

The Reachability and Related Decision Problems for Semi-Constructor TRSs

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Abstract

This paper shows that reachability is undecidable for confluent monadic and semi-constructor TRSs, and joinability and confluence are undecidable for monadic and semi-constructor TRSs. Here, a TRS is monadic if the height of the right-hand side of each rewrite rule is at most 1, and semi-constructor if all defined symbols appearing in the right-hand side of each rewrite rule occur only in its ground subterms.

1 Introduction

In this paper, we consider the reachability problem for confluent monadic and semi-constructor TRSs posed by our previous paper [4]. Here, a TRS is monadic if the height of the right-hand side of each rewrite rule is at most 1, and semi-constructor if all defined symbols appearing in the right-hand side of each rewrite rule occur only in its ground subterms. We give a negative answer to this problem. This undecidability result is compared with the decidability results of joinability and unification for the same class [4, 3].

Moreover, we show that joinability and confluence are undecidable for monadic and semi-constructor TRSs.

2 Preliminaries

We assume that the reader is familiar with standard definitions of rewrite systems [1] and we just recall here the main notations used in this paper.

Let F be a finite set of operation symbols graded by an arity function $ar: F \rightarrow \mathbb{N}(= \{0, 1, 2, \dots\})$, $F_n = \{f \in F \mid ar(f) = n\}$. We use x, y as variables, f as an operation symbol, r, s, t as terms. Let $V(s)$ be the set of variables occurring in s . The *height* of a term is defined as follows: $height(a) = 0$ if a is a variable or a constant and $height(f(t_1, \dots, t_n)) = 1 + \max\{height(t_1), \dots, height(t_n)\}$ if $n > 0$. The

root symbol of a term is defined as $root(a) = a$ if a is a variable and $root(f(t_1, \dots, t_n)) = f$.

A position in a term is expressed by a sequence of positive integers, and positions are partially ordered by the prefix ordering $\leq$. Let $\mathcal{O}(s)$ be the set of positions of s . For a set of positions W , let $\text{Min}(W)$ be the set of its minimal positions(w.r.t. $\leq$).

Let $s|_p$ be the subterm of s at position p . For a sequence $(p_1, \dots, p_n)$ of pairwise parallel positions and terms $t_1, \dots, t_n$, we use $s[t_1, \dots, t_n]_{(p_1, \dots, p_n)}$ to denote the term obtained from s by replacing each subterm $s|_{p_i}$ by t_i ($1 \leq i \leq n$). For a set of function symbols F , let $\mathcal{O}_F(s) = \{p \in \mathcal{O}(s) \mid root(s|_p) \in F\}$. For a string of unary function symbols $u = a_1 a_2 \dots a_k$ and a term t , let $u(t)$ be an abbreviation for $a_1(a_2(\dots a_k(t)))$.

A *rewrite rule* $\alpha \rightarrow \beta$ is a directed equation over terms. A *TRS* R is a set of rewrite rules. Let $\leftarrow$ be the inverse of $\rightarrow$, $\leftrightarrow = \rightarrow \cup \leftarrow$, and $\downarrow = \rightarrow^* \cdot \leftarrow^*$. t is *reachable* from s if $s \rightarrow^* t$. r is *confluent* on TRS R if for every $s \leftarrow_R^* r \rightarrow_R^* t$, $s \downarrow t$. A TRS R is *confluent* if every r is confluent on R . Let $\gamma: s_1 \xrightarrow{p_1} s_2 \dots \xrightarrow{p_{n-1}} s_n$ be a *rewrite sequence*. This sequence is abbreviated to $\gamma: s_1 \leftrightarrow^* s_n$. Let $|\gamma|$ be the number of steps of γ . γ is called *p-invariant* if $q > p$ for any redex position q of γ , and we write $\gamma: s_1 \xrightarrow[p\text{-inv}]{} s_n$.

The set D_R of *defined symbols* for a TRS R is defined as $D_R = \{root(\alpha) \mid \alpha \rightarrow \beta \in R\}$. A term s is *semi-constructor* if for every subterm t of s , t has no variable or $root(t)$ is not a defined symbol.

Definition 1 A rule $\alpha \rightarrow \beta$ is *monadic* if $height(\beta) \leq 1$, *semi-constructor* if β is semi-constructor. A TRS R is *monadic* if every rule in R is monadic, *semi-constructor* if every rule in R is semi-constructor.

3 Undecidability of joinability for monadic and semi-constructor TRSs

We have shown that joinability is undecidable for linear semi-constructor TRSs [4]. In this section, we show that joinability for monadic and semi-constructor TRSs is undecidable by a reduction from the Post's Correspondence Problem (PCP). Let $P = \{\langle u_i, v_i \rangle \in \Sigma^* \times \Sigma^* \mid 1 \leq i \leq n\}$ be an instance of the PCP. The corresponding TRS R_P is constructed as follows. Let $F = F_0 \cup F_1 \cup F_2$ where $F_0 = \{0, c, d, \$\}$, $F_1 = \{e_i \mid 1 \leq i \leq n\} (= E) \cup \Sigma$, $F_2 = \{f, g\}$.

$$\begin{aligned} R_P = & \{0 \rightarrow e_i(0) \mid 1 \leq i \leq n\} \cup \{0 \rightarrow f(c, d)\} \\ & \cup \{b \rightarrow a(b), b \rightarrow a(\$) \mid b \in \{c, d\}, a \in \Sigma\} \\ & \cup \{f(x, x) \rightarrow g(x, x)\} \\ & \cup \{e_i(g(u_i(x), v_i(y))) \rightarrow g(x, y) \mid 1 \leq i \leq n\} \end{aligned}$$

R_P is monadic. Here, $D_{R_P} = \{0, c, d, f\} \cup E$, so R_P is semi-constructor.

Lemma 2 $0 \rightarrow_{R_P}^* g(\$, \$)$ iff PCP P has a solution.

Proof. $0 \rightarrow_{R_P}^* g(\$, \$)$ iff there exists $i_1 \dots i_m \in \{1, \dots, n\}^*$ such that $0 \xrightarrow{m+1} e_{i_m} \dots e_{i_1}(f(c, d)) \xrightarrow{+} e_{i_m} \dots e_{i_1}(f(u_{i_1} \dots u_{i_m}(\$), u_{i_1} \dots u_{i_m}(\$))) \rightarrow e_{i_m} \dots e_{i_1}(g(u_{i_1} \dots u_{i_m}(\$), u_{i_1} \dots u_{i_m}(\$))) \xrightarrow{m} g(\$, \$)$ iff $u_{i_1} \dots u_{i_m} = v_{i_1} \dots v_{i_m}$. $\square$

Since $g(\$, \$)$ is a normal form, the following theorem holds.

Theorem 3 Both joinability and reachability for monadic and semi-constructor TRSs are undecidable.

4 Undecidability of reachability for confluent monadic and semi-constructor TRSs

We give a stronger result for reachability, that is, reachability for confluent monadic and semi-constructor TRSs is undecidable. Note that joinability is decidable for the same class [4, 3]. Let $\hat{F} = F \cup \{1\}$.

$$\begin{aligned} \hat{R}_P = R_P \cup & \{\$ \rightarrow 1\} \cup \{a(1) \rightarrow 1 \mid a \in \Sigma\} \\ & \cup \{e_i(g(1, v_i(y))) \rightarrow g(1, y), \\ & e_i(g(u_i(x), 1)) \rightarrow g(x, 1), \\ & e_i(g(1, 1)) \rightarrow g(1, 1) \mid 1 \leq i \leq n\} \end{aligned}$$

$\hat{R}_P$ is monadic. Here, $D_{\hat{R}_P} = D_{R_P} \cup \{\$ \} \cup \Sigma$, so $\hat{R}_P$ is semi-constructor. First, we show the confluence of $\hat{R}_P$.

4.1 Confluence of $\hat{R}_P$

To show the confluence of $\hat{R}_P$, we need some definitions and lemmata.

Definition 4 The set of Σ -strings is defined as follows.

- $1, c, d$ and $\$$ are Σ -strings.
- $a(t)$ is a Σ -string if t is a Σ -string and $a \in \Sigma$.

Lemma 5 For any Σ -string s , the following properties hold.

- (1) For any $\gamma : s \leftrightarrow^* t$, t is a Σ -string.
- (2) $s \rightarrow^* 1$.

Proof.

- (1) By induction on $|\gamma|$.
- (2) By induction on the structure of s . $\square$

Corollary 6 Every Σ -string is confluent.

Lemma 7 Let $\gamma : u(s) \rightarrow^* t$ where $u \in \Sigma^+$. Then, if $\text{root}(s) \notin \{1, c, d, \$\} \cup \Sigma$ and $u(s)|_{p1} = s$ then γ is p -invariant.

Proof. By induction on $|\gamma|$. $\square$

Definition 8 The set of E -strings is defined as follows.

- $0, f(t_1, t_2)$ and $g(t_1, t_2)$ are E -strings if t_1, t_2 are Σ -strings.
- $e_i(t)$ is an E -string if t is an E -string and $i \in \{1, \dots, n\}$.

Lemma 9 For any E -string s , the following properties hold.

- (1) For any $\gamma : s \leftrightarrow^* t$, t is an E -string.
- (2) $s \rightarrow^* g(1, 1)$.

Proof.

- (1) By induction on $|\gamma|$.
- (2) By induction on the structure of s . **Basis :** For any Σ -strings s_1, s_2 , $f(s_1, s_2) \rightarrow^* f(1, 1) \rightarrow g(1, 1)$ and $g(s_1, s_2) \rightarrow^* g(1, 1)$ by Lemma 5(2), and $0 \rightarrow f(c, d) \rightarrow^* g(1, 1)$. Thus, $s \rightarrow^* g(1, 1)$ if $s = f(s_1, s_2), g(s_1, s_2)$ or 0 . **Induction step :** Let $s = e_i(s')$ for some $i \in \{1, \dots, n\}$. By the induction hypothesis, $s' \rightarrow^* g(1, 1)$. Thus, $e_i(s') \rightarrow^* g(1, 1)$. $\square$

Corollary 10 Every E -string is confluent.

The following lemma is used as a component of the proof of Lemma 12.

Lemma 11 For any $i \in \{1, \dots, n\}$ and terms r_1, r_2 , the following properties hold.

- (1) If $s \xrightarrow{\varepsilon} e_i(g(r_1, r_2)) \xrightarrow{\varepsilon\text{-inv}}^* t$ then there exist terms t_1, t_2 such that $t \rightarrow^* g(t_1, t_2)$.
- (2) If $g(s_1, s_2) \leftarrow^* e_i(g(r_1, r_2)) \rightarrow^* g(t_1, t_2)$ and $g(r_1, r_2)$ is confluent then $g(s_1, s_2) \downarrow g(t_1, t_2)$.

Proof.

- (1) Let $t = e_i(g(t'_1, t'_2))$. If r_1 is a Σ -string then $t'_1 \rightarrow^* 1$ by Lemma 5. Otherwise, $r_1 \neq 1$. Thus, $r_1 = u_i(r'_1)$ for some term r'_1 by $e_i(g(r_1, r_2)) \xrightarrow{\varepsilon} s$. By Lemma 7, $t'_1 = u_i(t''_1)$, where $r'_1 \rightarrow^* t''_1$. Similarly, $t'_2 \rightarrow^* 1$ or $t'_2 = v_i(t''_2)$ for some term t''_2 . Thus, $t \rightarrow^* g(t_1, t_2)$, where $t_1 \in \{1, t''_1\}$ and $t_2 \in \{1, t''_2\}$.

- (2) By the definition of R_P , $e_i(g(r_1, r_2)) \xrightarrow{\varepsilon\text{-inv}}^* e_i(g(s'_1, s'_2)) \rightarrow g(s''_1, s''_2) \xrightarrow{\varepsilon\text{-inv}}^* g(s_1, s_2)$ and $e_i(g(r_1, r_2)) \xrightarrow{\varepsilon\text{-inv}}^* e_i(g(t'_1, t'_2)) \rightarrow g(t''_1, t''_2) \xrightarrow{\varepsilon\text{-inv}}^* g(t_1, t_2)$. Thus, $s'_1 \leftarrow^* r_1 \rightarrow^* t'_1$, $s'_1 \rightarrow^* s_1$ and $t''_1 \rightarrow^* t_1$. First, we show that $s_1 \downarrow t_1$.

Case of $s'_1 = t'_1 = 1$: Obviously, $s'_1 = s_1 = t''_1 = t_1 = 1$.

Case of $s'_1 = 1$ and $t'_1 = u_i(t''_1)$: Obviously, $s'_1 = s_1 = 1$. By Lemma 5, t_1 is a Σ -string and $t_1 \rightarrow^* 1$.

Case of $s'_1 = u_i(s''_1)$ and $t'_1 = 1$: Similar to the previous one.

Case of $s'_1 = u_i(s''_1)$ and $t'_1 = u_i(t''_1)$: By confluence of $g(r_1, r_2)$, r_1 is confluent. Thus, $u_i(s_1) \downarrow u_i(t_1)$. If s_1 is a Σ -string then $s_1 \downarrow t_1$ by Corollary 6. Otherwise, $s_1 \downarrow t_1$ by Lemma 7.

Similarly, $s_2 \downarrow t_2$. Thus, $g(s_1, s_2) \downarrow g(t_1, t_2)$. $\square$

Now, we show the confluence of $\hat{R}_P$.

Lemma 12 $\hat{R}_P$ is confluent.

Proof. We show that for any $\gamma : s \leftarrow^* r \rightarrow^* t$, $s \downarrow t$ by induction on height(r).

Basis: If $r \in \{c, d, 1\}$ then $s \downarrow t$ by Corollary 6, else if $r = 0$ then $s \downarrow t$ by Corollary 10. Otherwise, $s = r = t$ since r is a normal form.

Induction step: If γ is ε -invariant then $s \downarrow t$ by the induction hypothesis. So, we consider that γ has an ε -reduction. Let $\gamma_s : r \rightarrow^* s$ and $\gamma_t : r \rightarrow^* t$. Without loss of generality, we assume that γ_s has an ε -reduction and $\text{root}(r) \in \Sigma \cup \{f\} \cup E$.

Case of $\text{root}(r) \in \Sigma : \gamma_s : r = a(r_1) \xrightarrow{\varepsilon\text{-inv}}^* a(1) \rightarrow 1 = s$ holds for some $a \in \Sigma$ and r_1 . By Lemma 5, $t \rightarrow^* 1$.

Case of $\text{root}(r) = f : \gamma_s : r = f(r_1, r_2) \xrightarrow{\varepsilon\text{-inv}}^* f(r', r') \rightarrow g(r', r') \xrightarrow{\varepsilon\text{-inv}}^* g(s_1, s_2) = s$ holds for some terms r_1, r_2, r', s_1, s_2 . If γ_t is ε -invariant then $t = f(t_1, t_2)$ where $r_1 \rightarrow^* t_1$ and $r_2 \rightarrow^* t_2$. In this case, $s \rightarrow^* g(r_0, r_0) \leftarrow^* t$ for some r_0 by Figure 1(i). If γ_t has an ε -reduction then $\gamma_t : r = f(r_1, r_2) \xrightarrow{\varepsilon\text{-inv}}^* f(r'', r'') \rightarrow g(r'', r'') \xrightarrow{\varepsilon\text{-inv}}^* g(t_1, t_2) = t$ holds for some terms r'', t_1, t_2 . In this case, $s \rightarrow^* g(r_0, r_0) \leftarrow^* t$ for some r_0 by Figure 1(ii).

Case of $\text{root}(r) \in E : \gamma_s : r = e_i(r_1) \xrightarrow{\varepsilon\text{-inv}}^* e_i(g(s'_1, s'_2)) \rightarrow g(s''_1, s''_2) \xrightarrow{\varepsilon\text{-inv}}^* g(s_1, s_2) = s$ holds for some terms $r_1, s'_1, s'_2, s''_1, s''_2, s_1, s_2$ and $i \in \{1, \dots, n\}$. If γ_t is ε -invariant then $t = e_i(t_1)$ where $r_1 \rightarrow^* t_1$. By the induction hypothesis, there exists a term t' such that $e_i(g(s'_1, s'_2)) \xrightarrow{\varepsilon\text{-inv}}^* t' \leftarrow^* t$. By Lemma 11(1), $t' \rightarrow^* g(t'_1, t'_2)$ for some t'_1, t'_2 . Here, $g(s'_1, s'_2)$ is confluent by the induction hypothesis and $r_1 \rightarrow^* g(s'_1, s'_2)$. Thus, $s \downarrow g(t'_1, t'_2)$ by Lemma 11(2). (See Figure 1(iii).) If γ_t has an ε -reduction then $\gamma_t : r = e_i(r_1) \xrightarrow{\varepsilon\text{-inv}}^* e_i(g(t'_1, t'_2)) \rightarrow g(t''_1, t''_2) \xrightarrow{\varepsilon\text{-inv}}^* g(t_1, t_2) = t$ holds for some terms $t'_1, t'_2, t''_1, t''_2, t_1, t_2$. There exists a term s' such that $s \rightarrow^* s' \leftarrow^* e_i(g(t'_1, t'_2))$ as shown in Figure 1(iii). Here, $\text{root}(s') = g$ by $\text{root}(s) = g$. By the induction hypothesis and $r_1 \rightarrow^* g(t'_1, t'_2)$, $g(t'_1, t'_2)$ is confluent. Thus, $s' \downarrow t$ by Lemma 11(ii). (See Figure 1(iv).) $\square$

4.2 Reachability for confluent monadic and semi-constructor TRSs

Lemma 13 For any $\gamma : s \rightarrow^*_{\hat{R}_P} t$, if s has 1 as its subterm then so does t .

Proof. Since for any $\alpha \rightarrow \beta \in \hat{R}_P$, $V(\alpha) = V(\beta)$ and if α has 1 as its subterm then so does β . $\square$

Lemma 14 $0 \rightarrow^*_{\hat{R}_P} g(\$ \$)$ iff $0 \rightarrow^*_{\hat{R}_P} g(\$ \$)$.

Proof. Only if part: Let $\gamma : 0 \rightarrow^*_{\hat{R}_P} g(\$ \$)$. We assume to the contrary that γ must have $\hat{R}_P \setminus R_P$ reduction, i.e., $\gamma : 0 \rightarrow^*_{\hat{R}_P} s \rightarrow_{\hat{R}_P \setminus R_P} t \rightarrow^*_{\hat{R}_P} g(\$ \$)$ for some s, t . By the definition of $\hat{R}_P$, t has 1 as its subterm. By Lemma 13, $g(\$ \$)$ has 1 as its subterm, a contradiction. If part: By $R_P \subseteq \hat{R}_P$. $\square$

By Lemmata 2, 12, and 14, the following theorem holds.

Theorem 15 Reachability for confluent monadic and semi-constructor TRSs is undecidable.

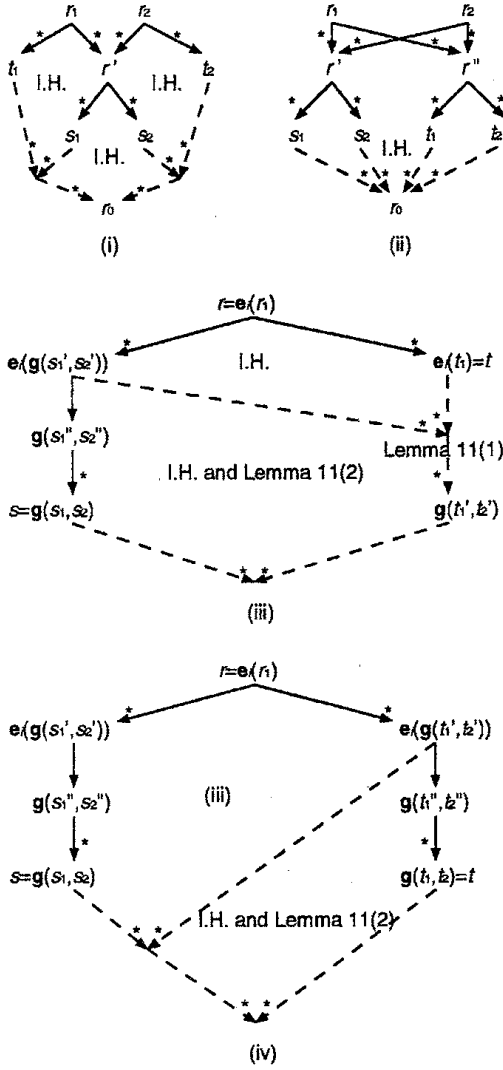


Figure 1:

5 Undecidability of confluence of monadic and semi-constructor TRSs

We show that confluence of monadic and semi-constructor TRSs is undecidable.

Let $F' = F'_0 \cup F'_1$ where $F'_0 = \{2\}$, $F'_1 = \{h\}$.

$$R = \{h(x) \rightarrow h(0), h(g(\$,\$)) \rightarrow 2\}$$

$\hat{R}_P \cup R$ is monadic. Here, $D_R = \{h\}$, so $\hat{R}_P \cup R$ is semi-constructor.

Lemma 16 For any s with $\text{root}(s) \in F'$, the following properties hold.

- (1) If $s \rightarrow_{\hat{R}_P \cup R} t$ then $\text{root}(t) \in F'$.
- (2) If $0 \rightarrow_{\hat{R}_P}^* g(\$,\$)$ then $s \rightarrow_{\hat{R}_P \cup R}^* 2$.

The proof is straightforward, so omitted.

Lemma 17 Let $s \rightarrow_{\hat{R}_P \cup R} t$, $\text{Min}(\mathcal{O}_{F'}(s)) = \{p_1, \dots, p_m\}$, and $\text{Min}(\mathcal{O}_{F'}(t)) = \{q_1, \dots, q_n\}$. Then, $s[2, \dots, 2]_{(p_1, \dots, p_m)} \rightarrow_{\hat{R}_P} t[2, \dots, 2]_{(q_1, \dots, q_n)}$ or $s[2, \dots, 2]_{(p_1, \dots, p_m)} = t[2, \dots, 2]_{(q_1, \dots, q_n)}$.

Proof. Let $s \xrightarrow{p}_{\hat{R}_P \cup R} t$. If there exists $i \in \{1, \dots, m\}$ such that $p_i \leq p$ then $\{p_1, \dots, p_m\} = \{q_1, \dots, q_n\}$ by Lemma 16(1). Thus, $s[2, \dots, 2]_{(p_1, \dots, p_m)} = t[2, \dots, 2]_{(q_1, \dots, q_n)}$. Otherwise, obviously $s \rightarrow_{\hat{R}_P} t$. Since every function symbol in F' does not occur in $\hat{R}_P$, $s[2, \dots, 2]_{(p_1, \dots, p_m)} \rightarrow_{\hat{R}_P} t[2, \dots, 2]_{(q_1, \dots, q_n)}$. $\square$

Lemma 18 $\hat{R}_P \cup R$ is confluent iff $0 \rightarrow_{\hat{R}_P}^* g(\$,\$)$.

Proof. Only if part: By $h(0) \leftarrow_{\hat{R}_P} h(g(\$,\$)) \rightarrow_{\hat{R}_P} 2$, confluence ensures that $h(0) \downarrow_{\hat{R}_P \cup R} 2$. Since 2 is a normal form, $h(0) \rightarrow_{\hat{R}_P \cup R}^* 2$. Thus, there exists a shortest sequence γ that satisfies $\gamma : h(0) \rightarrow_{\hat{R}_P \cup R}^* 2$. Since γ is shortest, $h(0) \xrightarrow{\varepsilon\text{-inv}}_{\hat{R}_P \cup R}^* h(g(\$,\$))$. Thus, there exists $\gamma' : 0 \rightarrow_{\hat{R}_P \cup R}^* g(\$,\$)$. Obviously, every function symbol occurring in γ' belongs to $\hat{F}$. Thus, $0 \rightarrow_{\hat{R}_P}^* g(\$,\$)$. By Lemma 14, $0 \rightarrow_{\hat{R}_P}^* g(\$,\$)$.

If part: Let $s \leftarrow_{\hat{R}_P \cup R}^* r \rightarrow_{\hat{R}_P \cup R}^* t$. By Lemma 17, $s[2, \dots, 2]_{(p_1, \dots, p_m)} \leftarrow_{\hat{R}_P}^* r[2, \dots, 2]_{(q_1, \dots, q_n)} \rightarrow_{\hat{R}_P}^* t[2, \dots, 2]_{(q_1, \dots, q_n)}$, where $\text{Min}(\mathcal{O}_{F'}(r)) = \{o_1, \dots, o_l\}$, $\text{Min}(\mathcal{O}_{F'}(s)) = \{p_1, \dots, p_m\}$, and $\text{Min}(\mathcal{O}_{F'}(t)) = \{q_1, \dots, q_n\}$. Since $\hat{R}_P$ is confluent by Lemma 12, $s[2, \dots, 2]_{(p_1, \dots, p_m)} \downarrow_{\hat{R}_P} t[2, \dots, 2]_{(q_1, \dots, q_n)}$. By $0 \rightarrow_{\hat{R}_P}^* g(\$,\$)$ and Lemma 16(2), $s \rightarrow_{\hat{R}_P \cup R}^* s[2, \dots, 2]_{(p_1, \dots, p_m)}$ and $t \rightarrow_{\hat{R}_P \cup R}^* t[2, \dots, 2]_{(q_1, \dots, q_n)}$. Thus, $s \downarrow_{\hat{R}_P \cup R}^* t$. $\square$

By Lemmata 2 and 18, the following theorem holds.

Theorem 19 Confluence of monadic and semi-constructor TRSs is undecidable.

6 Confluence of flat TRSs

In [2], the undecidability of confluence of flat TRSs has been claimed, but we found that the proof is incorrect. In this section, we explain its flaw.

Definition 20 [2] A rule $\alpha \rightarrow \beta$ is *flat* if $\text{height}(\alpha) \leq 1$ and $\text{height}(\beta) \leq 1$.

In [2], first the undecidability of reachability has been obtained by showing that $0 \rightarrow_{R_1}^* 1$ iff there

exists a solution for PCP for the following TRS R_1 .

$$\begin{aligned} R_1 = R_0 \cup \\ \{ & 0 \rightarrow f(q_A^{(3)}, q_A^{(4)}, q_A^{(5)}, q_B^{(13)}, q_B^{(14)}, q_A^{(6)}, q_B^{(15)}, q_B^{(16)}), \\ & f(x_1, x_2, x_1, y_{11}, y_{12}, x_2, y_{11}, y_{12}) \rightarrow \\ & g(x_1, x_2, x_1, y_{11}, y_{12}, x_2, y_{11}, y_{12}), \\ & g(x_0, x_0, y_{17}, y_{17}, y_{18}, y_{18}, y_{10}, y_{10}) \rightarrow 1 \} \end{aligned}$$

Here, R_0 has many rules, so omitted (see [[2], p.267]).

Next, the undecidability of confluence has been obtained by showing the claim that $R_1 \cup R_2$ is confluent iff $0 \rightarrow_{R_1}^* 1$ for the following TRS R_2 .

$$\begin{aligned} R_2 = & \{2 \rightarrow 0, 2 \rightarrow 1\} \cup \{c \rightarrow 0 \mid c \in \Xi_0 \setminus \{0, 1\}\} \\ & \cup \{d(x) \rightarrow 0, d(1) \rightarrow 1 \mid d \in \Xi_1\} \\ & \cup \{f(z_1, \dots, z_8) \rightarrow 1, g(z_1, \dots, z_8) \rightarrow 1 \mid \\ & \quad \text{one of the } z_i \text{ is } 1, \\ & \quad \text{the others are distinct variables}\} \end{aligned}$$

Here, $\Xi = \Xi_0 \cup \Xi_1 \cup \{f, g\}$, which is a set of function symbols occurring in R_1 . Ξ_0, Ξ_1 have many symbols, so omitted (see [[2], p.267]). Note that Ξ_0 has $q_A^{(3)}, q_A^{(4)}, q_A^{(5)}, q_A^{(6)}, q_B^{(13)}, q_B^{(14)}, q_A^{(6)}, q_B^{(15)}, q_B^{(16)}$.

However, the proof of the only-if part of the claim is incorrect. The proof claims that if $0 \rightarrow_{R_1}^* 1$ does not hold then $R_1 \cup R_2$ is not confluent because of the peak $0 \xleftarrow{R_2} 2 \xrightarrow{R_2} 1$. But, the claim overlooks that $0 \xrightarrow{R_1} f(q_A^{(3)}, q_A^{(4)}, q_A^{(5)}, q_B^{(13)}, q_B^{(14)}, q_A^{(6)}, q_B^{(15)}, q_B^{(16)}) \xrightarrow{R_2} f(0, 0, 0, 0, 0, 0, 0, 0) \xrightarrow{R_1} g(0, 0, 0, 0, 0, 0, 0, 0) \xrightarrow{R_1} 1$. Thus, the undecidability of confluence of flat TRSs has not been shown. Now, Jacquemard claims that the proof can be corrected.

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