

On the stability of incompressible viscous fluid motions
past objects

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Let E be the exterior domain in 3-space. Let us consider
the steady flow in E governed by

$$(1) \quad \begin{cases} -\nu \Delta w + (w \cdot \nabla) w + \nabla p = 0, \\ \operatorname{div} w = 0 \end{cases}$$

$$(2) \quad w(x) \longrightarrow w^\infty \quad (|x| \longrightarrow \infty)$$

$$(3) \quad w(x) = b(x) \quad (x \in \partial E)$$

where the viscosity coefficient ν is a positive constant,
 w^∞ is some fixed constant vector, b is some prescribed
function on E .

R. Finn showed that if $w^\infty - b$ is "small" enough, then
there exists a smooth solution w with

$$\sup_{x \in E} |x| |w(x) - w^\infty| < \infty$$

$$\nabla w \in L^3(E)$$

Given the disturbance $u_0 (\in L^2(E))$ to w . Then the perturbed flow v is governed by

$$(4) \quad \begin{cases} \frac{\partial v}{\partial t} - \nabla \Delta v + (v \cdot \nabla) v - \nabla p_2 = 0 \\ \operatorname{div} v = 0 \end{cases}$$

$$(5) \quad \begin{cases} \lim_{|x| \rightarrow \infty} v(x, t) = w^\infty, & v(x, t) = b(x) \quad (x \in \partial E, t > 0) \\ \lim_{t \downarrow 0} v(x, t) = w(x) + u_0(x) \end{cases}$$

Now our result is:

Assume that

- (i) $\sup_{x \in E} |x| |w(x) - w^\infty| < \frac{1}{2}$
- (ii) $\nabla w \in L^3(E)$
- (iii) $\operatorname{div} w = 0$

Then every weak solution v of (4), (5) becomes analytic (in t and x) after some definite time T_0 , and then converges to steady flow w uniformly in x on E like

$$|v(x, t) - w(x)| \leq M t^{-1/8}, \quad (t \rightarrow \infty)$$

(M ; constant)