

# A characterization of $\text{PSL}(2, 11)$ and $S_5$

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The symmetric group  $S_5$  of degree five and the two dimensional projective special linear group  $\text{PSL}(2, 11)$  over the field of eleven elements are doubly transitive permutation groups of degree five and eleven, respectively, in which the stabilizer of two points is isomorphic to the symmetric group  $S_3$  of degree three.

Let  $\Omega$  be the set of points  $1, 2, \dots, n$ , where  $n$  is odd. Let  $G$  be a doubly transitive permutation group in which the stabilizer  $G_{1,2}$  of the points 1 and 2 has even order and a Sylow 2-subgroup  $K$  of  $G_{1,2}$  is cyclic. In the case  $G_{1,2}$  is cyclic, Kantor-O'Nan-Seitz and the author proved independently that  $G$  contains a regular normal subgroup ([4] and [8]). In this lecture we shall study the case  $G_{1,2}$  is not cyclic. Let  $\tau$  be the unique involution in  $K$ . By a theorem of Witt ([10]) the centralizer  $C_G(\tau)$  of  $\tau$  in  $G$  acts doubly transitively on the set  $I(\tau)$  consisting of points in  $\Omega$  fixed by  $\tau$ .

The purpose of this lecture is to prove the following theorem.

Theorem. Let  $G$ ,  $G_{1,2}$ ,  $\tau$  and  $I(\tau)$  be above. Assume that all Sylow subgroups of  $G_{1,2}$  are cyclic, the image of the doubly transitive permutation representation of  $C_G(\tau)$  on  $I(\tau)$  contains a regular normal subgroup and that  $G$  does not contain a regular normal subgroup. If  $G$  has two classes of involutions, then  $G$  is isomorphic to  $S_5$  and  $n = 5$ . If  $G$  has one class of involutions and  $\tau$  is not contained in the center of  $G_{1,2}$ , then  $G$  is isomorphic to  $PSL(2, 11)$  and  $n = 11$ .

In [7] we proved this theorem in the case that the order of  $G_{1,2}$  equals  $2p$  for an odd prime number  $p$ .

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