

Recent topics on Multiplier Ideals

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Introduction to Multiplier Ideals.

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- (X, D) X : normal variety, $D \geq 0$: $\mathbb{Q}$ -divisor on X
 $(D = \sum d_i D_i, d_i \in \mathbb{Q}_{\geq 0}, D_i \subset X: \text{prime divisor})$

$K_X + D$ is $\mathbb{Q}$ -Cartier

i.e. $\exists r \in \mathbb{N}$ s.t. $r(K_X + D)$ is Cartier

- $(X, \mathcal{O}_X(t))$ X : $\mathbb{Q}$ -Goren. normal, $\mathcal{O}_X \in \mathcal{O}_X$, $t > 0$
 i.e. $\exists r \in \mathbb{N}$ s.t. rK_X is Cartier

- $(X, D; \mathcal{O}_X(t)) \dots$

Our main reference is Lazarsfeld's book [La2] for the theory of multiplier ideals.

log resol.

$f: \tilde{X} \rightarrow X$ log resol. of D (resp. $\mathcal{O}_X$)

$\Leftrightarrow$ f : proper birat^l

$\tilde{X}$: smooth

$\text{Supp}(\underbrace{f_* D}_{\text{strict transform of } D}) \cup \text{Exc.}(f)$: SNC divisor

(resp. $\mathcal{O}_X = \mathcal{O}_X(-F)$ inv., $\text{Supp } F \cup \text{Exc.}(f)$ is SNC.)

Def. of multiplier ideals

$$\bullet f(D) = f(X, D) := f_* \mathcal{O}_X(K_X - \underbrace{L(f^*(K_X + D))}_{\cong \frac{1}{n} f^*(nK_X + D)}) \subset \mathcal{O}_X$$

$$\bullet f(\mathcal{O}_X^t) = f(X, \mathcal{O}_X^t) := f_* \mathcal{O}_X(K_X - L(f^*K_X + tF)) \subset \mathcal{O}_X$$

We can define $f(X, \mathcal{O}_X^{t_1} \dots \mathcal{O}_X^{t_R})$, $f(X, D; \mathcal{O}_X^t)$ similarly.

$$(X, D) : \text{Klt (Kawamata log terminal)} \Leftrightarrow f(X, D) = \mathcal{O}_X$$

$$(X, D) : \text{Klt at } x \Leftrightarrow f(X, D)_x = \mathcal{O}_{X, x}$$

$$X \text{ has only lt sing. at } x \in X \Leftrightarrow (X, 0) \text{ is Klt at } x \in X \\ (\text{log terminal})$$

• Assume X has only lt sing. at $x \in X$

$$\text{lct}(D; x) := \sup \{ t \in \mathbb{Q} \mid f(X, t \cdot D)_x = \mathcal{O}_{X, x} \}$$

$$\text{lct}(\mathcal{O}_X^t; x) := \sup \{ t \in \mathbb{Q} \mid f(X, \mathcal{O}_X^t)_x = \mathcal{O}_{X, x} \}$$

Basic properties

(1) $f(D)$, $f(\mathcal{O}_X^t)$, etc. are indep. of the choice of the log resol. f . In particular,

$$\begin{array}{l} X : \text{smooth} \\ \text{Supp}(D) : \text{SNC} \end{array} \Rightarrow f(D) = \mathcal{O}_X(-LD)$$

$$(2) D_1 \geq D_2 \Rightarrow f(D_1) \leq f(D_2)$$

$$\mathcal{O}_1 \leq \mathcal{O}_2 \Rightarrow f(\mathcal{O}_1^t) \leq f(\mathcal{O}_2^t)$$

$$\text{Moreover if } \mathcal{O}_2 \leq \overline{\mathcal{O}}_1 \Rightarrow f(\mathcal{O}_1^t) = f(\mathcal{O}_2^t)$$

$$\left(\begin{array}{l} g: Y \rightarrow X: \text{normalized blow-up along } \mathcal{O} \text{ s.t. } \mathcal{O}_Y = \mathcal{O}_Y(-E) \\ \overline{\mathcal{O}} := g_* \mathcal{O}_Y(-E) \end{array} \right)$$

(3) Assume X has only lt sing.

$$\Rightarrow f(\mathcal{O}) \geq \mathcal{O}$$

$$\text{Moreover if } D \text{ is a cartier int. div.} \Rightarrow f(D) = \mathcal{O}(-D)$$

$$\text{if } \mathcal{O} \text{ is of pure ht } 1 \Rightarrow f(\mathcal{O}) = \mathcal{O}$$

$$(\odot) \text{ if } \mathcal{O} \text{ is of pure ht } 1 \Rightarrow \mathcal{O} \text{ is reflexive.}$$

(4) X : $\mathbb{Q}$ -Goren. affine var., $\mathcal{O}_x \subseteq \mathcal{O}_x, t > 0$

Fix $t \in \mathbb{R} \cap \mathbb{N}$. Take general elements $x_1, \dots, x_R \in \mathcal{O}$

$$A_i := \text{div } x_i, \quad D := \frac{1}{t} \sum A_i$$

$$\Rightarrow f(\mathcal{O}^t) = f(t \cdot D)$$

$$(5) \text{ lct}(D; x), \text{ lct}(\mathcal{O}; x) \in \mathbb{Q}_{>0}$$

Example

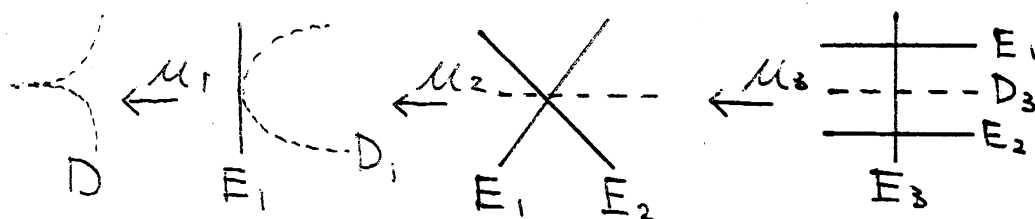
(1) X : smooth var. of dim. n , $x \in X$, $m := m_{x, x}$

$$f(m^t) = m^{L_{t+1}+1-n}, \quad \text{lct}(m; x) = n$$

☺ $f: \hat{X} \rightarrow X$: blow-up at x

$$\begin{cases} m \mathcal{O}_{\hat{X}} = \mathcal{O}_{\hat{X}}(-E), \quad K_{\hat{X}/X} = (n-1)E \\ f(m^t) = f_* \mathcal{O}_{\hat{X}}((n-1-L_{t+1})E) = m^{L_{t+1}+1-n} \end{cases}$$

$$(2) \quad X = \mathbb{C}^2, \quad D = (x^2 + y^3 = 0)$$



$$K_{X_1/X} = E_1, \quad K_{X_2/X} = E_1 + 2E_2, \quad K_{X_3/X} = E_1 + 2E_2 + 4E_3$$

$$f_1^* D = 2E_1 + D_1, \quad f_2^* D = 2E_1 + 3E_2 + D_2, \quad f_3^* D = 2E_1 + 3E_2 + 6E_3 + D_3$$

$$(f_i := \mu_i \circ \dots \circ \mu_1: X_i \rightarrow X)$$

$$f(t \cdot D) = f_{3X} \mathcal{O}_{X_3}(\pi - 2t^7 E_1 + \pi - 3t^7 E_2 + \pi - 6t^7 E_3 - L_{t+1} D_3)$$

$$\therefore \text{lct}(D; 0) = \frac{5}{6}, \quad f\left(\frac{5}{6} \cdot D\right) = (x, y)$$

(3) $X = \text{Spec } \mathbb{K}[\sigma \vee M]$: affine toric var.

$\mathcal{O} \subseteq \mathbb{K}[\sigma \vee M]$: monomial ideal

$\mathcal{O} \rightsquigarrow P(\mathcal{O}) \subset M_{\mathbb{R}}$: Newton polytope of $\mathcal{O}$

i.e. convex hull of the set of exponents of the monomials in $\mathcal{O}$

ex

$$\mathcal{O} = (x^4, xy, y^4) \subset \mathbb{C}[x, y] \Rightarrow \begin{array}{c} \text{graph of } P(\mathcal{O}) \end{array}$$

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Thm. (Blickle [Bl], Hara-Yoshida [HY])

Assume $X = \text{Spec } \mathbb{R}[\sigma^\vee \cap M]$ is $\mathbb{Q}$ -Goren.

$\exists u \in M$ s.t. $\text{div } \chi^u = -rK_X$ i.e. $\exists r \in \mathbb{N}$ s.t. rK_X : Cartier

$$w := \frac{u}{r}$$

$$\Rightarrow f(\sigma^t) = \langle \chi^v \mid v + w \in \text{Int}(t \cdot P(\sigma)) \rangle \subseteq M_{\mathbb{R}}$$

Cor. (Howald [Ho1])

$X = \mathbb{C}^n$, $\sigma \subseteq \mathbb{C}[x_1, \dots, x_n]$: monomial ideal

$$\Rightarrow f(\sigma^t) = \langle \chi^v \mid v + \mathbb{1} \in \text{Int}(t \cdot P(\sigma)) \rangle \subseteq \mathbb{R}^n$$

Proof

$\mu: Y \rightarrow X$: toric log resol. of σ s.t. $\sigma_Y = \sigma_Y(-F)$

$\rightsquigarrow f(\sigma^t)$: monomial ideal torus inv.

$$v + w \in \text{Int}(t \cdot P(\sigma))$$

$$\Leftrightarrow v + w - \varepsilon v' \in t \cdot P(\sigma) \quad (0 < \varepsilon < 1, \forall v' \in \text{Int}(\sigma^\vee \cap M))$$

$$\Leftrightarrow \mu^* \text{div } \chi^v - \mu^* K_X - \varepsilon \mu^* \text{div } \chi^{v'} \geq tF$$

$$\Leftrightarrow \mu^* \text{div } \chi^v + K_Y - \lceil K_Y + \varepsilon \mu^* \text{div } \chi^{v'} + \mu^* K_X + tF \rceil \geq 0$$

$$(\Leftrightarrow \chi^v \in f(\sigma^t)) \quad \begin{array}{c} \parallel \\ \lfloor \mu^* K_X + tF \rfloor \end{array}$$

- $$\left\{ \begin{array}{l} \textcircled{1} K_Y = -\sum D_i < 0 \text{ (} D_i \text{'s are all the torus inv. div)} \\ \textcircled{2} 1 > \chi \text{coeff. of } \varepsilon \mu^* \text{div } \chi^{v'} \geq 0 \\ \textcircled{3} \chi^{v'} \in \omega_X \text{ i.e. } \mu^* \text{div } \chi^{v'} \text{ and } K_Y \text{ have the same support} \end{array} \right.$$
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$f \in \mathbb{C}[X] \leadsto \mathcal{O}_f \subset \mathbb{C}[X]$: ideal gen. by the mono. appearing in f

$$j(t \cdot \text{div}(f)) \subseteq j(\mathcal{O}_f^t)$$

If f is "general" $\Rightarrow j(t \cdot \text{div}(f)) = j(\mathcal{O}_f^t)$ ($0 < \forall t < 1$)

Q. How "general"?

Thm. (Howald [Ho2])

$f \in \mathbb{C}[X]$: non-degenerate.

(i.e. $f \leadsto f_\sigma$ σ : face of $P(f) := P(\mathcal{O}_f)$
 $d f_\sigma$ is nowhere vanishing on $(\mathbb{C}^*)^n$
 for $\forall$ face σ of $P(f)$)

$$\Rightarrow j(t \cdot \text{div}(f)) = j(\mathcal{O}_f^t), \quad 0 < \forall t < 1$$

Ex.

$f = x_1^{d_1} + \dots + x_n^{d_n}$ is non-degenerate. Assume $\sum \frac{1}{d_i} < 1$.

since $P(f) = \{(u_1, \dots, u_n) \in \mathbb{R}^n \mid \sum u_i/d_i \geq 1\}$

$$\text{lc}(t(\text{div}(f)); 0) = \text{lc}(t(\mathcal{O}_f); 0) = \sum 1/d_i$$

Vanishing thm

(i) (local vanishing)

$f: \tilde{X} \rightarrow X$: log resol. of D

(resp. $\mathcal{O}$ s.t. $\mathcal{O} \mathcal{O}_{\tilde{X}} = \mathcal{O}_{\tilde{X}}(-F)$)

$$\Rightarrow R^i f_* \mathcal{O}_{\tilde{X}}(K_{\tilde{X}} - L f^*(K_X + D)) = 0, (\forall j > 0)$$

$$(\text{resp. } R^i f_* \mathcal{O}_{\tilde{X}}(K_{\tilde{X}} - L f^* K_X + t F) = 0, (\forall j > 0, \forall t > 0))$$

(ii) (Nadel vanishing)

L : cartier int. div. on X s.t. $L - D$ is nef and big

$$\Rightarrow H^i(X, \mathcal{O}_X(K_X + L) \otimes f(D)) = 0, (\forall i > 0)$$

Cor.

X : proj. normal var.

B : very ample divisor on X

L : cartier int. div. on X s.t. $L - D$ is nef and big

$\Rightarrow \mathcal{O}_X(K_X + L + mB) \otimes f(D)$ is gl. gen. if $m \geq \dim X$

☹ $\left[\begin{array}{l} \text{Lem. (Munford [La1, Theorem 1.8.5])} \\ F: \text{coherent s.t. } H^i(X, F \otimes \mathcal{O}_X(-iB)) = 0, \forall i > 0 \\ \Rightarrow F \text{ is gl. gen.} \end{array} \right.$

$$\text{Nadel} \Rightarrow H^i(X, \mathcal{O}_X(K_X + L + (m-i)B) \otimes f(D)) = 0, \forall i > 0$$

$$\Rightarrow 0.K. \quad \blacksquare$$

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Multiplier ideals and inversion of adjunction

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Recall adjunction formula
i.e. X : smooth, $Y \subset X$: smooth divisor

$$K_X + Y|_Y = K_Y$$

generalize $(X, S+B)$ X : normal var. $S \subset X$: reduced divisor $B \geq 0$: $\mathbb{Q}$ -cartier on X s.t. S has no common comp. with $\text{Supp}(B)$. $K_X + S + B$ is $\mathbb{Q}$ -cartier $\nu: S^\nu \rightarrow S$: normalization $\exists B^\nu \geq 0$ $\mathbb{Q}$ -divisor on S^ν (different of B on S^ν)

$$\text{s.t. } K_{S^\nu} + B^\nu = \nu^*(K_X + S + B|_S)$$

$$\begin{array}{ccc} \tilde{X} \supset \tilde{S} & \xrightarrow{g} & S^\nu \\ f \downarrow & & \swarrow \nu \\ X \supset S & & \end{array}$$

 f : embedded resol.

$$K_{\tilde{X}} + \tilde{S} \equiv f^*(K_X + S + B) + \sum a_i E_i$$

$$K_{\tilde{S}} \equiv g^*(K_{S^\nu} + B^\nu) + \sum a_i E_i|_{\tilde{S}}$$

ex. S : normal cartier $\Rightarrow B^\vee = B|_S$

$$(X, S+B) \begin{array}{c} \xrightarrow{\text{adjunction}} \\ \xleftarrow{\text{inv. of adj.}} \end{array} (S^\vee, B^\vee)$$

$$d := \min \{a_i \mid f(E_i) \subset S\}$$

$$d_S := \min \{a_i \mid E_i|_{\tilde{S}} \neq \emptyset\}$$

Note

In general $d \leq d_S$

Thm

(i) (Kollár [K+], Shokurov [Sh])

$$d > -1 \Leftrightarrow d_S > -1$$

(ii) (Kawakita [Ka])

$$d \geq -1 \Leftrightarrow d_S \geq -1$$

Def.

(i) $(X, D, \mathcal{O}_X(t))$ X : normal, $D \geq 0$: $\mathbb{Q}$ -divisor on X , $\mathcal{O}_X \subseteq \mathcal{O}_X(t)$, $t > 0$
 $K_X + D$ is $\mathbb{Q}$ -cartier,

$f: \tilde{X} \rightarrow X$: log resol. of $(D, \mathcal{O}_X)$, $\mathcal{O}_{\tilde{X}}(-F) = f^* \mathcal{O}_X(-D)$

$$f(X, D, \mathcal{O}_X(t)) := f_* \mathcal{O}_{\tilde{X}}(K_{\tilde{X}} - L(f^*(K_X + D) + tF)) \subset \mathcal{O}_X$$

(ii) $(X, S; B, \mathcal{O}_X^t) := (X, S, B)$ as above, $\mathcal{O}_X \subseteq \mathcal{O}_X$, $t > 0$.

$f: \tilde{X} \rightarrow X$: log resol. of $(S+B, \mathcal{O}_X)$ s.t. $\tilde{S} := f_*^{-1} S$ is smooth

$$\text{adj}(X, S; B, \mathcal{O}_X^t) := f_* \mathcal{O}_{\tilde{X}}(K_{\tilde{X}} - \lfloor f^*(K_X + S + B) + tF \rfloor + \tilde{S}) \subset \mathcal{O}_X$$

$$f(X, S+B, \mathcal{O}_X^t)$$

Remark

(0) $B=0$, $\mathcal{O}_X = \mathcal{O}_X \Rightarrow \text{adj}(X, S) := \text{adj}(X, S; B, \mathcal{O}_X^t)$

(i) $\text{adj}(X, S; B, \mathcal{O}_X^t)$ is indep. of the choice of f .

(ii) $X: \mathbb{Q}$ -Goren. affine, $h + \dim \mathcal{O}_X \geq 2$, $f \in \mathcal{O}_X$ is general

$$\Rightarrow f(X, \mathcal{O}_X) = \text{adj}(X, \text{div}(f))$$

Note

$$\bullet d_S > -1 \Leftrightarrow f(S^\vee, B^\vee) = \mathcal{O}_{S^\vee}$$

$$\Leftrightarrow (S^\vee, B^\vee) = \mathbb{R} \mathbb{Q}^+$$

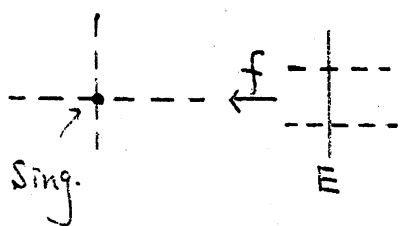
$$\bullet d > -1 \Leftrightarrow \text{adj}(X, S; B) = \mathcal{O}_X \text{ near } S$$

$$\Leftrightarrow (X, S+B) : \text{plt near } S$$

(purely log terminal)

Ex

(i) $X = \mathbb{C}^2$, $S = (xy=0)$, $B=0$, $\mathcal{O}_X = \mathcal{O}_X$



$$K_{\tilde{X}/X} = E, \quad f^* S = \tilde{S} + 2E$$

$$\text{adj}(X, S) = f_* \mathcal{O}_{\tilde{X}}(K_{\tilde{X}} - f^* K_X - f^* S + \tilde{S})$$

$$= f_* \mathcal{O}_{\tilde{X}}(-E) = (x, y)$$

(ii) X : $\mathbb{Q}$ -Goren. normal surface, $S \subset X$: reduced cartier divisor.

$$\Rightarrow \text{adj}(X, S) \mathcal{O}_S = \mathcal{C}(S) := \text{Ann}(\nu_* \mathcal{O}_{S^\vee} / \mathcal{O}_S)$$

$$\left(\begin{array}{l} \nu: S^\vee \rightarrow S: \text{normalization} \\ \tilde{X} \rightarrow X: \text{embedded resol} \end{array} \right)$$

Thm (Restriction thm, c.f. [La2, Theorem 9.5.1])

$(X, S; B, \mathcal{O}_X^t)$ as above, Assume $\mathcal{O}_X \not\in I_S = \mathcal{O}_X(-S)$

$$\Rightarrow \text{adj}(X, S; B, \mathcal{O}_X^t) \mathcal{O}_S = \nu_* \mathcal{f}(S^\vee, B^\vee, \mathcal{O}_{S^\vee}) \subset \mathcal{O}_S$$

In particular

$$d > -1 \Leftrightarrow d_S > -1 \quad \text{in this case } S^\vee \cong S \text{ (i.e. } S \text{ normal)}$$

proof

For simplicity assume $\mathcal{O}_X = \mathcal{O}_X^t$

$$\begin{array}{ccc} \tilde{X} \supset \tilde{S} & & \\ f \downarrow \quad \downarrow \text{ } g & \searrow & S^\vee \\ X \supset S & \swarrow & \end{array} \quad f: \text{embedded resol.}$$

$$\begin{aligned} 0 \rightarrow \mathcal{O}_{\tilde{X}}(K_{\tilde{X}} - Lf^*(K_X + S + B))_{\perp} &\rightarrow \mathcal{O}_{\tilde{X}}(K_{\tilde{X}} - Lf^*(K_X + S + B) + \tilde{S}) \\ &\xrightarrow{\cdot \mathcal{O}_{\tilde{S}}} \mathcal{O}_{\tilde{S}}(K_{\tilde{S}} - Lg^*(K_{S^\vee} + B^\vee))_{\perp} \rightarrow 0 \end{aligned}$$

$\downarrow f_*$

$$\begin{aligned} 0 \rightarrow \mathcal{f}(X, S+B) &\rightarrow \text{adj}(X, S; B) \xrightarrow{\cdot \mathcal{O}_S} \nu^* \mathcal{f}(S^\vee, B^\vee) \\ &\rightarrow R^1 f_* \mathcal{O}_{\tilde{X}}(K_{\tilde{X}} - Lf^*(K_X + S + B))_{\perp} \xrightarrow{\uparrow} 0 \end{aligned}$$

local vanishing $\square$

proof of Thm (ii) (Kawakita)

The question is local $\leadsto$ discuss over a germ at
a closed pt. $x \in S \subset X$
($X = \text{Spec } R$, R : local)

Assume $d_S \geq -1$ ($\Leftrightarrow (S^\vee, B^\vee)$ is lc)

$$\mathcal{O}_0 := \text{adj}(X, S; B) \subset \mathcal{O}_x$$

$$\mathcal{O}_{n+1} := \text{adj}(X, S; B, \mathcal{O}_n^{1-\varepsilon_n}), \quad 0 < \varepsilon_n < 1$$

$$\mathcal{O}_1 \mathcal{O}_S = \text{adj}(X, S; B, \mathcal{O}_0^{1-\varepsilon}) \mathcal{O}_S = \nu_* f(S^\vee, B^\vee, \mathcal{O}_0 \mathcal{O}_S^{1-\varepsilon})$$

$(\odot d_S \geq -1, \mathcal{O}_0 \mathcal{O}_S = \nu_* f(S^\vee, B^\vee)) \xrightarrow{\quad} \bigcup \mathcal{O}_0 \mathcal{O}_S$

Since $\mathcal{O}_0 \supset \mathcal{O}_1$,

$$\mathcal{O}_1 \mathcal{O}_S = \mathcal{O}_0 \mathcal{O}_S, \quad \text{i.e. } \mathcal{O}_1 + I_S = \mathcal{O}_0 + I_S$$

Thus $\begin{cases} \mathcal{O}_0 \supset \mathcal{O}_1 \supset \mathcal{O}_2 \supset \dots \\ \mathcal{O}_0 + I_S = \mathcal{O}_1 + I_S = \mathcal{O}_2 + I_S = \dots \end{cases}$

Suppose $d < -1$, $(k\tilde{x} + \tilde{S} \equiv f^*(kx + S + B) + \sum a_i E_i)$

$\Rightarrow \exists E_i$: f -exc. divisor on $\tilde{X}$ s.t. $a_i < -1$.

$$\mathcal{O}_0 = \text{adj}(X, S; B) \subset f_* \mathcal{O}_{\tilde{X}}(\lceil a_i \rceil E_i) = f_* \mathcal{O}_{\tilde{X}}(-E_i)$$

$$\begin{aligned} \mathcal{O}_1 &= \text{adj}(X, S; B, \mathcal{O}_0^{1-\varepsilon}) \subset \text{adj}(X, S; B, f_* \mathcal{O}_{\tilde{X}}(-E_i)^{1-\varepsilon}) \\ &\subset f_* \mathcal{O}_{\tilde{X}}(\lceil a_i - (1-\varepsilon) \rceil E_i) \\ &= f_* \mathcal{O}_{\tilde{X}}(-2E_i) \end{aligned}$$

($\odot \varepsilon < 1$)

$$\therefore \mathcal{O}_n \subset f_* \mathcal{O}_{\tilde{X}}(-(n+1)E_i), \quad \forall n \geq 0$$

On the other hand, by Nagata's thm,

$$\forall l \in \mathbb{N}, \exists k(l) \in \mathbb{N} \text{ s.t. } f_* \mathcal{O}_{\tilde{X}}(-k(l)E) \subset m_{x,x}^l$$

$$\therefore \mathcal{O}_0 \subset \bigcap_{n \in \mathbb{N}} (\mathcal{O}_n + I_S) \subset \bigcap_{l \in \mathbb{N}} (m_{x,x}^l + I_S) = I_S$$

this implies $\nu_* f(S^\vee, B^\vee) = 0$ contradiction.

$$\therefore d \geq -1$$



Conj (Kollár, Shokurov) (See [K+] and [Sh])

$\forall Z \subset S$: closed subset

$$d(Z) := \min \{a_i \mid f(E_i) \subset Z\}$$

$$d_S(Z) := \min \{a_i \mid E_i|_{\tilde{S}} \neq \emptyset, f(E_i|_{\tilde{S}}) \subset Z\}$$

$$d(Z) = d_S(Z) ?$$

($\leq$ o.k.)

known case (Eiñ-Mustaţ & [EM], c.f. [EMY])

X : l.c.i., S : normal cartier

Higher codimension

X : $\mathbb{Q}$ -Goren. normal var./c

$Y := \sum_{i=1}^r t_i Y_i$, $t_i > 0$, $Y_i \subset X$: closed subscheme

$\mathcal{O}_i \subset \mathcal{O}_X$: def. ideal of Y_i

$f: \tilde{X} \rightarrow X$: log resol. of $\mathcal{O}_1, \dots, \mathcal{O}_r$

$$\mathcal{O}_i \mathcal{O}_{\tilde{X}} = \mathcal{O}_{\tilde{X}}(-F_i)$$

$$K_{\tilde{X}/X} - \sum t_i F_i \equiv \sum a_j E_j$$

$$(X, Y): \mathbb{Q} \text{ lt} \Leftrightarrow a_j > -1, \forall j$$

$$(X, Y): \text{lc} \Leftrightarrow a_j \geq -1, \forall j$$

Thm (T- [Ta1])

(X, Y) as above. Assume X is smooth

$Z \subsetneq X$: $\mathbb{Q}$ -Goren. closed subvar. s.t. $Z \not\subset \cup Y_i$

$(Z, Y|_Z): \text{lc} \Rightarrow (X, Y+Z): \text{lc near } Z$

proof

For simplicity, assume $Y=0$

$L \subset Z$: locus of lc sing.

i.e. L is defined by $f(Z) = f(Z, \mathcal{O}_Z) \subset \mathcal{O}_Z$

(since Z is lc, L is reduced)

$(I_Z \subset) I_L \subset \mathcal{O}_X$: def. ideal of L in X

i.e. the lift of $f(Z)$

We have the following two restriction thm.

$$\textcircled{1} f(Z, (\mathcal{O}_Z \mathcal{O}_Z)^t) \subset I_L f(X, \mathcal{O}_X^t) \mathcal{O}_Z, \forall \mathcal{O}_X \subset \mathcal{O}_X, \forall t > 0$$

$$\textcircled{2} f(Z, (\mathcal{O}_Z \mathcal{O}_Z)^t) \subset f(X, \mathcal{O}_X^t I_Z^{1-\varepsilon}) \mathcal{O}_Z, \forall \mathcal{O}_X \subset \mathcal{O}_X, \forall t > 0, \\ 0 < \forall \varepsilon < 1$$

(We prove these by char. $p > 0$ method, later)

$$z: \ell_c \Rightarrow f(z, (I_L \cup z)^{1-\varepsilon}) \supset I_L \cup z, \quad 0 < \varepsilon < 1$$

$\underset{f(z)}{I_L \cup z} \quad \quad \quad \underset{f(z)}{I_L \cup z}$

$$\text{by ①, } I_L \cup z \subset f(z, (I_L \cup z)^{1-\varepsilon}) \subset I_L f(X, I_L^{1-\varepsilon}) \cup z \\ \Rightarrow f(X, I_L^{1-\varepsilon}) = \emptyset_X$$

$$\text{by ②, } I_L \cup z \subset f(z, (I_L \cup z)^{1-\varepsilon}) \stackrel{②}{\subset} f(X, I_L^{1-\varepsilon} I_z^{1-\varepsilon}) \cup z$$

$$\text{since } f(X, I_L^{1-\varepsilon} I_z^{1-\varepsilon}) \supset I_z f(X, I_L^{1-\varepsilon}) = I_z,$$

$$I_L \subset f(X, I_L^{1-\varepsilon} I_z^{1-\varepsilon}), \quad 0 < \varepsilon < 1$$

$$\leadsto (X, z): \ell_c \text{ near } z$$



Sketch of proof ② (we can prove ① similarly)

Assume $X = \text{Spec } R$ $((R, m): \text{complete RLR of char. 0})$

$$z = \text{Spec } S \quad (S = R/I, I = \sqrt{I} \subset R: \text{unmixed})$$

char. $p > 0$

$$\underline{\text{ETS}} \left(\tau(S, (\mathcal{O}_S)^t) \subset \tau(R, \mathcal{O}_R^t I^{1-\varepsilon}) S \right)$$

$$\forall \mathcal{O} \in R, \forall t > 0, 0 < \varepsilon < 1$$

$$\text{dual} \left(\begin{array}{c} \uparrow \\ \mathcal{O}_{E_S}^{*(\mathcal{O}_S)^t} \supset \mathcal{O}_{E_R}^{*\mathcal{O}_R^t I^{1-\varepsilon}} \cap E_S \end{array} \right)$$

$$E_S := E_S(S/m_S), \quad E_R := E_R(R/m),$$

$$E_S \cong (0:I)_{E_R} \subset E_R$$

$$\mathcal{O}_R^{[t]g} I^{[g(1-\varepsilon)]} F_R^e(z) = 0 \in F_R^e(E_R) \cong E_R \quad \forall g = p^e \gg 0$$

$$F_R^e: E_R \rightarrow F_R^e(E_R) \cong E_R$$

$$F_S^e: E_S \rightarrow F_S^e(E_S)$$

$$(\delta = p^e)$$

$$\forall z \in E_s, F_s^e(z) = 0 \in F_s^e(E_s) \Leftrightarrow (I^{[\delta]}: I) F_R^e(z) = 0 \in E_R$$

$$\text{Since } I^{[\delta]}: I \subset I^{\delta-1} \subset I^{[\delta(1-\varepsilon)]}, \delta = p^e \gg 0, 0 < \varepsilon < 1$$

$$\sigma^{[\tau\delta]}(I^{[\delta]}: I) F_R^e(z) = 0, \delta = p^e \gg 0,$$

$$\Leftrightarrow (\sigma S)^{[\tau\delta]} F_s^e(z) = 0, \delta = p^e \gg 0$$

$$\Rightarrow z \in O_{E_s}^{*(\sigma S)^t}$$



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A char. p analog of adjoint ideals (Appendix)

(R, m) : F -finite normal local of char. $p > 0$

$f \neq 0, \mathfrak{a} \subset R, t > 0$

$$\tau^{\text{div}}(R, f; \mathfrak{a}^t) := \text{Ann}_R O_E^*(f; \mathfrak{a}^t)$$

$$E := E_R(R/m) \cong H_m^d(W_R)$$

$$z \in O_E^*(f; \mathfrak{a}^t) \stackrel{\text{def}}{=} \exists c \in \forall \text{ min. prime of } R/f$$

$$\text{s.t. } cf^{q-1} \mathfrak{a}^{tq} z^q = 0, q = p^e \gg 0$$

$$\left(F^e: E \rightarrow F^e(E) := eR \otimes_R E \right. \\ \left. z \mapsto z^e := 1 \otimes z \right)$$

If R is $\mathbb{Q}$ -Goren.

R/f is $\mathbb{Q}$ -Goren, normal

$$\Rightarrow \tau(R/f, (\mathfrak{a} R/f)^t) = \tau^{\text{div}}(R, f; \mathfrak{a}^t) R/f$$

See [Ta3] for details

Ex

$$R = k[[X, Y]], f = XY, \mathfrak{a} = R$$

$$\Rightarrow \tau^{\text{div}}(R, f) = (X, Y)$$

$$\stackrel{!}{=} \tau^{\text{div}}(R, f; R)$$

Thm (T- [Ta3])

(R, m) : normal local ring ess. of finite type / k

$f \neq 0 \in R, \mathfrak{a} \subset R, t > 0$

$(\tilde{R}, \tilde{f}, \tilde{\mathfrak{a}})$: reduction to char. $p \gg 0$ of $(R, f, \mathfrak{a})$

$$\Rightarrow \text{adj}(\text{Spec } R, \text{div}(f); \mathfrak{a}^t) = \tau^{\text{div}}(\tilde{R}, \tilde{f}; \tilde{\mathfrak{a}}^t)$$

Applications of asymptotic multiplier ideals

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local properties of multiplier ideals

(1) (Restriction thm)

 X : normal $\mathbb{Q}$ -Goren. var./ $\mathbb{C}$ $S \subset X$: normal $\mathbb{Q}$ -Goren. cartier divisor $\mathcal{O}_S \subset \mathcal{O}_X$, $t > 0$. Assume $S \notin \text{Zeros}(\mathcal{O}_S)$.

$$\Rightarrow j(S, (\mathcal{O}_S \mathcal{O}_S)^t) = \text{adj}(X, S; \mathcal{O}_S^t) \mathcal{O}_S \subset j(X, \mathcal{O}_S^t) \mathcal{O}_S$$

(2) (Subadditivity) (Demailly-Ein-Lazarsfeld [DEL])

 X : smooth

$$\Rightarrow j(\mathcal{O}_S^s \mathcal{O}_T^t) \subset j(\mathcal{O}_S^s) j(\mathcal{O}_T^t), \quad \forall \mathcal{O}_S, \mathcal{O}_T \subset \mathcal{O}_X, \quad \forall s, t > 0$$

$$\left(\begin{array}{l} \text{More generally} \\ x \in X, \quad j(\mathcal{O}_S^s \mathcal{O}_T^t)_x \subset \sum_{\substack{\lambda + \mu = \dim x \\ \lambda, \mu \geq 0}} j(\mathcal{O}_S^s m_{x,x}^\lambda)_x j(\mathcal{O}_T^t m_{x,x}^\mu)_x \\ \qquad \qquad \qquad \subset j(\mathcal{O}_S^s)_x j(\mathcal{O}_T^t)_x \end{array} \right)$$

(3) (Summation) (Mustață [Mu])

X : smooth

$$\Rightarrow g(X, (\mathcal{O}_X + \mathcal{B})^t) \subset \sum_{\substack{\lambda + \mu = t \\ \lambda, \mu \geq 0}} g(X, \mathcal{O}_X^\lambda) g(X, \mathcal{B}^\mu)$$

In particular

$$g(X, (\mathcal{O}_X + \mathcal{B})^{s+t}) \subset g(X, \mathcal{O}_X^s) + g(X, \mathcal{B}^t)$$

Sketch of proof (2)

$$\begin{array}{ccc} X \cong \Delta & \hookrightarrow & X \times X \\ & \searrow p_1 & \searrow p_2 \\ & X & X \end{array}$$

since X : smooth, $\Delta \hookrightarrow X \times X$ c.i. diagonal embedding

$$\begin{array}{ccccc} \tilde{X}_1 & \leftarrow & \tilde{X}_1 \times \tilde{X}_2 & \rightarrow & \tilde{X}_2 \\ \downarrow & & \downarrow & & \downarrow \\ X_1 & \leftarrow & X_1 \times X_2 & \rightarrow & X_2 \end{array}$$

$$g(X, \mathcal{O}_X^s \mathcal{B}^t) \subset g(X \times X, (p_1^{-1} \mathcal{O}_X)^s (p_2^{-1} \mathcal{B})^t) \cap \Delta$$

↑
repeated applications of
Restriction thm

$$p_1^{-1} g(X, \mathcal{O}_X^s) \cdot p_2^{-1} g(X, \mathcal{B}^t)$$

$$\Rightarrow g(X, \mathcal{O}_X^s \mathcal{B}^t) \subset g(X, \mathcal{O}_X^s) g(X, \mathcal{B}^t) \quad \square$$

Ex (c.f. [TW])

$$X = \text{Spec } \mathbb{C}[x, y, z] / (xy - z^5) \quad A_4\text{-sing.}$$

$$\mathcal{O}_X = (x, y^4, y^3 z, y^2 z^2, y z^3, z^4)$$

$$g(\mathcal{O}_X) = \mathcal{O}_X, \quad g(\mathcal{O}_X^{\frac{1}{2}}) = (x, y^2, y z, z^2)$$

$$\leadsto g(\mathcal{O}_X) \not\subset g(\mathcal{O}_X^{\frac{1}{2}})^2$$

$\mathcal{O}_X^{\frac{1}{2}} \mathcal{O}_X^{\frac{1}{2}}$

$$x \in g(\mathcal{O}_X), \quad x \notin g(\mathcal{O}_X^{\frac{1}{2}})^2$$

Sing. case (T- [Ta 2])

(2)' (Subadditivity)

X : $\mathbb{Q}$ -Goren. normal var./ $\mathbb{C}$

$$\Rightarrow J \cdot g(\mathcal{O}_X^s \mathcal{I}^t) \subset g(\mathcal{O}_X^s) g(\mathcal{I}^t), \quad \forall \mathcal{O}_X, \mathcal{I} \subset \mathcal{O}_X, \forall s, t > 0$$

($J \subset \mathcal{O}_X$: Jacobian ideal sheaf)
(We cannot replace J by $\sqrt{J}$)

(3)' (Summation)

X : $\mathbb{Q}$ -Goren. normal var.

$$\Rightarrow g(X, (\mathcal{O}_X + \mathcal{I})^t) = \sum_{\substack{\lambda + \mu = t \\ \lambda, \mu \geq 0}} g(X, \mathcal{O}_X^\lambda \mathcal{I}^\mu)$$

In particular

$$J \cdot g(X, (\mathcal{O}_X + \mathcal{I})^t) \subset \sum_{\substack{\lambda + \mu = t \\ \lambda, \mu \geq 0}} g(X, \mathcal{O}_X^\lambda) g(X, \mathcal{I}^\mu)$$

(J : Jacobian)

Sketch of proof (2)'

Assume $X = \text{Spec } R$, R : complete local of char. 0
 $\leadsto$ char. $p > 0$.

$$\text{ETS } J \cdot \tau(\mathcal{O}_X^s \mathcal{I}^t) \subset \tau(\mathcal{O}_X^s) \tau(\mathcal{I}^t)$$

$$\text{dual } \left(\begin{array}{l} \tau(\mathcal{I}^t) := \text{Ann } \mathcal{O}_E^{*\mathcal{I}^t}, \quad E := E_R(R/\mathfrak{m}) \\ (\mathcal{O}_E^{*\mathcal{O}_X^s \mathcal{I}^t} : J)_E \supset (\mathcal{O}_E^{*\mathcal{I}^t} : \tau(\mathcal{O}_X^s))_E \end{array} \right)$$

$\downarrow$
 $\mathbb{Z}$

$$\leadsto \tau(\mathcal{O}_S)z \in \mathcal{O}_E^* \mathcal{O}^t$$

$$\text{i.e. } \exists c \in R^0 \text{ s.t. } c \mathcal{O}^{\tau \mathcal{O}} \tau(\mathcal{O}_S)^{[\mathcal{O}]} z = 0 \in \mathbb{F}^e(E),$$

$$(R^0 := R \setminus \bigcup_{P: \text{minimal prime}} P)$$

$$\forall g = p^e \gg 0$$

claim $\exists d \in R^0$ s.t. $d \mathcal{O}^{\tau \mathcal{O}} J^{[\mathcal{O}]} \subset \tau(\mathcal{O}_S)^{[\mathcal{O}]}, \forall g = p^e \gg 0.$

If we accept this claim

$$\Rightarrow cd \mathcal{O}^{\tau \mathcal{O}} \mathcal{O}^{\tau \mathcal{O}} J^{[\mathcal{O}]} z = 0 \in \mathbb{F}^e(E) \quad \forall g = p^e \gg 0$$

$$\Rightarrow Jz \subset \mathcal{O}_E^* \mathcal{O}^t \quad \square$$

Ex.

$X, \mathcal{O}$ as above Ex.

$$J = (x, y, z^4), \quad \sqrt{J} = (x, y, z)$$

$$(x, y, z^4) \not\subset \mathcal{O} \subset \mathcal{O}^{\frac{1}{2}}$$

$$(x, y, z) \not\subset \mathcal{O} \not\subset \mathcal{O}^{\frac{1}{2}}$$

$$\cup_{xz}$$

$$\not\subset$$

Asymptotic multiplier ideals (See [ELS] or [La2] for details)

X : $\mathbb{Q}$ -Goren. normal var.

$\mathcal{O}_\bullet$: graded family of ideals on X

$$\text{i.e. } \mathcal{O}_\bullet = \{\mathcal{O}_m\}_{m \in \mathbb{N}}$$

$$\mathcal{O}_0 = \mathcal{O}_X, \quad \mathcal{O}_1 \neq 0, \quad \mathcal{O}_m \subset \mathcal{O}_X$$

$$\mathcal{O}_R \cdot \mathcal{O}_L \subset \mathcal{O}_{R+L}, \quad \forall R, L \in \mathbb{N}$$

$t > 0$ fix.

$$f(\mathcal{O}_{\mathbb{R}}^{\frac{t}{\mathbb{R}}}) = f((\mathcal{O}_{\mathbb{R}})^{\frac{t}{\mathbb{R}}}) = f(\mathcal{O}_{\mathbb{R}}^t)$$

$\leadsto \{f(\mathcal{O}_m^{\frac{t}{m}})\}_{m \in \mathbb{N}}$ has a unique max. element
w.r.t. inclusion.

Denote it by $f(\mathcal{O}^t)$

Ex.

(1) $\mathcal{O}_m := \mathcal{O}^m$, $\mathcal{O} \subseteq \mathcal{O}_x$

$$\Rightarrow f(\mathcal{O}^t) = f(\mathcal{O}^t)$$

(2) L : linear system,

$$\mathcal{O}_m := \mathfrak{b}(\text{Im } L): \text{base ideal of } \text{Im } L$$

$$\Rightarrow f(\mathcal{O}^t) =: f(t \cdot \|L\|)$$

(3) $X = \text{Spec } R$, $P \subset R$: prime ideal

$$\mathcal{O}_m := P^{(m)} := P^m R_P \cap R$$

$$\Rightarrow f(\mathcal{O}^t) =: f(t \cdot P^{(\cdot)})$$

Basic properties

(1). $t_1 > t_2 \Rightarrow f(\mathcal{O}^{t_1}) \subset f(\mathcal{O}^{t_2})$

(2). $\mathcal{O}.$, $\mathfrak{b}.$,

If $0 \neq e \in \mathcal{O}_x$ s.t. $e \mathcal{O}_m \subset \mathfrak{b}_m$, $\forall m \gg 0$

$$\Rightarrow f(\mathcal{O}^t) \subset \mathfrak{b}^t$$

$$(3). \mathcal{O}_X^{\otimes k} g(\mathcal{O}_X^{\otimes l}) \subset g(\mathcal{O}_X^{\otimes k+l}), \quad k, l \in \mathbb{N}$$

In particular, if X has only lt sing.

$$(\Leftrightarrow g(\mathcal{O}_X) = \mathcal{O}_X)$$

$$\Rightarrow \mathcal{O}_X^{\otimes k} \subset g(\mathcal{O}_X^{\otimes k}), \quad \forall k \in \mathbb{N}$$

(4). (Restriction) S : normal $\mathbb{A}^1$ -Goren Cartier divisor on X

$$g(S, (\mathcal{O}_S^{\otimes t})^{\otimes t}) \subset g(X, \mathcal{O}_X^{\otimes t}) \otimes_{\mathcal{O}_S} (\mathcal{O}_S^{\otimes t})^{\otimes t} \quad (\forall t > 0)$$

(5). (Subadditivity)

$J \subset \mathcal{O}_X$: Jacobian ideal

$$J^{\otimes l-1} g(\mathcal{O}_X^{\otimes k}) \subset g(\mathcal{O}_X^{\otimes k})^{\otimes l}, \quad k, l \in \mathbb{N}$$

(6). (Summation)

$$(\mathcal{O}_X^{\otimes k} + \mathcal{O}_X^{\otimes l})^{\otimes m} := \sum_{k+l=m} \mathcal{O}_X^{\otimes k} \cdot \mathcal{O}_X^{\otimes l}$$

$$g((\mathcal{O}_X^{\otimes k} + \mathcal{O}_X^{\otimes l})^{\otimes t}) \subset \sum_{k+l=t} g(\mathcal{O}_X^{\otimes k})^{\otimes t} \cdot g(\mathcal{O}_X^{\otimes l})^{\otimes t}$$

$$(7). 0 \neq e \in \mathcal{O}_X \text{ s.t. } e \mathcal{O}_X^{\otimes m} \subset g(\mathcal{O}_X^{\otimes m}), \quad \forall m \gg 0$$

$$\Rightarrow J \cdot \mathcal{O}_X^{\otimes m} \subset g(\mathcal{O}_X^{\otimes m}), \quad \forall m \in \mathbb{N}$$

($J \subset \mathcal{O}_X$: Jacobian ideal)

short proof

$$(2). g(\mathcal{O}_X^{\otimes t}) = g(\mathcal{O}_X^{\otimes \frac{t}{m}})^{\otimes m} = g(e^{\otimes \frac{t}{m}} \mathcal{O}_X^{\otimes \frac{t}{m}})^{\otimes m} \\ \subset g(\mathcal{O}_X^{\otimes \frac{t}{m}})^{\otimes m} \subset g(\mathcal{O}_X^{\otimes t}) \quad m \gg 0$$

$$(7). e J^{\otimes l} \mathcal{O}_X^{\otimes m} \subset e J^{\otimes l} \mathcal{O}_X^{\otimes m} \quad \text{use subadditivity} \\ \subset J^{\otimes l} g(\mathcal{O}_X^{\otimes m}) \subset g(\mathcal{O}_X^{\otimes m})^{\otimes l}, \quad l \gg 0$$

$$\Rightarrow J \cdot \mathcal{O}_X^{\otimes m} \subset \overline{g(\mathcal{O}_X^{\otimes m})} = g(\mathcal{O}_X^{\otimes m})$$

Symbolic powers

(Swanson [Sw]) R : normal domain

$R \supset P$: prime

$\Rightarrow \bar{R} = \bar{R}(P) \in \mathbb{N}$ s.t. $P^{(\bar{R}m)} \subset P^m$, $\forall m \in \mathbb{N}$

Q. What is $\bar{R}$?

Thm (Ein-Lazarsfeld-Smith [ELS])

R : regular affine domain / $\mathbb{C}$ (f.g. alg. over $\mathbb{C}$)

$P \subset R$: prime of ht. h

$\Rightarrow P^{(hm)} \subset P^m$, $\forall m \in \mathbb{N}$ (i.e. $\bar{R}(P) = \text{ht. } P$)

Thm (Hochster-Huneke [HH1])

R : regular ring of equal char., $P \subset R$: prime of ht. h .

$P^{(hm)} \subset P^m$, $\forall m \in \mathbb{N}$

Singular case

Thm (T - [Ta2])

R : affine domain / $\mathbb{K}$, $\mathbb{K}$: perfect field of char. $p > 0$

$P \subset R$: prime of ht. h , $J \subset R$: Jacobian ideal

$\Rightarrow J^{m-1} \tau(R) P^{(hm)} \subset P^m$, $\forall m \in \mathbb{N}$

proof of ELS

$$P^{(\cdot)} := \{P^{(m)}\}_{m \in \mathbb{N}}$$

$$P^{(hm)} \subset J(hm \cdot P^{(\cdot)}) \subset J(h \cdot P^{(\cdot)})^m$$

ETS $J(h \cdot P^{(\cdot)}) \subset P$

$$\begin{aligned} J(h \cdot P^{(\cdot)})_P &= J(h \cdot P^{(\cdot)} R_P) \\ &= J((P R_P)^h) \subset P R_P \\ &\leadsto J(h \cdot P^{(\cdot)}) \subset P \quad \square \end{aligned}$$

proof of sing. case

$$P^{(\cdot)} := \{P^{(m)}\}_{m \in \mathbb{N}}$$

$$\begin{aligned} J^{m-1} J(R) P^{(hm)} &\subset J(hm \cdot P^{(\cdot)}) J^{m-1} \\ &\subset \underbrace{J(h \cdot P^{(\cdot)})^m}_P \quad \square \end{aligned}$$

If P is special

Can we get a better bound?

Thm (Hochster-Huneke [HH2], T-Yoshida [TY])

R : regular ring of equal char.

$P \subset R$: prime of ht $h \geq 2$

If R/P is F-pure or of dense F-pure type (see [Hu])

$$\Rightarrow P^{(hm-1)} \subset P^m \quad \forall m \in \mathbb{N}$$

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