. Then the total amount of bets on L given B is exactly $1 - B$ by Walras law. Therefore, when $1/2 < y < p$, B must be equal to y in equilibrium, because all of type E bet on F and all of type V bet on L .

When $y \geq p$, B must be equal to p in a competitive equilibrium. Indeed, if $B > p$, all type E prefer to bet on L , so we get $p < B \leq 1 - y$, which is inconsistent with $y \geq p$ and $1 - p < p$. If $B < p$, then all type E prefer to bet on F , so we get $y \leq B < p$, which is inconsistent with $y \geq p$. At $B = p$, type E is indifferent between F and L , and type V prefers L , so this is a competitive equilibrium where exactly p of type E bet on F .

Finally, when $y \leq 1/2$, $B = \frac{1}{2}$ holds in equilibrium. Indeed, if $B > \frac{1}{2}$ ($\geq y$), all type V bet on L , thus $y \geq B$ would result. If $B < \frac{1}{2}$ instead, all agents bet on F , thus $B = 1$ would result. At $B = \frac{1}{2}$, type V is indifferent between F and L , and type E prefers F , so a competitive equilibrium occurs where exactly $\frac{1}{2}$ ($\leq 1 - y$) of type V bet on L .

The payout in a competitive equilibrium for a unit of bet on F is $(1 - \tau)/y$ with probability p if $1/2 < y < p$, and $(1 - \tau)/p$ if $p \leq y$. It means that this static market exhibits the *favorite longshot bias* (FLB) if and only if $y < p$, and conforms with the efficient market hypothesis if and only if $y \geq p$. In words, in this market, FLB can be explained if and only if the population fraction of type V agents exceeds the winning probability of the longshot horse. The following table summarizes the discussion so far:

range of y	equilibrium	type E bet	type V bet	market bias
$p < y$	$B = p$	p on F , $y - p$ on L	bet on L	no bias
$\frac{1}{2} \leq y \leq p$	$B = y$	on F	bet on L	FLB
$y < \frac{1}{2}$	$B = \frac{1}{2}$	on F	$\frac{1}{2}$ on L , $\frac{1}{2} - y$ on F	FLB

(1)

3 Simple Evolutionary Dynamics

3.1 The model and FLB in the long run.

We are interested in whether or not FLB arises in the long run. In the context of the setup in the previous section, it is equivalent to ask if the fraction of type E agents stay smaller than the winning probability p of the favorite horse as the population evolves over time. So denote by $y(t)$ the fraction of type E and $1 - y(t)$ be that of type V in day $t = 1, 2, \dots$. Since the outcome of the competitive model is uniquely determined by the fraction of type E at the time, it suffices to describe how $y(t)$ changes over time, as follows.

At the end of each day, some agents become unable to bet any more and quit from the race tracks for good. Denote by q_E and q_V the rates of agents quitting the betting market for type E and type V , respectively. Agents might be forced to exit for financial reasons or might lose interests on betting after bad experience: we shall elaborate on how these rates are related to the equilibrium odds in the betting market in the next section. Since the behavior of agents is independent on the past history by definition, we might as well assume that the quit rates in day t depend only on $y(t)$. Assume in addition that these rates depend continuously on the fraction of type E and that both $q_E(y)$ and $q_V(y)$ belongs to $(0, 1)$ for any $y \in [0, 1]$. The total size of quitting agents at the end of day t , $z(t)$, is therefore given by:

$$z(t) = q_E(y(t))y(t) + q_V(y(t))(1 - y(t)). \quad (2)$$

At the beginning of each day, new agents arrive to keep the total population equal to one. Denote by f the fraction of type E agents in the arriving agents replacing those who have just quit. We assume that f depends only on the prevailing returns on the horses, which equivalently means that f depends only on the fraction of type E agents, denoted by y . So assume that it is a continuous function of y , and that $f(y) \in (0, 1)$ for any $y \in [0, 1]$. We shall refer to function f as the *replacement rule*. To fix the idea, we offer two examples for the replacement rule, which we shall study in Section 4.

Example 1 (simple replicator) *New agents are chosen according to the relative fitness of the previous day, measured by the quit rates:*

$$f(y) = \frac{q_V(y)}{q_V(y) + q_E(y)}. \quad (3)$$

Here, the size of new type E agents increases if type E agents are more resilient (i.e., smaller q_E) in the previous day. This can be regarded as an environment where it is more likely for the new comers to adopt the behavior of well performing agents.⁶

Example 2 (fixed rate) New agents are chosen at random from an underlying pool of potential agents where the fraction of type E is δ , $\delta \in (0, 1)$:

$$f(y) = \delta. \quad (4)$$

This may be interpreted as an environment that the new comers do not take their predecessors' performance into account.

The fraction of type E agents therefore changes as follows:

$$y(t+1) = (1 - q_E(y(t)))y(t) + f(y(t))z(t). \quad (5)$$

Substituting (2) to (5), we have an evolutionary dynamics given by

$$y(t+1) = \Psi(y(t)), \quad t = 1, 2, \dots \quad (6)$$

where the policy function Ψ is given by

$$\begin{aligned} \Psi(y) &= (1 - q_E(y))y + f(y)\{q_E(y)y + q_V(y)(1 - y)\} \\ &= y - (1 - f(y))q_E(y)y + f(y)q_V(y)(1 - y). \end{aligned} \quad (7)$$

To see the long run outcome of our betting environment, let y^* be a steady state of dynamics (6). As is discussed in the previous section, FLB is exhibited in the long run if and only if $y^* < p$ holds. Setting $y^* = y(t) = y(t+1)$ in equation (6) and collecting terms, we find that $y^* \in (0, 1)$ is obtained as a solution to the following equation:

$$y^* = \frac{f(y^*)q_V(y^*)}{(1 - f(y^*))q_E(y^*) + f(y^*)q_V(y^*)}. \quad (8)$$

Note that y^* belongs to $(0, 1)$ because $0 < q_E, q_V, f < 1$.

⁶ Alternatively, $f(y) = \frac{1 - q_E(y)}{(1 - q_V(y)) + (1 - q_E(y))}$ can be motivated similarly.

3.2 Existence and Stability.

We shall report a sufficient condition which implies that a unique steady state exists and it is within an interval $[\underline{y}, \bar{y})$. Thus in particular FLB is exhibited if $\bar{y} = p$ in addition. Consider first the following assumption:

Assumption 1 (i) $q_E(y)$ is increasing and $q_V(y)$ is decreasing in y on $[\underline{y}, \bar{y}]$; (ii) $f(y)$ is non-increasing in y on $[\underline{y}, \bar{y}]$.

Part (i) of Assumption 1 says that the quit rate of a type goes up if its relative population size increases; the environment becomes increasingly unfavorable for type E and favorable for type V as it gets crowded with type E agents. Part (ii) of Assumption 1 means that the arriving agents of each type are not increasing in its relative population size, which excludes the uninteresting possibility of one type dominating the other simply because of increasing number of arrivals. The following lemma shows that $\Psi(y) - y$ is decreasing in $[\underline{y}, \bar{y}]$ under Assumption 1, which means in particular that a steady state in $[\underline{y}, \bar{y}]$ is unique if it exists.

Lemma 3 Under Assumption 1, $\Psi(y) - y$ is decreasing in y on $[\underline{y}, \bar{y}]$.

Proof. From (7),

$$\Psi(y) - y = -(1 - f(y))q_E(y)y + f(y)q_V(y)(1 - y).$$

$(1 - f(y))q_E(y)y$ is increasing and $f(y)q_V(y)(1 - y)$ is decreasing in $[\underline{y}, \bar{y}]$ under (i) and (ii) of Assumption 1. Hence, $\Psi(y) - y$ is decreasing in y on $[\underline{y}, \bar{y}]$. ■

The monotonicity result above implies that the boundary behavior of the functions is the key. So consider next the following assumption:

Assumption 2 (i) $q_V(\hat{y}) = q_E(\hat{y})$ for some $\hat{y} \in [\underline{y}, \bar{y})$; (ii) $f(\underline{y}) \geq \underline{y}$ and $f(\bar{y}) \leq \bar{y}$.

Part (i) of Assumption 2 says that the chance of quitting is the same for both types at some $\hat{y} \in [\underline{y}, \bar{y})$, i.e., the range $[\underline{y}, \bar{y})$ in question should contain a state where both types are equally fit, and hence neither type is uniformly dominant on the range. Notice that with part (i) of Assumption 1, it implies that at $y = \bar{y}$, type E is *less fit*. Thus, other things being equal, the type E

population will diminish more than the type V population at $y = \bar{y}$, hinting that the long run limit will be less than $\bar{y}$. Obviously this is the key condition for FLB, which we shall discuss in depth in the next section.

Part (ii) of Assumption 2 roughly means that type E should be scarce enough at $\underline{y}$ so that the population of type E should be non-decreasing at $\underline{y}$, and it should be abundant enough so that its population should be non-increasing at $\bar{y}$. The next result shows that these are indeed sufficient for identifying a steady state.

Lemma 4 *Under assumptions 1 and 2, $\Psi(\underline{y}) - \underline{y} \geq 0 > \Psi(\bar{y}) - \bar{y}$ holds and there is a unique steady state y^* in $[\underline{y}, \bar{y}]$.*

Proof. (i) of Assumption 1 and (i) of Assumption 2 imply $q_V(\underline{y}) \geq q_E(\underline{y})$ and $q_V(\bar{y}) < q_E(\bar{y})$. Hence, we have

$$\begin{aligned}\Psi(\underline{y}) - \underline{y} &= -(1 - f(\underline{y}))q_E(\underline{y})\underline{y} + f(\underline{y})q_V(\underline{y})(1 - \underline{y}) \\ &\geq -(1 - f(\underline{y}))q_V(\underline{y})\underline{y} + f(\underline{y})q_V(\underline{y})(1 - \underline{y}) \\ &= q_V(\underline{y})(f(\underline{y}) - \underline{y}) \geq 0,\end{aligned}$$

and similarly

$$\begin{aligned}\Psi(\bar{y}) - \bar{y} &= -(1 - f(\bar{y}))q_E(\bar{y})\bar{y} + f(\bar{y})q_V(\bar{y})(1 - \bar{y}) \\ &< -(1 - f(\bar{y}))q_V(\bar{y})\bar{y} + f(\bar{y})q_V(\bar{y})(1 - \bar{y}) \\ &= q_V(\bar{y})(f(\bar{y}) - \bar{y}) \leq 0.\end{aligned}$$

Hence, $\Psi(\bar{y}) - \bar{y} \geq 0$ and $\Psi(\bar{y}) - \bar{y} < 0$ hold. Since $\Psi(y) - y$ is decreasing in y by Lemma 3, we conclude that the steady state y^* uniquely exists and $\underline{y} \leq y^* < \bar{y}$ ■

Remark 5 *An examination of the proof of Lemma 4 reveals that when $f(\bar{y}) = \bar{y}$ and $q_V(\bar{y}) = q_E(\bar{y})$, we have $\Psi(\bar{y}) - \bar{y} = 0$ and so $y^* = \bar{y}$.*

As for the stability of the unique steady state, assume that all the relevant functions are continuously differentiable on $(\underline{y}, \bar{y})$, and observe the following property:⁷

⁷Differentiability is not essential, but it makes the analysis very simple.

Lemma 6 Assume assumptions 1 and 2. Suppose that there is a constant $\eta > 0$ such that

$$\max \{|q'_E(y)|, |q'_V(y)|\} \leq \eta \max \{q_E(y), q_V(y)\}$$

at any $y \in (\underline{y}, \bar{y})$ and $|f'|$ is bounded by κ on $(\underline{y}, \bar{y})$. Then, for any $y \in (\underline{y}, \bar{y})$,

$$1 > \Psi'(y) > 1 - (1 + \kappa + \eta) \max_{y \in [\underline{y}, \bar{y}]} (\max \{q_E(y), q_V(y)\}).$$

Proof. Since f is non increasing, $0 \leq -f'(y) \leq \kappa$ must hold for all $y \in (\underline{y}, \bar{y})$. Write $\epsilon := \max_{y \in [\underline{y}, \bar{y}]} (\max \{q_E(y), q_V(y)\})$. So at any $y \in [\underline{y}, \bar{y}]$, $0 \leq q_E(y), q_V(y) \leq \epsilon$ and $0 \leq q'_E(y) \leq \eta\epsilon, 0 \geq q'_V(y) \geq -\eta\epsilon$.

From (7), $\Psi'(y)$ can be computed directly as follow:

$$\begin{aligned} \Psi'(y) &= 1 - \{(1 - f(y))q_E(y) + f(y)q_V(y)\} \\ &\quad + \{f'(y)q_E(y)y + f'(y)q_V(y)(1 - y)\} \\ &\quad - \{(1 - f(y))yq'_E(y) - f(y)q'_V(y)(1 - y)\} \end{aligned}$$

Observe that since $q_E(y), q_V(y) \leq \epsilon$, we have

$$\begin{aligned} -\{(1 - f(y))q_E(y) + f(y)q_V(y)\} &\geq -\epsilon, \\ \{yq_E(y) + (1 - y)q_V(y)\} &\leq \epsilon, \end{aligned}$$

and since $-f'$ is bounded as above, the second inequality gives us:

$$\{f'(y)q_E(y)y + f'(y)q_V(y)(1 - y)\} \geq -\kappa\epsilon.$$

Finally, note that using $0 \leq q'_E(y) \leq \eta\epsilon, 0 \geq q'_V(y) \geq -\eta\epsilon$, we obtain

$$\begin{aligned} &-\{(1 - f(y))yq'_E(y) - f(y)q'_V(y)(1 - y)\} \\ &\geq -(1 - f(y))y\eta\epsilon - f(y)(1 - y)\eta\epsilon \\ &= -\eta\epsilon((1 - f(y))y + f(y)(1 - y)) \\ &\geq -\eta\epsilon((1 - f(y)) \max \{y, 1 - y\} + f(y) \max \{y, 1 - y\}) \\ &\geq -\eta\epsilon. \end{aligned}$$

The result is established by combining the inequalities above. ■

As is well known, if $0 < \Psi'(y) < 1$ on $(\underline{y}, \bar{y})$ and $y^* = \Psi(y^*) \in (\underline{y}, \bar{y})$, then starting with any initial point y_0 in $(\underline{y}, \bar{y})$, the dynamics $y(t) = \Psi(y(t - 1))$ $t = 1, 2, \dots$, converges monotonically to a unique steady state y^* . Thus, Lemma 6 says that if quitting probabilities $q_E(y)$ and $q_V(y)$ are small enough uniformly in $(\underline{y}, \bar{y})$, the unique steady state is globally stable.

4 Gambler's fate under positive track take

The dynamic analysis in the previous section takes the rates of quitting, q_E and q_V , as well as the replacement rule f , exogenously given. As we have promised, we shall now relate these rates to the basic static competitive market model.

With a positive race track, any type of gamblers will lose on average and the expected loss gets indefinitely large if an agent continues playing and the markets are efficient. Thus technically, a standard criterion of the long run growth in wealth is not sensible for survival in gambling markets. We postulate instead that the gamblers at the race track are there to enjoy the races, and they are satisfied if they “win” after a certain period of time, i.e., they are ahead of some target. To put it differently, quitting gamblers must be among those who are short of the target.

To formalize this idea in our set up, imagine that there are many races in one day and let K be the number of races in each day. We regard one day as the time period after which the agents review their performance. The races are identical, and the outcomes are independent.⁸ Each agent bets one unit of money at each race and has a target rate of return from gambling $\hat{x}$. Since the races are iid, assume that the betting strategy is the same throughout the day, and the same equilibrium occurs at every race.

If an agent has won more than a target of wealth $\hat{x}K$ at the end of the day, it is an enjoyable day for him and he is determined to return to the race track. On the other hand, if an agent's gain is not more than $\hat{x}K$, he is severely discouraged and doubts if he should ever come back to the race track. Call such an agent a loser, and we assume that some losers actually quit gambling. In reality, the size of the loss might influence the decision making, but for simplicity we assume that exit takes place with a common probability ϵ for all losers independently. Technically, the size of ϵ concerns the stability, and it is not crucial for eliciting FLB. If a loser happens to stay in, then he forgets all the troubles and comes back to the race track with a fresh mind and a fresh pocket.

Let us then find the number of exiting agents in this scenario. Let W_F and W_L be the terminal wealth after betting all day on horse F and L , respectively. Denote the chance of an agent exiting the race track by $\rho_j(B)$

⁸Obviously if for a fixed race, the outcomes of F and L horses are perfectly negatively correlated, so implicitly we are assuming that this model is an abstraction of many races at many different race tracks.

when a agent bets on horse $j = F, L$ on the whole day. According to our postulate about the exiting rule described above, $\rho_j(B)$ is given by

$$\rho_j(B) = \epsilon \Pr[W_j \leq \hat{x}K]$$

for $j = F, L$.⁹

Recall that when the fraction of bet on F is B , then mean μ_j and variance σ_j^2 of the return betting on horse j ($j = F, L$) in each race are given by

$$\begin{aligned} \mu_F &= \frac{1-\tau}{B}p - 1, & \sigma_F^2 &= \left(\frac{1-\tau}{B}\right)^2 p(1-p), \\ \mu_L &= \frac{1-\tau}{1-B}(1-p) - 1, & \sigma_L^2 &= \left(\frac{1-\tau}{1-B}\right)^2 p(1-p). \end{aligned} \quad (9)$$

By assumption there are many races, let us further assume that W_F and W_L are in fact independent normal random variables; that is, W_j can be regarded as the normal distribution of mean $\mu_j K$ and variance $\sigma_j^2 K$.¹⁰ Standardizing the wealth per bet by

$$z_j(B) = \frac{KB - \mu_j K}{\sigma_j \sqrt{K}}, \quad (10)$$

one can express $\rho_j(B)$ as

$$\rho_j(B) = \epsilon \Phi(z_j(B)) = \epsilon \int_{-\infty}^{z_j(B)} \phi(u) du, \quad (11)$$

for $j = F, L$, where Φ and ϕ are the cumulative probability distribution function and the density function for the standard normal distribution, respectively. Substituting (9) to (10), z_j can be written as

$$\begin{aligned} z_F(B) &= \sqrt{\frac{K}{p(1-p)}} \left\{ \frac{1+\hat{x}}{1-\tau} B - p \right\}, \\ z_L(B) &= \sqrt{\frac{K}{p(1-p)}} \left\{ \frac{1+\hat{x}}{1-\tau} (1-B) - (1-p) \right\}. \end{aligned} \quad (12)$$

Notice in (9) that when $p = B$ hence there is no bias, the expected rate of return is $-\tau$. Hence, if $\hat{x} \leq -\tau$, the target can be thought as a modest one. We believe that gamblers tend to have a more optimistic target, and so it will be sensible to imagine that $\hat{x}$ is greater than $-\tau$.

⁹This may be seen as an extremely simplified version of the so called *Gambler's Ruin problem*. In principle, one should be able to replace ρ_j with the probability of reaching $\hat{x}k$ at some $k \leq K$, which will make the analysis more complicated. But we conjecture that the basic message remains the same.

¹⁰Of course, if these agents bet on exactly the same races, their wealth will be correlated. In principle the analysis can be carried out taking care of correlation, but we believe that such an analysis just blurs our message.

Remark 7 If $\hat{x} > -\tau$, $z_F(p) > z_L(p)$ and hence $\rho_F(p) > \rho_L(p)$ holds because $p > \frac{1}{2}$, which means that when the odds are fair, an agent betting on F is more likely to be a loser than an agent betting on L . This is intuitive: when $\hat{x} = -\tau$, then the chance of not making the target is half irrespective of the variance, while when $\hat{x}$ exceeds $-\tau$, the higher the variance is, i.e., the flatter the distribution is, the less is the chance of not making the target.

Now we relate the chance of exit to the rate of quitting for each type. We assume that the law of large number allows us to equate the probability of exit to the fraction of quitting agents in the population. Then as is summarized in (1), when $\frac{1}{2} \leq y \leq p$, all type E bet on F and all type V bet on L , we let $q_E(y) = \rho_F(y)$ and $q_V(y) = \rho_L(y)$. In contrast, when $y > p$ or $\frac{1}{2} > y$, the equilibrium odds do not depend on y but agents of the same type behave differently in equilibrium, and we need to adjust the quitting rates accordingly. When $y > p$, suppose that $\frac{p}{y}$ of type E keep betting on F and $\left(1 - \frac{p}{y}\right)$ of them keep betting on L , and then the implied quit rate is $\frac{p}{y}\rho_F(p) + \left(1 - \frac{p}{y}\right)\rho_L(p)$.¹¹ The case of $y < \frac{1}{2}$ is worked out similarly to obtain the following relation:

$$\begin{aligned} q_E(y) &= \begin{cases} \rho_F(\frac{1}{2}) & y < \frac{1}{2}, \\ \rho_F(y) & \frac{1}{2} \leq y \leq p, \\ \frac{p}{y}\rho_F(p) + \left(1 - \frac{p}{y}\right)\rho_L(p) & y > p, \end{cases} \\ q_V(y) &= \begin{cases} \frac{1}{2(1-y)}\rho_L(\frac{1}{2}) + \frac{1-2y}{2(1-y)}\rho_F(\frac{1}{2}) & y < \frac{1}{2} \\ \rho_L(y) & \frac{1}{2} \leq y \leq p, \\ \rho_L(p) & y > p. \end{cases} \end{aligned} \quad (13)$$

Now we shall verify that the key assumptions are satisfied in this set up.

Lemma 8 *Assumption 1 holds for $[y, \bar{y}] = [\frac{1}{2}, p]$ when the replacement rule is given by a simple replicator (3) or a constant (4). If $\hat{x} > -\tau$ in addition, then Assumption 2 holds on $[\frac{1}{2}, p]$ for a simple replicator (3) and for a constant (4) with $\frac{1}{2} \leq \delta \leq p$.*

Proof. On $[1/2, p]$, $q_E(y)$ and $q_V(y)$ are identical to $\rho_F(y)$ and $\rho_L(y)$, respectively.

¹¹Or one may assume that agents take turns to bet on F or L to justify a different rate but still it must be a convex combination of $\rho_F(p)$ and $\rho_L(p)$. The essence of our analysis remains the same as long as it is such a combination.

(i) of Assumption 1 holds; recall (12). Since z_F is increasing and z_L is decreasing in B , $\rho_F(B)$ is increasing and $\rho_L(B)$ is decreasing by (11).

(ii) of Assumption 1 trivially holds if f is constant (4). When f is given by a simple replicator (3), (ii) also holds because $\rho_E(y)$ is increasing and $\rho_V(y)$ is decreasing in y , and hence $f(y) = \frac{q_V(y)}{q_V(y) + q_E(y)} = \frac{1}{1 + \rho_E(y)/\rho_V(y)}$ is decreasing in y .

Now we assume that $\hat{x} > -\tau$ and show that Assumption 2 holds. A candidate for $\hat{y}$ can be found by directly solving $\rho_F(\hat{y}) = \rho_L(\hat{y})$ in (12), as follows:

$$\hat{y} = \frac{1}{2} + \frac{(2p-1)(1-\tau)}{2(1+\hat{x})}. \quad (14)$$

Note that $\hat{x} > -\tau$ implies that $\hat{y} < p$ (recall Remark 7). Since $p > 1/2$ and $\tau < 1$, we also conclude that $\hat{y} > 1/2$. Hence (i) of Assumption 2 holds for both rules.

It remains to verify (ii) of Assumption 2. When f is given by (4) and $\frac{1}{2} \leq \delta \leq p$, it is clearly satisfied. When f is given by (3), since $\rho_L(p) < \rho_F(p)$ when $\hat{x} > -\tau$, by the construction of q_j , we find that

$$\frac{q_V(p)}{q_V(p) + q_E(p)} < \frac{q_V(p)}{q_V(p) + q_V(p)} = \frac{1}{2},$$

hence, $f(p) \leq p$ since $1/2 < p$. Also $f(\frac{1}{2}) \geq \frac{1}{2}$ follows since $\rho_F(\frac{1}{2}) < \rho_L(\frac{1}{2})$.
■

We shall now prove the main result, which says that when target return $\hat{x}$ exceeds $-\tau$, i.e., the gamblers are ambitious enough, these functions satisfy assumptions 1 and 2, and therefore FLB is exhibited at a unique steady state.

Proposition 9 *Suppose that the replacement rule f is given by a simple replicator (3) or a constant (4) with $\frac{1}{2} \leq \delta \leq p$. If $\hat{x}$ is greater than $-\tau$, then FLB is exhibited at a unique steady state.*

Proof. By Lemma 8, there is a steady state y^* in $[\frac{1}{2}, p)$, and it is unique in $[\frac{1}{2}, p)$. So the result is established if we verify that there is no steady state outside $[\frac{1}{2}, p)$.

If $y \geq p$, substituting (13) to (7), we have

$$\begin{aligned} \Psi(y) - y &= -(1 - f(y)) \left(\frac{p}{y} \rho_F(p) + \left(1 - \frac{p}{y} \right) \rho_L(p) \right) y + f(y) \rho_L(p)(1 - y) \\ &= f(y) \{ p(\rho_F(p) - \rho_L(p)) + \rho_L(p) \} - p(\rho_F(p) - \rho_L(p)) - \rho_L(p)y. \end{aligned}$$

So if f is a constant (4), then $\Psi(y) - y$ is decreasing in y on $(p, 1]$. Since assumptions 1 and 2 hold by Lemma 8, $\rho_F(p) > \rho_V(p)$ holds, and so evaluating the equation above at $y = p$, we see that $\Psi(p) - p < 0$ follows. Thus there cannot be a steady state in $[p, 1]$. If f is a simple replicator (3) and $\hat{x} > -\tau$, then $\rho_F(p) > \rho_L(p)$ holds and so $f(y) \leq \frac{1}{2}$ on $[p, 1]$. Then, for any $y \in [p, 1]$,

$$\begin{aligned}\Psi(y) - y &\leq \frac{1}{2} (p(\rho_F(p) - \rho_L(p)) + \rho_L(p)) - p(\rho_F(p) - \rho_L(p)) - \rho_L(p)y \\ &= -\frac{1}{2}p(\rho_F(p) - \rho_L(p)) + \left(\frac{1}{2} - y\right) \rho_L(p) \\ &< 0,\end{aligned}$$

since $y \geq p > \frac{1}{2}$. Consequently, there cannot be a steady state in $[p, 1]$.

Similarly, if $y \leq \frac{1}{2}$, we have

$$\begin{aligned}\Psi(y) - y &= -(1 - f(y))\rho_F\left(\frac{1}{2}\right)y + f(y) \left(\frac{1}{2(1-y)}\rho_L\left(\frac{1}{2}\right) + \frac{1-2y}{2(1-y)}\rho_F\left(\frac{1}{2}\right) \right) (1-y) \\ &= \frac{f(y)}{2} \left(\rho_F\left(\frac{1}{2}\right) + \rho_L\left(\frac{1}{2}\right) \right) - \rho_F\left(\frac{1}{2}\right)y.\end{aligned}$$

Again, it is decreasing in y if f is a constant, and $\rho_L\left(\frac{1}{2}\right) > \rho_F\left(\frac{1}{2}\right)$ by assumption so $\Psi(y) - y > 0$ holds on $\left[0, \frac{1}{2}\right]$. If f is a simple replicator (3) and $\hat{x} > -\tau$, $\rho_F\left(\frac{1}{2}\right) < \rho_L\left(\frac{1}{2}\right)$ and so $f(y) \geq \frac{1}{2}$ on $\left[0, \frac{1}{2}\right]$. Then, for $y \in \left[0, \frac{1}{2}\right]$,

$$\begin{aligned}\Psi(y) - y &\geq \frac{1}{4} \left(\rho_F\left(\frac{1}{2}\right) + \rho_L(p) \right) - \frac{1}{2}\rho_F\left(\frac{1}{2}\right) \\ &= \frac{1}{4}\rho_L\left(\frac{1}{2}\right) - \frac{1}{4}\rho_F\left(\frac{1}{2}\right) \\ &> 0.\end{aligned}$$

Therefore, there cannot be a steady state in $\left[0, \frac{1}{2}\right]$. ■

Next, we shall establish a result which ensures the stability of the unique steady state if the chance of quitting, ϵ , for losers is sufficiently small.

Proposition 10 *Suppose that the replacement rule f is given by a simple replicator (3) or a constant (4) with $\frac{1}{2} \leq \delta \leq p$. Then there exists $\bar{\epsilon} > 0$ such that for any chance of quitting $\epsilon < \bar{\epsilon}$, the corresponding unique steady state $y^* \in \left(\frac{1}{2}, p\right)$ is globally stable in $\left(\frac{1}{2}, p\right)$.*

Proof. Recall that q_E and q_V are given by quitting probability ϵ times the cumulative distribution function of the standard normal distribution, so we can find a constant $\eta > 0$ independently of ϵ such that $\max \{|q'_E(y)|, |q'_V(y)|\} \leq \eta \max \{q_E(y), q_V(y)\}$ on $(\frac{1}{2}, p)$. Moreover, $\max_{y \in [y, \bar{y}]} (\max \{q_E(y), q_V(y)\}) \leq \epsilon$. By Lemma 6, it therefore suffices to show that a constant κ can be found independently of ϵ to bound f' on $(\frac{1}{2}, p)$.

If the replacement rule f is a constant (4) with $\delta \leq p$, then $f' = 0$ and so this can be trivially done. So it remains to establish it for the replacement rule f is given by a simple replicator (3). But notice that in this case f itself is independent of ϵ and so is f' , because both $q_V(y)$ and $q_E(y)$ are standard normal distributions multiplied by the same ϵ . Also as $\frac{1}{\epsilon}(q_V(y) + q_E(y))$ is bounded away from zero, $f'(y)$ can in fact continuously extended on $[\frac{1}{2}, p]$, and hence in particular it is bounded on $(\frac{1}{2}, p)$. ■

5 Comparative Statics: Role of track take

To facilitate a comparative statics analysis on the steady state, let functions q_E , q_V , and f depend on an exogenous variable α in some prespecified interval $I \subseteq \mathbb{R}$, and denote them by $q_E(y, \alpha)$, $q_V(y, \alpha)$ and $f(y, \alpha)$. The corresponding policy function (7) is denoted by $\Psi(y, \alpha)$. Assume that assumptions 1 and 2 hold at any $\alpha \in I$, and write $y^*(\alpha)$ for the unique steady state when the exogenous parameter is set at α . Letting $\xi(y, \alpha) = \Psi(y, \alpha) - y$, equation (8) implies that the steady state $y^*(\alpha)$ satisfies

$$\xi(y^*(\alpha), \alpha) = 0. \quad (15)$$

Apply the Implicit Function Theorem to (15), and we have

$$\xi_y(y^*(\alpha), \alpha) \frac{d}{d\alpha} y^*(\alpha) + \xi_\alpha(y^*(\alpha), \alpha) = 0,$$

where ξ_y and ξ_α is partial derivatives of ξ by y and α , respectively, as long as ξ_y does not vanish at $(y^*(\alpha), \alpha)$. As Lemma 3, $\xi_y(y^*(\alpha), \alpha) < 0$ under Assumption 1. Hence, $\frac{d}{d\alpha} y^*(\alpha)$ and $\xi_\alpha(y^*(\alpha), \alpha)$ have the same sign, and hence we have shown the following result:

Lemma 11 *Under Assumption 1, the steady state $y^*(\alpha)$ is increasing (decreasing) as an exogenous parameter $\alpha \in I$ increases, if $\xi_\alpha(y^*(\alpha), \alpha)$ is positive (negative).*

Now we are ready to examine how the track take affects FLB in the set up of normally distributed wealth. Let

$$\alpha = (1 + \hat{x})/(1 - \tau) \quad (16)$$

and substitute α to (12). Then, the critical values z_F and z_L can be rewritten as

$$\begin{aligned} z_F(B, \alpha) &= \sqrt{\frac{K}{p(1-p)}}(\alpha B - p), \\ z_L(B, \alpha) &= \sqrt{\frac{K}{p(1-p)}}(\alpha(1 - B) - (1 - p)). \end{aligned} \quad (17)$$

Write ρ_F and ρ_L as functions of y and α accordingly.

When the replacement rule is a constant (4), we find the following result.

Lemma 12 *Suppose that $f(y, \alpha) = \delta$ where $\delta \in [\frac{1}{2}, p]$ and let $I \subseteq [1, +\infty)$ be an open interval. Then, $y^*(\cdot)$ is decreasing (resp. increasing) at α in I , if $-(1 - \delta)\phi(z_F(y^*(\alpha), \alpha))(y^*(\alpha))^2 + \delta\phi(z_L(y^*(\alpha), \alpha))(1 - y^*(\alpha))^2 < 0$ holds (resp. $>$ holds).*

Proof. By Lemma 11, we have only to examine the sign of $\xi_\alpha(y^*(\alpha), \alpha)$. Since y^* belongs to $[1/2, p]$, we may set $q_E(y^*(\alpha), \alpha) = \rho_F(y^*(\alpha), \alpha)$ and $q_V(y^*(\alpha), \alpha) = \rho_L(y^*(\alpha), \alpha)$. Hence, $\xi(y, \alpha)$ is expressed as

$$\xi(y, \alpha) = -(1 - \delta)\rho_F(y, \alpha)y + \delta\rho_L(y, \alpha)(1 - y)$$

Differentiating $\rho_F(y, \alpha)$ and $\rho_L(y, \alpha)$ by α , we have

$$\frac{\partial \rho_F}{\partial \alpha} = \phi(z_F(y, \alpha)) \frac{\epsilon \sqrt{K}}{\sqrt{p(1-p)}} y, \quad \frac{\partial \rho_L}{\partial \alpha} = \phi(z_L(y, \alpha)) \frac{\epsilon \sqrt{K}}{\sqrt{p(1-p)}} (1 - y). \quad (18)$$

Therefore, differentiating $\xi(y, \alpha)$ with respect to α and evaluating it at $y = y^*(\alpha)$,

$$\begin{aligned} &\xi_\alpha(y^*(\alpha), \alpha) \\ &= -(1 - \delta) \frac{\partial}{\partial \alpha} \rho_F(y^*(\alpha), \alpha) y^*(\alpha) + \delta \frac{\partial}{\partial \alpha} \rho_L(y^*(\alpha), \alpha) (1 - y^*(\alpha)) \\ &= \frac{\epsilon \sqrt{K}}{\sqrt{p(1-p)}} \left\{ -(1 - \delta) \phi(z_F(y^*(\alpha), \alpha)) (y^*(\alpha))^2 + \delta \phi(z_L(y^*(\alpha), \alpha)) (1 - y^*(\alpha))^2 \right\} \end{aligned}$$

Hence the sign of $\xi_\alpha(y^*(\alpha), \alpha)$ is determined as is stated. ■

Now we shall focus on the special case of $\delta = p$, i.e., the arriving rate of type E is equal to the probability that horses F wins. Although the ratio of arriving agents can sustain “efficiency” of the betting market, Proposition 9 has shown that FLB emerges when $\hat{x} > -\tau$.¹² For this special case we have a clear comparative statics result as follows:

Proposition 13 *Suppose that $f(y, \alpha) = p$. Then, for sufficiently small $\eta > 0$, $y^*(\alpha)$ is decreasing in $\alpha \in [1, 1 + \eta]$.*

Proof. Notice that $\alpha = 1$ in (16) implies $\hat{x} = -\tau$. It is readily confirmed that $y^* = p$ is a unique solution to $\xi(y, 1) = 0$. So $y^*(1) = p$, and $z_F(y^*(1), 1) = z_L(y^*(1), 1)$.

To apply Lemma 12, it suffices to show that $-(1-p)\phi(z_F(y^*(\alpha), \alpha))(y^*(\alpha))^2 + p\phi(z_L(y^*(\alpha), \alpha))(1 - y^*(\alpha))^2$ is negative for α close to one. By direct computation, we have;

$$\begin{aligned} & -(1-p)\phi(z_F(y^*(1), 1))(y^*(1))^2 + p\phi(z_L(y^*(1), 1))(1 - y^*(1))^2 \\ & = -p^2(1-p)\phi(z_F(y^*(1), 1)) + p(1-p)^2\phi(z_F(y^*(1), 1)) \\ & = p(1-p)(1-2p)\phi(z_F(y^*(1), 1)) < 0. \end{aligned}$$

Hence the desired conclusion follows by continuity. ■

Proposition 13 is a local result and it does not assert that the bias is globally increasing in τ . One might expect that such an assertion would be true since the track take represents market friction. But this is not the case in general. The intuition is very simple as a matter of fact. Think of an extreme case where τ is so high that it is just impossible to win at all, and $\epsilon = 1$. Then the market is always filled with newly arriving agents even in the long run, and hence the steady state will inherit the property of the pool of potential agents, and the results of past races will not matter much; that is, the property of steady state will be governed by the property of f . In particular, FLB caused by market friction would rather diminish in the range where τ is extremely large.

For instance, under a constant replacement rule with $\delta = p$, the steady state will in fact approach p , which means that at some point the bias starts decreasing as τ increases. Indeed, let $\alpha \rightarrow +\infty$, and observe that $z_F(y^*(\alpha), \alpha)$ and $z_L(y^*(\alpha), \alpha)$ approach ∞ by (17), and $q_E(y^*(\alpha), \alpha)$ and $q_V(y^*(\alpha), \alpha)$

¹²When $\tau = 0$ and $\hat{x} = 0$, $q_V(p) = q_E(p)$ and so FLB does not occur by Remark 5.

approach 1. Then, by (8), $y^*(\alpha)$ approaches $f(y^*(\alpha))$, and so $y^*(\alpha) \rightarrow p$ holds when $f(y) = p$ everywhere.

To appreciate this phenomenon numerically, figure 1 depicts relationship between τ and y^* when $p = f(y, \alpha) = 0.9$, $\epsilon = 0.1$, $K = 30$ and $\hat{x} = 0$. So FLB occurs if y^* is smaller than 0.9. As the figure shows, FLB does not exist when $\tau = 0$. Then y^* decreases and FLB increases at the neighborhood of $\tau = 0$ when τ increase as Proposition 13 showed above. In contrast, y^* increases when $\tau \geq 1.3$ and the size of FLB decreases in τ .

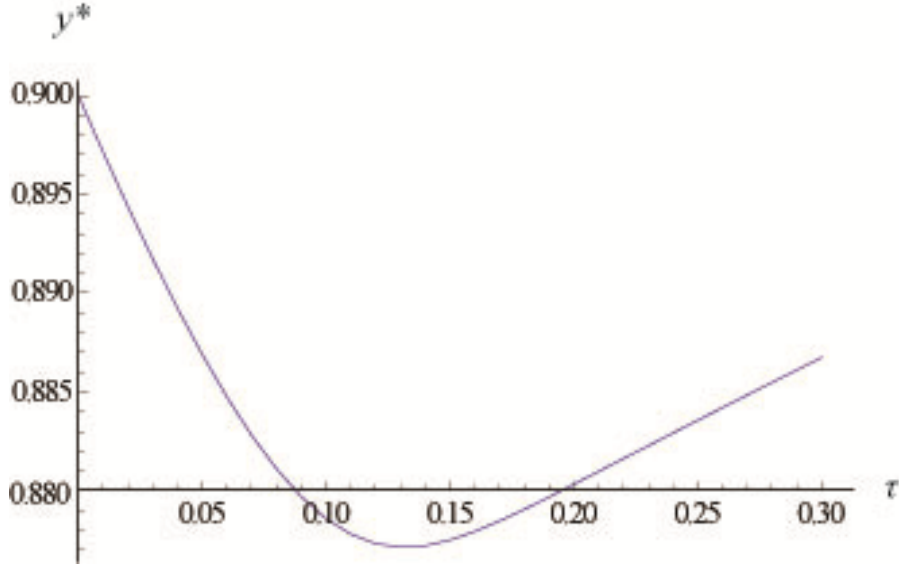


Figure 1: Equilibrium odds and track take for fixed replacement rate

Of course, the fact that FLB vanishes in this case does not imply that the market learns more to become more efficient as α increases. The limit probability approaches δ , rather than p . Thus it is more accurate to say that if as α gets larger, the market loses its learning power and the property of the entering population matters more.

The idea is similar for the case of a replicator type replacement. Both q_V and q_E go to one as nobody can survive in the limit, and then both types will appear badly but equally fit. Hence the replacement rule supplies roughly the same amounts for each type from the population. Consequently, when the target for survival is very severe, the steady state y^* will approach 0.5.

Figure 2 depicts relationship between α (see (16)) and y^* when $p = 0.75$, $\epsilon = 0.2$, $K = 30$, when the replacement rule is given by (3). So FLB occurs if y^* is smaller than 0.75. As is expected, FLB does not exist at $\alpha = 0$. As α goes up, i.e., as the target gets more severe, the size of FLB increases, and y^* approaches 0.5.

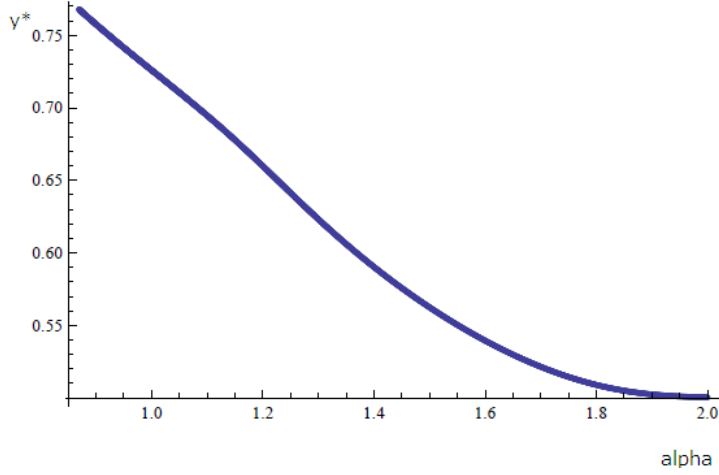


Figure 2: Equilibrium odds and survival target for replicator rule

6 Concluding Remarks

We have demonstrated in a simple evolutionary market model that FLB arises in the long run. The driving force for this result is that under the exit rule we postulated, the variance seekers, who may be considered as an extreme form of risk lovers, are better fit than the expected value maximizers when the markets are not biased.

Under our exit rule, one must earn more than a target value to survive. We justified this rule in the context of gambling markets, but it might be questionable when the market returns are positive. Nevertheless, let us tentatively suppose that the track take τ is negative and so the market returns are positive in absence of FLB, but the same exit rule applies. Notice that the steady state y^* is effectively determined by α in (16) and so even if τ is negative, FLB can still arise with high target $\hat{x}$. Intuition is simple, and the

same as before: without FLB, both types will earn the same average return, and the distribution of returns for the variance seekers have a fatter tail. So if the target is set to be higher than the average, then the variance seekers have a better chance of meeting the target and so they are better fit. A bias must arise to offset this advantage for survival. That is, we have an evolutionary model where wealth grows but some bias persists. Coming back to the discussion of the EMH in Section 1, the growth based justification of the EMH outlined there fails once the target based survival criterion is accepted.

We are therefore tempted to speculate more generally that in a market environment where some “large” agents’ survival is conditional on achieving some higher than average target value, the markets tend to exhibit some biases in favor of low risk alternatives. For instance, imagine an environment where the performance of fund managers are evaluated first by whether or not the fund outperformed some benchmark number which represents the average returns of some sort. Those managers who do not make the target are subject to some chance of career termination. To outperform the average, the fund managers are naturally interested in riskier alternatives, and the logic of our analysis indicate that such preferences will exaggerate the market returns of less risky assets.

In conclusion, we contend that the implication of the target driven behavior is worth investigating, beyond the race tracks.

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