

Pseudo-Multi Impulse for Simulating Critical Response of Elastic-Plastic High-Rise Buildings under Long-Duration, Long-Period Ground Motion

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Abstract

A pseudo-multi impulse (PMI) is proposed as an extension of the ordinary base-input multi impulse substituting a long-duration, long-period ground motion which can often be simulated by a multi-cycle sine wave (MSW). PMI is treated as a multitude of impulsive lateral forces and hardly excites the higher-mode responses of elastic multi-degree-of-freedom (MDOF) models because the undamped fundamental participation vector is adopted as the influence coefficient vector. It is shown that the critical time interval of PMI can be obtained without any repetition. This enables a smart derivation of the critical response. The displacement transfer functions are derived for elastic MDOF models, where two kinds of the influence coefficient vectors are adopted; 1) 1 at every component (conventional one), 2) the undamped fundamental-mode participation vector. It is demonstrated that the critical PMI can efficiently and accurately evaluate the maximum responses under MSW and the critical input period of MSW for elastic-plastic MDOF models. Furthermore, PMI is extended to treat the critical case for the second eigenmode. Finally, the critical PMI is applied to high-rise buildings for the investigation into the input energy and the cumulative plastic deformation ductility demand.

Keywords: Critical excitation, Elastic-plastic earthquake response, Nonlinear resonance, Multi impulse, Pseudo-multi impulse, Long-duration ground motion, Cumulative plastic deformation ductility.

1. Introduction

Recently, many researchers focused on the seismic responses of long-period structures under long-duration ground motions after the experience of actual ground motions^[1-5]. Since long-period components of earthquake ground motions propagate far from the epicenter, long-period structures on relatively soft ground may exhibit resonant responses and experience severe structural/non-structural damage. There have been several reports on observed long-period, long-duration ground motions and damaged structures and lifelines^[6-9]. In relation to this topic, Iwan derived transcendental equations for stationary responses of elastic-plastic models under harmonic excitation^[10-12]. Caughey approximately derived the nonlinear stationary responses by using the equivalent linearization technique^[13, 14]. However, these methods require iterative procedures to find the input critical (resonant) frequency for a specified input level. For enhancing the seismic safety of buildings, the critical responses should be considered and their smart derivation without repetition is strongly demanded.

The critical excitation method has been developed to rationally treat resonant responses of structures^[15-19]. Based on the concept of the critical excitation method, Kojima and Takewaki^[20] proposed a ‘multi impulse (MI)’ as another representative for a long-duration, long-period ground motion. MI consists of a multitude of impulses. They showed that the critical input period of MI can be easily obtained for elastic-plastic SDOF models based on the energy balance law. Moreover, it was demonstrated that the critical input period can also be simply calculated for elastic-plastic MDOF models without any repetitive procedure^[21]. The critical responses under MI have been investigated for elastic-plastic MDOF models so far^[22, 23]. However, the use of MI may lead to the overestimation of the acceleration responses under the corresponding multi-cycle sine wave (MSW) as a representative of long-duration ground motions due to its impulsive nature. Since an impulse input equally and simultaneously excites all the eigenmode responses, floor accelerations are apt to become excessive. Especially in the cases of MDOF models with small viscous damping, the displacement responses also become larger than those under the corresponding MSW. It is believed that the maximum structural responses should be accurately estimated for reasonable designs of structural frames and non-structural components^[24, 25].

On the other hand, an innovative approach has been introduced by Akehashi and Takewaki^[26] for controlling the excitation of higher-mode responses under impulsive inputs. Although a double impulse (DI) has been treated as a ground motion input substituting the fling-step near-fault ground motion^[27], a pseudo-double impulse (PDI) proposed by Akehashi and Takewaki^[26] was treated as a set of impulsive lateral forces. PDI mainly excites the fundamental eigenmode responses and hardly excites the higher-mode responses because the undamped fundamental participation vector is adopted as the influence coefficient vector.

In this paper, a pseudo-multi impulse (PMI) is proposed as an extension of the ordinary MI. Although the ordinary MI is treated as a ground acceleration, PMI is treated as a multitude of impulsive lateral forces. Since the participation vector in terms of the undamped fundamental mode is adopted as the influence coefficient vector, PMI hardly excites the higher-mode responses. In Section 2, the concept of PMI and the critical condition on the input timing of PMI are explained. Then, the way of the

time-history response analysis under the critical PMI is explained. In Section 3, displacement responses under a single impulse and displacement transfer functions are derived for elastic MDOF models. Two kinds of the influence coefficient vectors are adopted in this section; 1) 1 at every component (conventional one), 2) the undamped fundamental-mode participation vector. In Section 4, the time-history response analysis is conducted to compare the responses of elastic, and elastic-plastic MDOF models under MI, PMI, and the corresponding MSW. Furthermore, PMI is extended to treat the critical case for the second eigenmode. In Section 5, the input energies and the maximum interstory drifts are investigated for elastic/elastic-plastic MDOF models under the critical PMI. In addition, the cumulative plastic deformation ductility demand is investigated for high-rise buildings with different numbers of stories under the critical PMI.

2. Pseudo-multi impulse (PMI) and its critical timing

A multi impulse (MI) was introduced by Kojima and Takewaki^[20] as a substitute of a long-period, long-duration ground motion to capture the critical maximum responses of elastic-plastic structures. Although a multi-cycle sine wave (MSW) has been widely used as another direct substitute of long-period, long-duration ground motion, an iterative procedure with much computational load is required to obtain the critical responses of the elastic-plastic structures by parametrically changing the input period. On the other hand, the critical condition on the time interval has been analytically derived for the double impulse in the previous paper^[21]. This procedure for the double impulse can be extended conveniently to MI. Therefore, the use of MI provides the critical input period for the elastic-plastic MDOF models without repetition. However, the use of MI may lead to the overestimation of the responses (especially for acceleration) under the corresponding MSW due to its impulsive nature. This issue must be resolved. The relation between MSW and MI is shown in Figure 1 from the view point of the Fourier amplitude and the transfer function.

In this section, a pseudo-multi impulse (PMI) is newly introduced to precisely simulate critical responses for elastic-plastic MDOF models under long-period, long-duration ground motions (see Figure 1). PMI is an extended version of a pseudo-double impulse (PDI) proposed by Akehashi and Takewaki^[26]. PMI hardly excites the higher-mode responses of elastic MDOF models because each impulse input of PMI gives the change of the relative velocity response along height which is proportional to the undamped fundamental eigenmode. In Section 2.1, PMI is explained in detail. Then, in Section 2.2, the critical condition on the input timing of PMI and the way of the time-history response analysis under the critical PMI are explained.

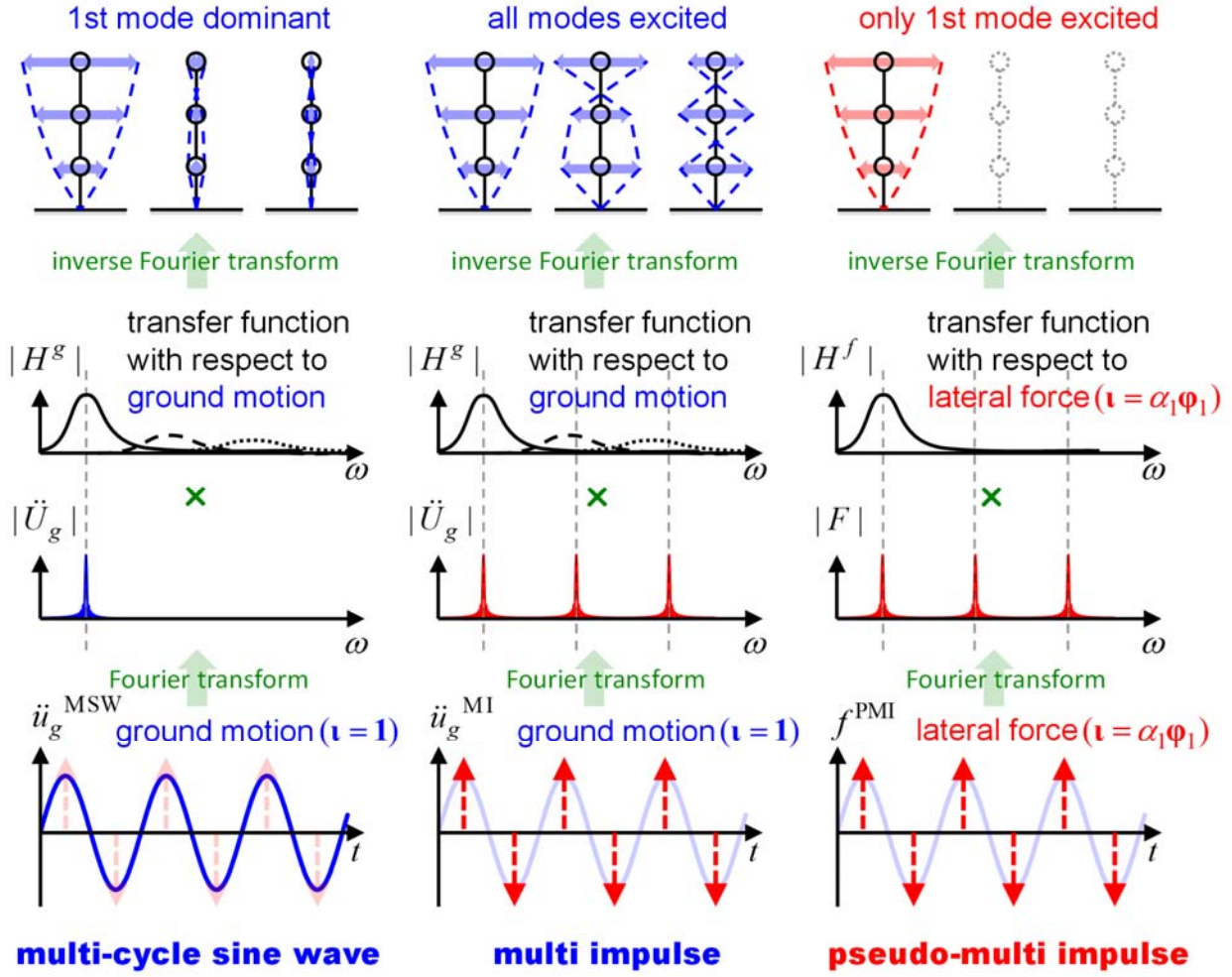


Figure 1 Input and response properties of multi-cycle sine wave (harmonic excitation), multi impulse and pseudo-multi impulse.

2.1 PMI

The ground accelerations $\ddot{u}_g^{\text{MI}}$, $\ddot{u}_g^{\text{MSW}}$ of MI and the corresponding MSW are expressed as follows.

$$\ddot{u}_g^{\text{MI}}(t) = \sum_{n=1}^{N^{\text{MI}}} (-1)^{n-1} V \delta\{t - (n-1)t_0\}, \quad (1)$$

$$\ddot{u}_g^{\text{MSW}}(t) = \begin{cases} A_l \sin(\pi t / t_0) & (0 \leq t \leq N^{\text{MI}} t_0) \\ 0 & \text{otherwise} \end{cases}, \quad (2)$$

where V , t_0 , N^{MI} , $\delta(t)$, A_l are the input velocity amplitude of MI, the time interval of the two adjacent impulses, the number of impulses, the Dirac delta function and the acceleration amplitude of

the corresponding MSW, respectively. The Fourier amplitudes $|\ddot{U}_g^{\text{MI}}|, |\ddot{U}_g^{\text{MSW}}|$ of $\ddot{u}_g^{\text{MI}}, \ddot{u}_g^{\text{MSW}}$ can be derived as

$$|\ddot{U}_g^{\text{MI}}| = V \left| \sum_{n=0}^{N^{\text{MI}}-1} (-1)^n e^{-i\omega n t_0} \right|, \quad (3)$$

$$|\ddot{U}_g^{\text{MSW}}| = \left| \int_{-\infty}^{\infty} \ddot{u}_g^{\text{MSW}} e^{-i\omega t} dt \right| = \left| \frac{\pi A_t t_0}{\pi^2 - (\omega t_0)^2} (1 - e^{-i\omega N^{\text{MI}} t_0}) \right|. \quad (4)$$

The maximum values of Equations (3), (4) are $N^{\text{MI}}V, N^{\text{MI}}A_t t_0$. By equating these, the relation between A_t and V can be expressed by

$$A_t = 2V / t_0. \quad (5)$$

Figure 2 illustrates $|\ddot{U}_g^{\text{MI}}|/V, |\ddot{U}_g^{\text{MSW}}|/V$ in the case of $N^{\text{MI}} = 20$. Although $|\ddot{U}_g^{\text{MI}}|$ and $|\ddot{U}_g^{\text{MSW}}|$ correspond well in the range of $0 \leq \omega t_0 / \pi \leq 2$, $|\ddot{U}_g^{\text{MI}}|$ has multiple peaks at $\omega t_0 / \pi = 1, 3, 5, \dots$. The use of MI may lead to the overestimation of the responses (especially acceleration) under the corresponding MSW since MI excites all the mode responses and MSW does not.

To overcome this difficulty, PMI is introduced in this paper. Although the ordinary MI is treated as a ground acceleration input, PMI is treated as a multitude of impulsive lateral forces. PMI hardly excites the higher-mode responses of elastic MDOF models because the undamped fundamental-mode participation vector is adopted as the influence coefficient vector of PMI. In other words, each impulse input of PMI gives the change of the relative velocity response along height which is proportional to the undamped fundamental-mode participation vector.

Consider an elastic N -story shear mass model. The equations of motions under MI and PMI are expressed as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\mathbf{1} \left(\sum_{n=1}^{N^{\text{MI}}} (-1)^{n-1} V \delta\{t - (n-1)t_0\} \right) = -\mathbf{M}\mathbf{1}\ddot{u}_g^{\text{MI}}, \quad (6a)$$

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}(\alpha_1 \boldsymbol{\phi}_1) \left(\sum_{n=1}^{N^{\text{MI}}} (-1)^{n-1} V \delta\{t - (n-1)t_0\} \right) = -\mathbf{M}(\alpha_1 \boldsymbol{\phi}_1) f^{\text{PMI}}, \quad (6b)$$

where $\mathbf{M}, \mathbf{C}, \mathbf{K}, \mathbf{u}, \boldsymbol{\phi}_n, \alpha_n \boldsymbol{\phi}_n$ denote the mass matrix, the damping matrix, the stiffness matrix, the displacement vector, the n -th eigenmode vector and the n -th undamped participation vector, respectively. Note that $\alpha_n \boldsymbol{\phi}_n$ satisfies $\alpha_1 \boldsymbol{\phi}_1 + \dots + \alpha_N \boldsymbol{\phi}_N = \mathbf{1}$. Hereafter, $\ddot{u}_g^{\text{MI}}$ indicates MI as a ground acceleration and f^{PMI} indicates the amplitude of PMI as an equivalent lateral force. It is noted again that PMI does not excite the higher-mode responses of elastic proportionally-damped MDOF models. In the cases of elastic non-proportionally damped MDOF models, PMI mainly excites the fundamental-mode responses and slightly excites the higher-mode responses.

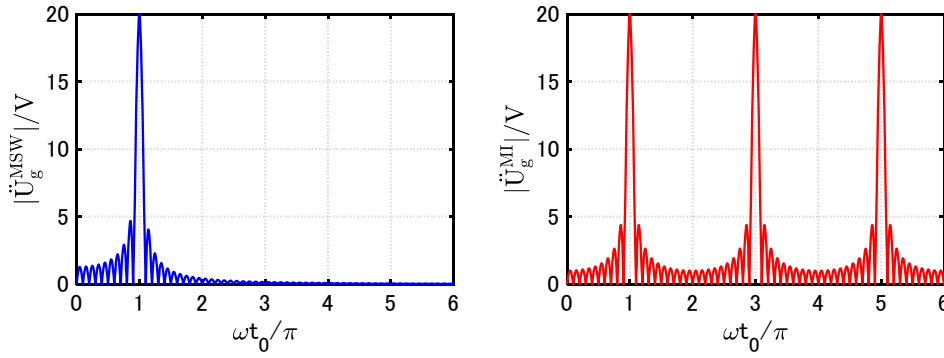


Figure 2 Fourier amplitudes of multi-cycle sine wave and multi impulse ($N^{\text{MI}} = 20$).

2.2 Critical condition on time interval of PMI

Akehashi and Takewaki [26] showed that the total input energy by PDI is maximized in the case that the second impulse acts at the timing when the sum $(\alpha_1 \phi_1^T) \mathbf{M} \ddot{\mathbf{u}}$ of the elastic fundamental-mode inertial forces (the story shear force in the first story) attains zero. This critical condition is applicable to elastic-plastic MDOF models and non-proportionally damped MDOF models although $\alpha_1 \phi_1^T$ does not strictly provide the modal decomposition. It should be noted that the corresponding facts also hold in the case of PMI.

The procedure to find the critical impulse timing of PMI is explained below. Each time interval of two adjacent impulses is changed sequentially so that each impulse input meets the critical condition. This procedure is conducted until the time interval converges and the model exhibits the stationary response. It is noted that this procedure is not a repetition, but a forward-direction procedure. The convergence value of the timing is adopted as the critical impulse timing. In addition, the time-history response analysis under PMI with the constant time interval is conducted in order to confirm that the impulse inputs meet the critical condition in the stationary response.

It is noted that the free vibration analysis without any external force is conducted in the time-history response analysis under PMI. Namely, the impulses are not treated as external forces, and the changes of the velocity responses by the impulses are directly given. It is also noted that the time intervals are not required to be set beforehand in the time-history response analysis under the critical PMI. In the analysis code, the critical condition of each impulse input is automatically judged and the change of the velocity response is also automatically provided at each input timing.

3. Displacement responses of elastic MDOF models under single impulse and transfer function with respect to ground motion and lateral force with $\mathbf{v} = \alpha_1 \phi_1$

In Section 3.1, the displacement responses under two different single impulses (SI) with $\mathbf{v} = \mathbf{1}, \alpha_1 \phi_1$ are derived for elastic proportionally-damped MDOF models and elastic non-proportionally damped

MDOF models. It is noted that SI with $\mathbf{u} = \alpha_1 \boldsymbol{\phi}_1$ corresponds to PMI for $N^{\text{MI}} = 1$. In Section 3.2, the displacement transfer functions with respect to ground motions and the transfer functions with respect to lateral forces with $\mathbf{u} = \alpha_1 \boldsymbol{\phi}_1$ are derived.

3.1 Elastic responses of proportionally damped MDOF models and non-proportionally damped MDOF models under single impulse

Consider an elastic proportionally-damped MDOF model with the undamped n -th natural circular frequency ω_n , the n -th damping ratio h_n and the n -th damped natural circular frequency $\omega_{Dn} = \omega_n \sqrt{1 - h_n^2}$. The displacement response under SI with the velocity amplitude V as a ground acceleration ($\mathbf{u} = \mathbf{1}$) can be expressed by

$$\mathbf{u}(t) = \sum_{i=1}^N (\alpha_i \boldsymbol{\phi}_i) \frac{-V e^{-h_i \omega_i t}}{\omega_{Di}} \sin \omega_{Di} t. \quad (7)$$

On the other hand, the displacement response of the model under another SI with $\mathbf{u} = \alpha_1 \boldsymbol{\phi}_1$ as a set of horizontal forces can be expressed by

$$\mathbf{u}(t) = (\alpha_1 \boldsymbol{\phi}_1) \frac{-V e^{-h_1 \omega_1 t}}{\omega_{D1}} \sin \omega_{D1} t. \quad (8)$$

SI with $\mathbf{u} = \alpha_1 \boldsymbol{\phi}_1$ as a set of horizontal forces excites only the fundamental eigenmode in the case of proportionally-damped MDOF models, although SI as a ground acceleration excites all the mode responses.

Consider next an elastic non-proportionally damped MDOF model with the n -th complex eigenvector $\boldsymbol{\psi}_n$, the n -th pseudo-undamped natural circular frequency ω_n^* , the n -th damping ratio h_n^* , the n -th eigenvalue $\lambda_n = -h_n^* \omega_n^* + i \omega_n^* \sqrt{1 - (h_n^*)^2}$. The displacement response under SI with the velocity amplitude V as a ground acceleration can be expressed by

$$\mathbf{u}(t) = \sum_{i=1}^N V e^{-h_i^* \omega_i^* t} \{ \boldsymbol{\beta}_i^g \cos(\omega_{Di}^* t) - \boldsymbol{\gamma}_i^g \sin(\omega_{Di}^* t) \}, \quad (9)$$

where

$$\boldsymbol{\beta}_n^g + i \boldsymbol{\gamma}_n^g = \frac{-2 \boldsymbol{\psi}_n^T \mathbf{M} \mathbf{1}}{2 \lambda_n \boldsymbol{\psi}_n^T \mathbf{M} \boldsymbol{\psi}_n + \boldsymbol{\psi}_n^T \mathbf{C} \boldsymbol{\psi}_n} \boldsymbol{\psi}_n. \quad (10)$$

The decomposition of $\boldsymbol{\beta}_n^g, \boldsymbol{\gamma}_n^g$ by the undamped eigenmodes leads to

$$\boldsymbol{\beta}_n^g = \beta_{n,1}^g (\alpha_1 \boldsymbol{\phi}_1) + \dots + \beta_{n,N}^g (\alpha_N \boldsymbol{\phi}_N), \quad (11a)$$

$$\gamma_n^g = \gamma_{n,1}^g(\alpha_1 \Phi_1) + \dots + \gamma_{n,N}^g(\alpha_N \Phi_N). \quad (11b)$$

Following the previous paper [26], new quantities $\mu_{n,j}^g, \theta_{n,j}^g$ are defined by

$$\mu_{n,j}^g = \omega_{Dn}^* \sqrt{(\beta_{n,j}^g)^2 + (\gamma_{n,j}^g)^2}, \quad (12a)$$

$$\theta_{n,j}^g = \arctan(\beta_{n,j}^g / \gamma_{n,j}^g). \quad (12b)$$

By substituting Equations (11a, b), (12a, b) into Equation (9), the following equation is obtained.

$$\mathbf{u}(t) = \sum_{i=1}^N \frac{-V e^{-h_i^* \omega_i^* t}}{\omega_{Di}^*} \sum_{j=1}^N \left\{ \mu_{i,j}^g(\alpha_j \Phi_j) \sin(\omega_{Di}^* t - \theta_{i,j}^g) \right\}. \quad (13)$$

From Equations (7), (13), it is understood that $\mu_{i,i}^g$ is dominant among $\mu_{i,1}^g, \dots, \mu_{i,N}^g$ and the value of $\theta_{i,i}^g$ is close to zero as far as the damping matrix has weak non-proportionality.

Consider next the case of SI with $\mathbf{u} = \alpha_1 \Phi_1$ as a set of horizontal forces. The displacement response can be expressed as

$$\mathbf{u}(t) = \sum_{i=1}^N V e^{-h_i^* \omega_i^* t} \{ \beta_i \cos(\omega_{Di}^* t) - \gamma_i \sin(\omega_{Di}^* t) \}, \quad (14)$$

where

$$\beta_n + i\gamma_n = \frac{-2\psi_n^T \mathbf{M}(\alpha_1 \Phi_1)}{2\lambda_n \psi_n^T \mathbf{M} \psi_n + \psi_n^T \mathbf{C} \psi_n} \psi_n. \quad (15)$$

The decomposition of β_n, γ_n by the undamped eigenmodes leads to

$$\beta_n = \beta_{n,1}(\alpha_1 \Phi_1) + \dots + \beta_{n,N}(\alpha_N \Phi_N), \quad (16a)$$

$$\gamma_n = \gamma_{n,1}(\alpha_1 \Phi_1) + \dots + \gamma_{n,N}(\alpha_N \Phi_N). \quad (16b)$$

As in the case of SI as a ground acceleration, new quantities $\mu_{n,j}, \theta_{n,j}$ are defined as

$$\mu_{n,j} = \omega_{Dn}^* \sqrt{(\beta_{n,j})^2 + (\gamma_{n,j})^2}, \quad (17a)$$

$$\theta_{n,j} = \arctan(\beta_{n,j} / \gamma_{n,j}). \quad (17b)$$

The substitution of Equations (16a, b), (17a, b) into Equation (14) leads to

$$\mathbf{u}(t) = \sum_{i=1}^N \frac{-V e^{-h_i^* \omega_i^* t}}{\omega_{Di}^*} \sum_{j=1}^N \left\{ \mu_{i,j}(\alpha_j \Phi_j) \sin(\omega_{Di}^* t - \theta_{i,j}) \right\}. \quad (18)$$

$\mu_{i,i}$ is dominant among $\mu_{i,1}, \dots, \mu_{i,N}$ and the value of $\theta_{i,i}$ is close to zero as far as the damping matrix has weak non-proportionality. Moreover, the values of $\mu_{i,j}$ are small when $i \neq 1$ and the higher-mode responses are negligible.

3.2 Transfer functions with respect to ground motion and lateral force with $\mathbf{u} = \alpha_1 \boldsymbol{\phi}_1$ for proportionally damped MDOF models and non-proportionally damped MDOF models

In this section, the displacement transfer function $\mathbf{H}^g = \mathbf{U} / \ddot{\mathbf{U}}_g^{\text{MI}}$ with respect to MI as a ground acceleration and the displacement transfer function $\mathbf{H}^f = \mathbf{U} / F^{\text{PMI}}$ with respect to PMI as the equivalent lateral force are derived.

The Fourier transformation of Equation (6a) provides

$$(-\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K})\mathbf{U} = -\mathbf{M}\mathbf{1}\ddot{\mathbf{U}}_g^{\text{MI}}. \quad (19)$$

$\mathbf{H}^g$ can be obtained straightforwardly as

$$\mathbf{H}^g = \mathbf{U} / \ddot{\mathbf{U}}_g^{\text{MI}} = -(-\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K})^{-1} \mathbf{M}\mathbf{1}. \quad (20)$$

In the case of a proportionally-damped MDOF model, the n -th mode displacement transfer function $\mathbf{H}_n^g$ can be expressed as

$$\mathbf{H}_n^g = \mathbf{U}_n / \ddot{\mathbf{U}}_g^{\text{MI}} = \frac{-1}{\omega_n^2 - \omega^2 + 2ih_n\omega_n\omega} \alpha_n \boldsymbol{\phi}_n. \quad (21)$$

The relation between $\mathbf{H}_n^g$ and $\mathbf{H}^g$ is expressed by

$$\mathbf{H}^g = \sum_{n=1}^N \mathbf{H}_n^g. \quad (22)$$

Consider next $\mathbf{H}^f$, which is derived through the Fourier transformation of Equation (6b) as

$$\begin{aligned} \mathbf{H}^f = \mathbf{U} / F^{\text{PMI}} &= -(-\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K})^{-1} \mathbf{M}(\alpha_1 \boldsymbol{\phi}_1) \\ &= \frac{-1}{\omega_1^2 - \omega^2 + 2ih_1\omega_1\omega} \alpha_1 \boldsymbol{\phi}_1. \end{aligned} \quad (23)$$

Since PMI excites only the fundamental eigenmode for elastic proportionally-damped MDOF models, $\mathbf{H}^f = \mathbf{H}_1^g$ is satisfied.

Consider next $\mathbf{H}^g$ of a non-proportionally damped MDOF model. Equation (6a) can be rewritten in the form of the first-order differential equation as follows.

$$\mathbf{a}\dot{\mathbf{z}} + \mathbf{b}\mathbf{z} = -\begin{pmatrix} \mathbf{0} \\ \mathbf{M}\mathbf{1}\ddot{u}_g^{\text{MI}} \end{pmatrix}, \mathbf{a} = \begin{pmatrix} \mathbf{O} & \mathbf{M} \\ \mathbf{M} & \mathbf{C} \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -\mathbf{M} & \mathbf{O} \\ \mathbf{O} & \mathbf{K} \end{pmatrix}, \mathbf{z} = \begin{pmatrix} \dot{\mathbf{u}} \\ \mathbf{u} \end{pmatrix}. \quad (24)$$

The m -th eigenvector $\boldsymbol{\kappa}_m$ can be expressed by

$$\boldsymbol{\kappa}_m = \begin{pmatrix} \lambda_m \boldsymbol{\psi}_m \\ \boldsymbol{\psi}_m \end{pmatrix} \quad (m=1, \dots, 2N). \quad (25)$$

The orthogonality of the eigenvectors with respect to $\mathbf{a}, \mathbf{b}$ provides

$$\boldsymbol{\kappa}_j^T \mathbf{a} \boldsymbol{\kappa}_m = 0, \boldsymbol{\kappa}_j^T \mathbf{b} \boldsymbol{\kappa}_m = 0 \quad (j \neq m). \quad (26)$$

The response $\mathbf{z}$ of an elastic, non-proportionally damped MDOF model can be expressed by

$$\mathbf{z} = \sum_{n=1}^N u_n(t) = \sum_{n=1}^N \{\boldsymbol{\kappa}_n z_n(t) + \bar{\boldsymbol{\kappa}}_n \bar{z}_n(t)\}, \quad (27)$$

where $\bar{\boldsymbol{\kappa}}_n \bar{z}_n$ is the complex conjugate of $\boldsymbol{\kappa}_n z_n$. By the Fourier transformation of Equation (24) and the premultiplication of $\boldsymbol{\kappa}_m^T = (\lambda_m \boldsymbol{\psi}_m^T \ \boldsymbol{\psi}_m^T)$, the following equation is obtained.

$$(\lambda_m \boldsymbol{\psi}_m^T \ \boldsymbol{\psi}_m^T)(i\omega \mathbf{a} + \mathbf{b}) \begin{pmatrix} \lambda_m \boldsymbol{\psi}_m \\ \boldsymbol{\psi}_m \end{pmatrix} Z_m = -\boldsymbol{\psi}_m^T \mathbf{M} \mathbf{1} \ddot{U}_g^{\text{MI}}. \quad (28)$$

The coefficient of Z_m in the left side of Equation (28) can be rewritten in the following manner (see Appendix 1).

$$\begin{aligned} & (\lambda_m \boldsymbol{\psi}_m^T \ \boldsymbol{\psi}_m^T)(i\omega \mathbf{a} + \mathbf{b}) \begin{pmatrix} \lambda_m \boldsymbol{\psi}_m \\ \boldsymbol{\psi}_m \end{pmatrix} \\ &= -\{\omega^2 - (i\lambda_m + \omega)^2\} \boldsymbol{\psi}_m^T \mathbf{M} \boldsymbol{\psi}_m + i\omega \boldsymbol{\psi}_m^T \mathbf{C} \boldsymbol{\psi}_m + \boldsymbol{\psi}_m^T \mathbf{K} \boldsymbol{\psi}_m \end{aligned} \quad (29)$$

From Equations (28), (29), $Z_m / \ddot{U}_g^{\text{MI}}$ can be expressed by

$$\frac{Z_m}{\ddot{U}_g^{\text{MI}}} = \frac{-\boldsymbol{\psi}_m^T \mathbf{M} \mathbf{1}}{-\{\omega^2 - (i\lambda_m + \omega)^2\} \boldsymbol{\psi}_m^T \mathbf{M} \boldsymbol{\psi}_m + i\omega \boldsymbol{\psi}_m^T \mathbf{C} \boldsymbol{\psi}_m + \boldsymbol{\psi}_m^T \mathbf{K} \boldsymbol{\psi}_m}. \quad (30)$$

The transfer function $\mathbf{H}^g$ is then obtained as

$$\begin{aligned} \mathbf{H}_n^g = \frac{\mathbf{U}_n}{\ddot{U}_g^{\text{MI}}} &= \frac{-\boldsymbol{\psi}_n^T \mathbf{M} \mathbf{1}}{-\{\omega^2 - (i\lambda_n + \omega)^2\} \boldsymbol{\psi}_n^T \mathbf{M} \boldsymbol{\psi}_n + i\omega \boldsymbol{\psi}_n^T \mathbf{C} \boldsymbol{\psi}_n + \boldsymbol{\psi}_n^T \mathbf{K} \boldsymbol{\psi}_n} \boldsymbol{\psi}_n \\ &+ \frac{-\bar{\boldsymbol{\psi}}_n^T \mathbf{M} \mathbf{1}}{-\{\omega^2 - (i\bar{\lambda}_n + \omega)^2\} \bar{\boldsymbol{\psi}}_n^T \mathbf{M} \bar{\boldsymbol{\psi}}_n + i\omega \bar{\boldsymbol{\psi}}_n^T \mathbf{C} \bar{\boldsymbol{\psi}}_n + \bar{\boldsymbol{\psi}}_n^T \mathbf{K} \bar{\boldsymbol{\psi}}_n} \bar{\boldsymbol{\psi}}_n, \end{aligned} \quad (31a)$$

$$\mathbf{H}^g = \sum_{n=1}^N \mathbf{H}_n^g. \quad (31b)$$

Similarly, $\mathbf{H}^f$ can be obtained as follows.

$$\mathbf{H}_n^f = \frac{\mathbf{U}_n}{F^{\text{PMI}}} = \frac{-\boldsymbol{\Psi}_n^T \mathbf{M}(\alpha_1 \boldsymbol{\Phi}_1)}{-\{\omega^2 - (i\lambda_n + \omega)^2\} \boldsymbol{\Psi}_n^T \mathbf{M} \boldsymbol{\Psi}_n + i\omega \boldsymbol{\Psi}_n^T \mathbf{C} \boldsymbol{\Psi}_n + \boldsymbol{\Psi}_n^T \mathbf{K} \boldsymbol{\Psi}_n} \boldsymbol{\Psi}_n + \frac{-\bar{\boldsymbol{\Psi}}_n^T \mathbf{M}(\alpha_1 \boldsymbol{\Phi}_1)}{-\{\omega^2 - (i\bar{\lambda}_n + \omega)^2\} \bar{\boldsymbol{\Psi}}_n^T \mathbf{M} \bar{\boldsymbol{\Psi}}_n + i\omega \bar{\boldsymbol{\Psi}}_n^T \mathbf{C} \bar{\boldsymbol{\Psi}}_n + \bar{\boldsymbol{\Psi}}_n^T \mathbf{K} \bar{\boldsymbol{\Psi}}_n} \bar{\boldsymbol{\Psi}}_n, \quad (32a)$$

$$\mathbf{H}^f = \sum_{n=1}^N \mathbf{H}_n^f. \quad (32b)$$

Although $\mathbf{H}^g$ and $\mathbf{H}^f$ have been derived, the Fourier transformation of the unit impulse response functions provides them in a different manner. It is noted that the unit impulse response functions are obtained by dividing Equations (13), (18) by V . Then $\mathbf{H}^g$ can be derived as

$$\mathbf{H}_n^g = \int_0^\infty \left\{ \frac{-e^{-h_n^* \omega_n^* t}}{\omega_{Dn}^*} \sum_{j=1}^N \mu_{n,j}^g(\alpha_j \boldsymbol{\Phi}_j) \sin(\omega_{Dn}^* t - \theta_{n,j}^g) \right\} e^{-i\omega t} dt$$

$$= \sum_{j=1}^N \left\{ -\mu_{n,j}^g(\alpha_j \boldsymbol{\Phi}_j) \frac{-(h_n^* \omega_n^* + i\omega) / \omega_{Dn}^* \sin(\theta_{n,j}^g) + \cos(\theta_{n,j}^g)}{(\omega_n^*)^2 - \omega^2 + 2ih_n^* \omega_n^* \omega} \right\}, \quad (33a)$$

$$\mathbf{H}^g = \sum_{n=1}^N \mathbf{H}_n^g. \quad (33b)$$

The detail of the derivation of Equation (33a) is shown in Appendix 2. Similarly, $\mathbf{H}^f$ can be obtained as follows.

$$\mathbf{H}_n^f = \sum_{j=1}^N \left\{ -\mu_{n,j}^f(\alpha_j \boldsymbol{\Phi}_j) \frac{-(h_n^* \omega_n^* + i\omega) / \omega_{Dn}^* \sin(\theta_{n,j}^f) + \cos(\theta_{n,j}^f)}{(\omega_n^*)^2 - \omega^2 + 2ih_n^* \omega_n^* \omega} \right\}, \quad (34a)$$

$$\mathbf{H}^f = \sum_{n=1}^N \mathbf{H}_n^f. \quad (34b)$$

Equations (33a, b), (34a, b) clearly indicate that $\mu_{i,j}^g, \mu_{i,j}^f$ can be regarded as the weight coefficient of the j -th undamped mode vector component in the i -th complex mode.

4. Comparison of responses for elastic and elastic-plastic MDOF models under MI, PMI and MSW through time-history response analysis

In Section 4.1, the time-history response analysis is conducted to compare the responses of elastic MDOF models under MI, PMI, and the corresponding MSW. In Section 4.2, the maximum responses

of elastic-plastic MDOF models under those inputs are treated. In Section 4.3, PMI is extended to treat the critical response for the second mode.

4.1 Elastic models

Consider two 24-story shear building models with different damping distributions. This setting of damping distribution can be understood from the placement of passive viscous dampers. Both models have a trapezoidal distribution of story stiffnesses ($k_1 / k_{24} = 2.5$). The undamped fundamental natural period is 2.4 [s]. All the floor masses have the same value ($=400 \times 10^3$ [kg]). The common story height is 4 [m]. The P-Model has stiffness-proportional type damping and the NP-Model has a uniform distribution of damping coefficients (non-proportionally damped). For both models, the sum of damping coefficients is set to 20×10^7 [Ns/m]. The fundamental-mode damping ratio of the P-Model is 0.019, and that of the NP-Model is 0.017 (complex eigenvalue analysis). Figure 3 shows the undamped natural periods $T_1, \dots, T_4$ and the 1-4th participation vectors $\alpha_1 \phi_1, \dots, \alpha_4 \phi_4$. Since the NP-model has non-proportionally damping, PMI excites all the mode responses of the NP-Model as stated in Sections 3.1 and 3.2. However, it will be demonstrated through the numerical examples that the higher-mode responses hardly contribute to the responses under PMI.

The number of the impulses in PMI is set to $N^{\text{MI}} = 100$, and the input velocity of PMI is set to $V = 0.1$ [m/s]. Three kinds of the time interval $t_0 = T_1 / 6, T_1 / 2, 3T_1 / 2$ are treated. Note that $t_0 = T_1 / 2$ is nearly the critical time interval.

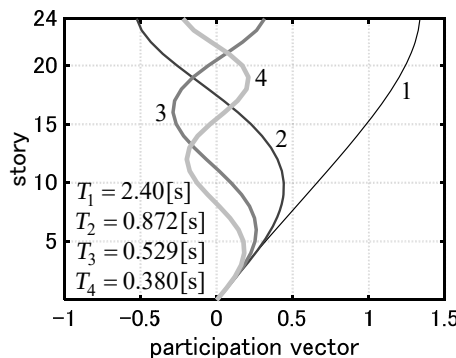


Figure 3 Participation vectors and natural periods of treated model

Figures 4a-9a show the distributions of the maximum interstory drifts and the distributions of the maximum floor accelerations under MI, PMI and the corresponding MSW. Figures 4b-9b illustrate the Fourier amplitudes of theses inputs and the amplitudes of the top relative acceleration transfer functions. The top relative acceleration transfer functions are adopted for visibility in place of those of the top displacement transfer functions. It can be observed from Figures 7b-9b that the higher-mode responses under PMI are hardly excited for the NP-Model. In common with all the cases of

Figures 4-9, the maximum floor accelerations in lower stories under MI are much larger than those under PMI and MSW. This is because the impulse input with $\mathbf{u} = \mathbf{1}$ equally and simultaneously excites all the mode responses. In other words, the instantaneous change $-V$ of the interstory velocity response is provided only to the first story since the impulse input causes the instantaneous change $-V\mathbf{1}$ of the horizontal velocity responses. Then the excessive relative acceleration response $-\mathbf{M}^{-1}\mathbf{C}(-V\mathbf{1})$ arises to balance with the damping force $\mathbf{C}(-V\mathbf{1})$. In the cases of $t_0 = T_1 / 2$ (Figures 4 and 7), the maximum interstory drifts and the maximum floor accelerations under PMI correspond well to those under MSW. Although the maximum interstory drifts under MI also correspond to those under PMI and MSW, the maximum floor accelerations under MI become larger in upper and lower stories than those under PMI and MSW. It can be observed from Figures 4b and 7b that the second eigenmode enlarges the maximum floor accelerations in upper stories. In the cases of $t_0 = T_1 / 6$ (Figures 5 and 8), the maximum responses under PMI are extremely small. This is because the higher modes are hardly excited and the fundamental eigenmode is also hardly excited. As seen in Figures 5b and 8b, the peaks of the transfer functions do not correspond to the peaks of the Fourier amplitude of PMI. On the other hand, the maximum interstory drifts under MI correspond well to those under MSW. This is because the second-mode responses are dominantly excited and the other mode-responses hardly contribute to the maximum interstory drifts. The maximum floor accelerations under MI and MSW also correspond well except for the lower stories. In the cases of $t_0 = 3T_1 / 2$ (Figures 6 and 9), the maximum responses under MSW are extremely small. This is because the peaks of the transfer functions do not correspond to the peaks of the Fourier amplitude of MSW, and all the modes are hardly excited. On the other hand, the correspondence of the maximum interstory drifts under MI and those under PMI is fairly good because both inputs almost equally excite the fundamental-mode responses. However, the maximum floor accelerations under MI are larger than those under PMI because of the contribution of the higher-mode responses.

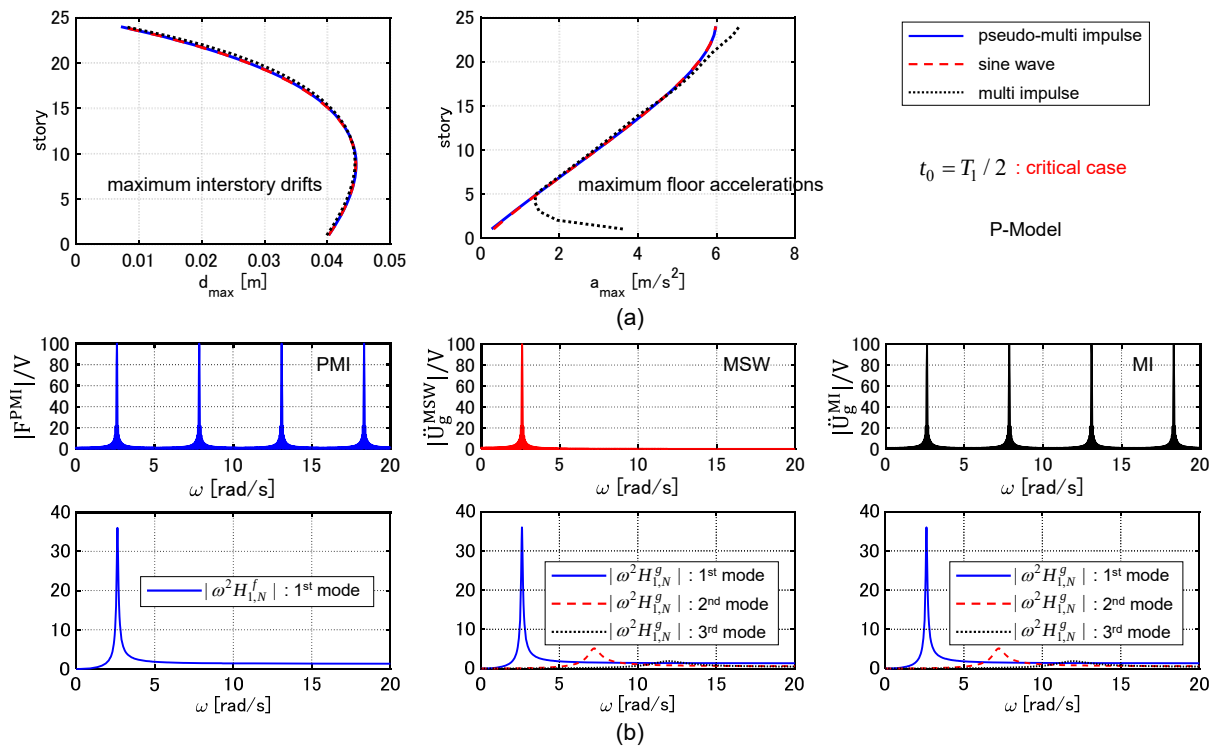


Figure 4 Maximum interstory drifts, maximum floor accelerations, relative acceleration transfer functions of top story for P-Model and Fourier amplitudes of PMI, MSW, MI with $t_0 = T_1 / 2$, (a) maximum responses, (b) transfer functions and Fourier amplitudes.

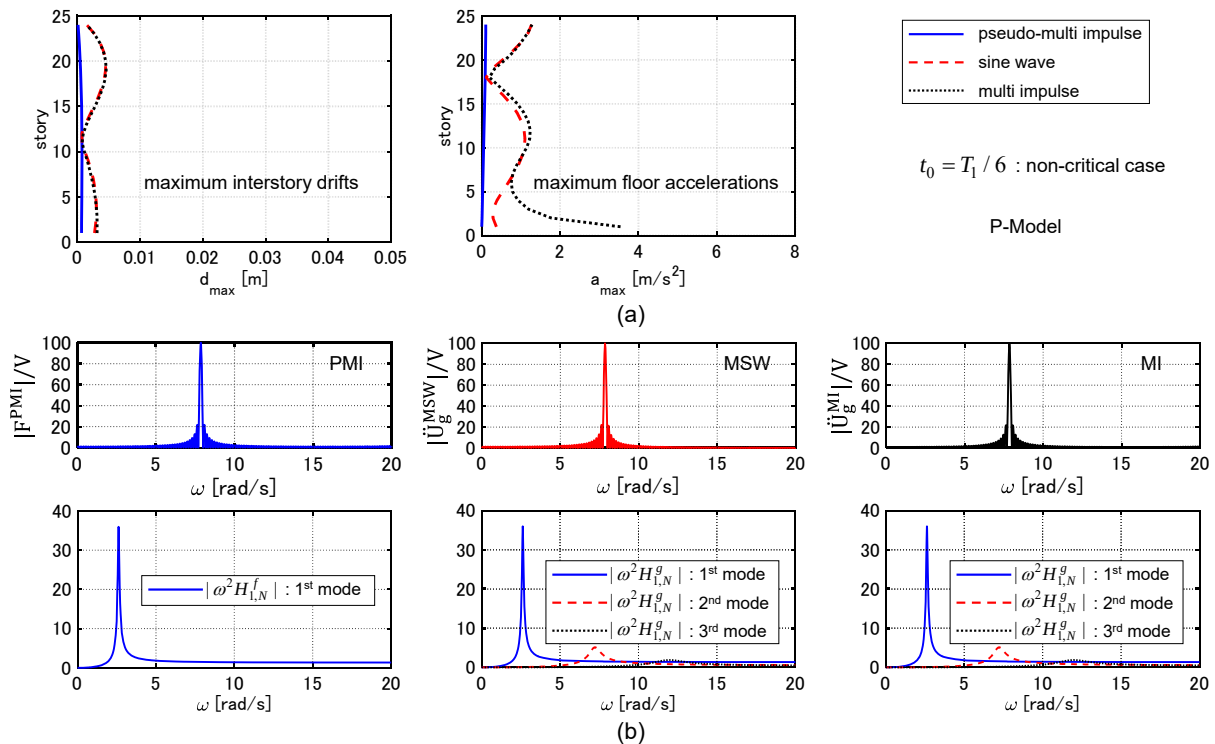


Figure 5 Maximum interstory drifts, maximum floor accelerations, relative acceleration transfer functions of top story for P-Model and Fourier amplitudes of PMI, MSW, MI with $t_0 = T_1 / 6$, (a) maximum responses, (b) transfer functions and Fourier amplitudes.

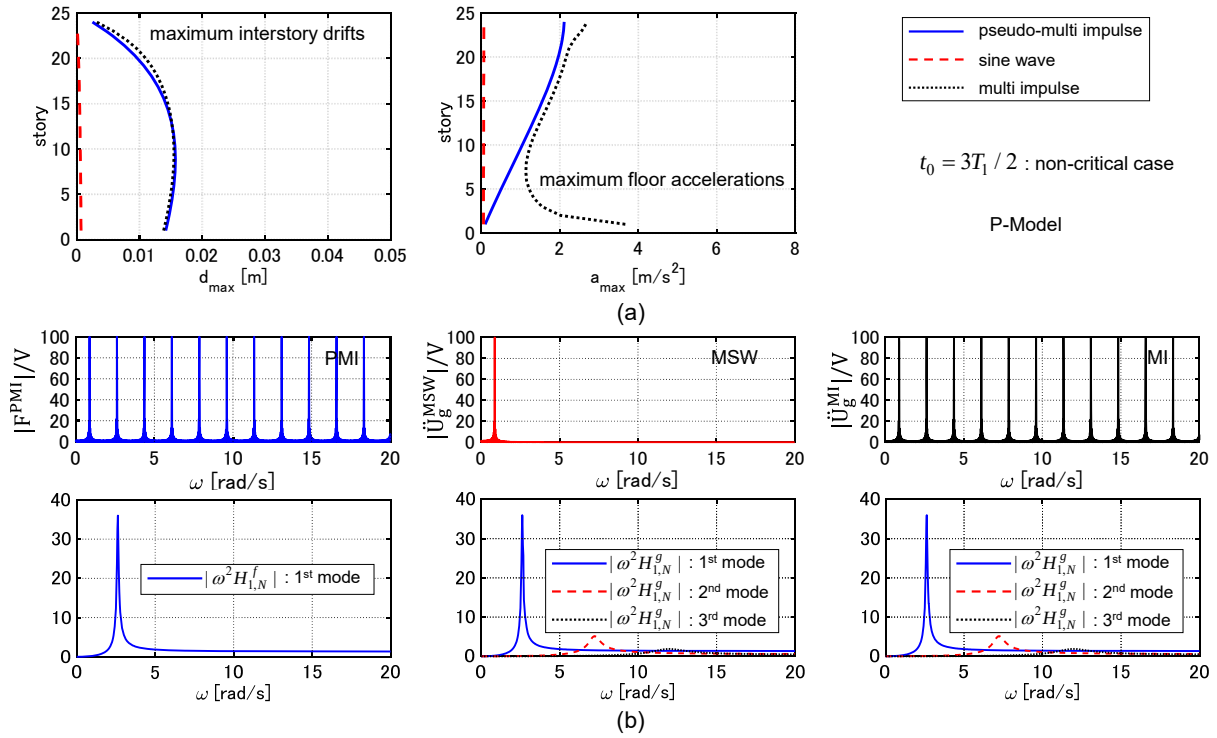


Figure 6 Maximum interstory drifts, maximum floor accelerations, relative acceleration transfer functions of top story for P-Model and Fourier amplitudes of PMI, MSW, MI with $t_0 = 3T_1 / 2$, (a) maximum responses, (b) transfer functions and Fourier amplitudes.

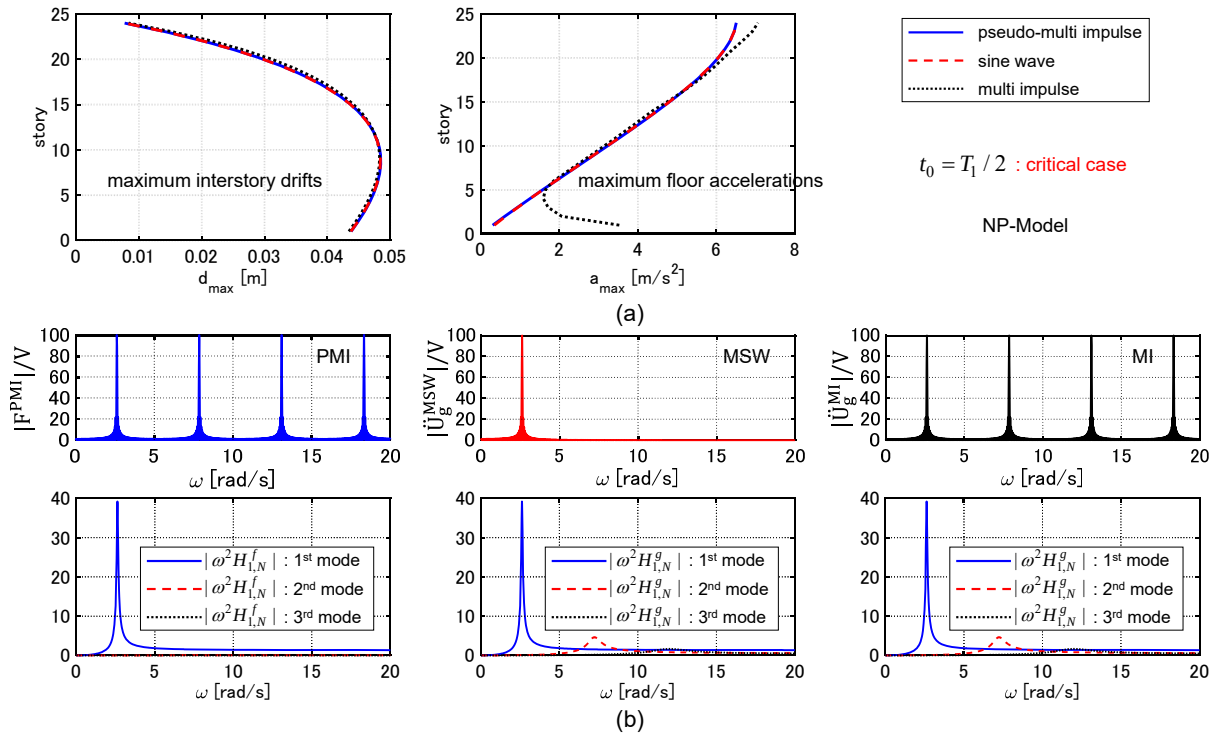


Figure 7 Maximum interstory drifts, maximum floor accelerations, relative acceleration transfer functions of top story for NP-Model and Fourier amplitudes of PMI, MSW, MI with $t_0 = T_1 / 2$, (a) maximum responses, (b) transfer functions and Fourier amplitudes.

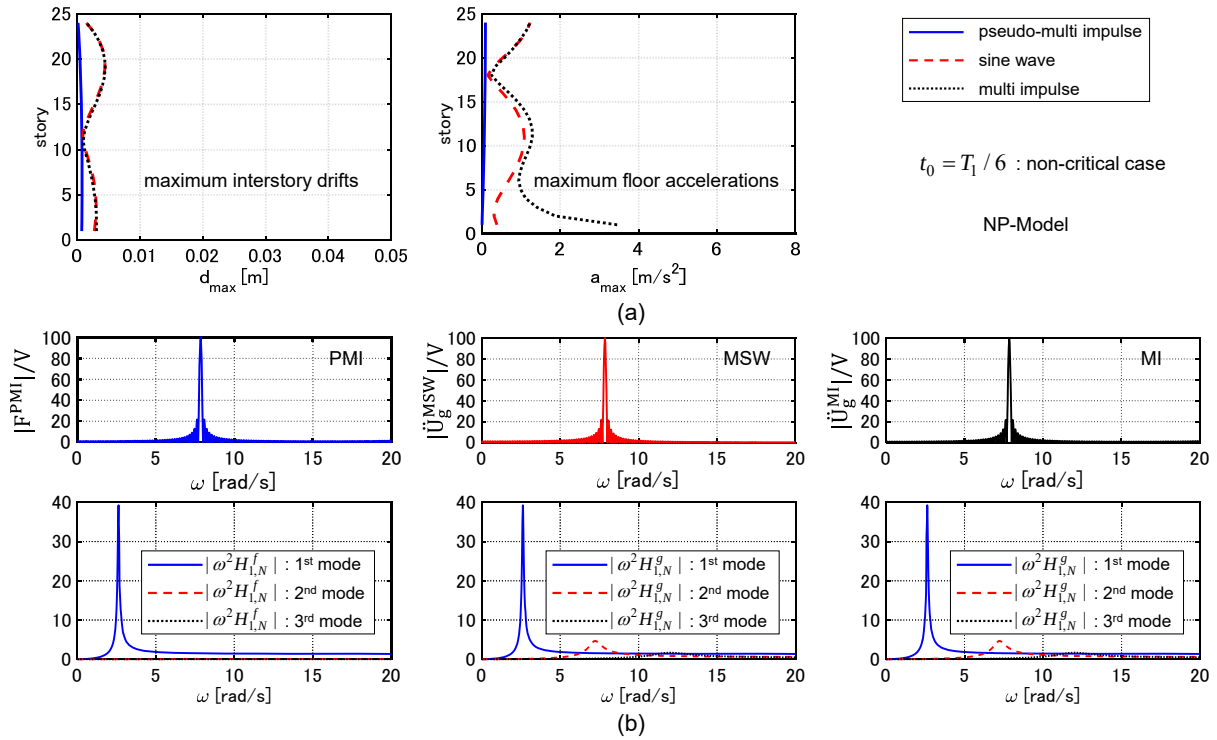


Figure 8 Maximum interstory drifts, maximum floor accelerations, relative acceleration transfer functions of top story for NP-Model and Fourier amplitudes of PMI, MSW, MI with $t_0 = T_1 / 6$, (a) maximum responses, (b) transfer functions and Fourier amplitudes.

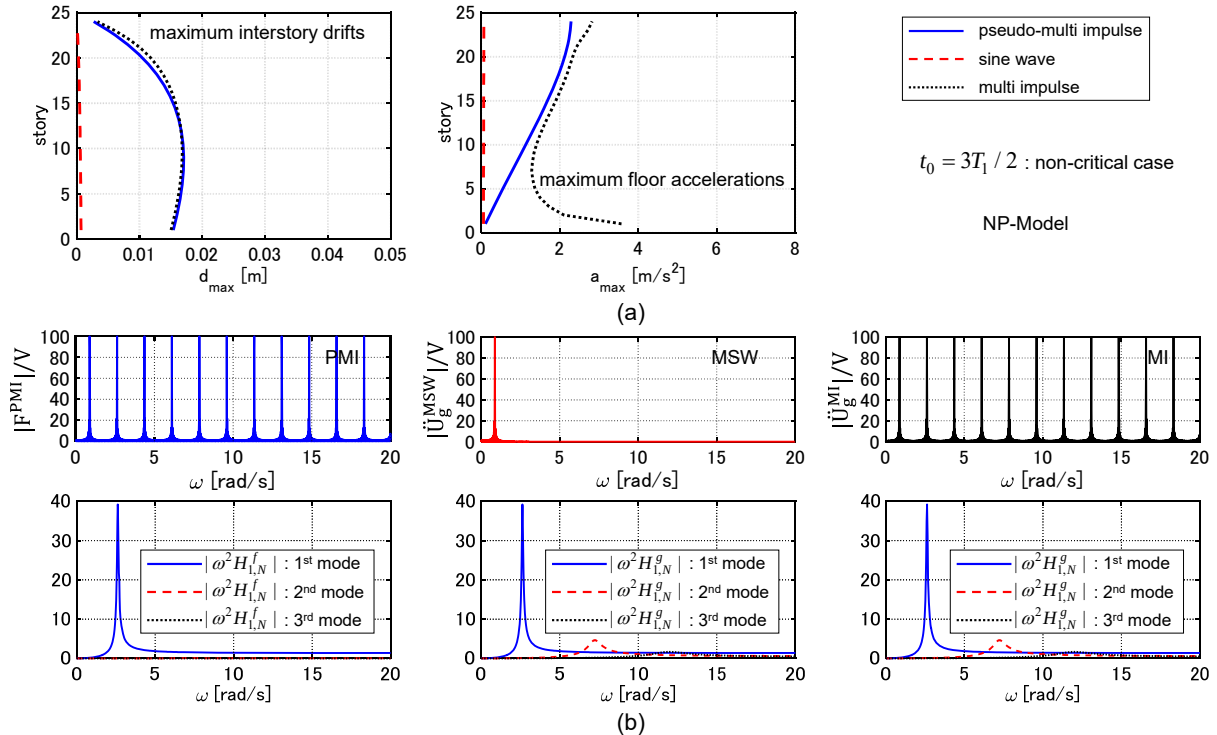


Figure 9 Maximum interstory drifts, maximum floor accelerations, relative acceleration transfer functions of top story for NP-Model and Fourier amplitudes of PMI, MSW, MI with $t_0 = 3T_1 / 2$, (a) maximum responses, (b) transfer functions and Fourier amplitudes

4.2 Elastic-plastic models

In this section, the time-history response analysis is conducted to compare the maximum responses of elastic-plastic MDOF models under MI, PMI, and the corresponding MSW. P-Model and NP-Model are used again. The common yield interstory drift is $d_y = 4/150$ [m] and the bilinear hysteresis (kinematic hardening) in the story shear-interstory drift relation is assumed. The post-yield stiffness ratio is set to 0.1. It is noted that the maximum interstory drifts $d_{\max,i}$ in the i -th story is evaluated by $d_{\max,i} = (d_{\max,i}^+ - d_{\max,i}^-) / 2$, where $d_{\max,i}^+, d_{\max,i}^-$ are the positive and negative peak values of the interstory drifts in the stationary response (see Figure 10). It is pointed out that the value of $(d_{\max,i}^+ + d_{\max,i}^-)$ does not always become zero due to the residual drift. In the case of Figure 10, the story yields before attaining the stationary response, and the story exhibits linear elastic response in the stationary response.

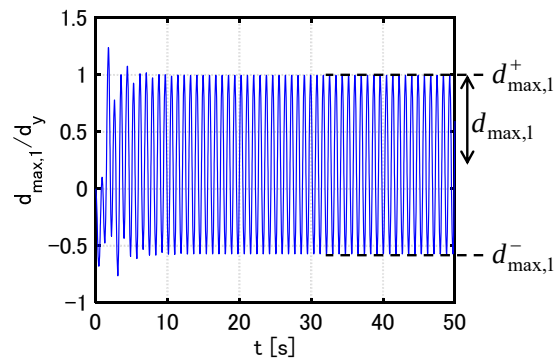


Figure 10 Example of time-history displacement response at 1st floor.

The durations of MI, PMI and MSW are set to $50T_1$ ($=120$ [s]), and the input velocities of PMI are set to $V = 0.05, 0.1, 0.2, 0.4, 0.7$ [m/s]. Various kinds of the time interval $T_1/6 \leq t_0 \leq 3T_1/4$ are treated.

Figures 11a, b and 12a, b show the maximum interstory drifts $d_{\max}$ and the maximum top accelerations $a_{\max,N}$ under MI, PMI, and MSW with respect to t_0 . The critical timing for PMI at each input level is also plotted (blue circle). Figures 11c, d, and 12c, d present the distributions of the maximum interstory drifts and the distributions of the maximum floor accelerations under the critical PMI, and those under the corresponding MSW and MI. It can be observed from Figures 11a, b and 12a, b that the critical PMI and the corresponding MSW almost maximize $d_{\max}$ and locally maximize $a_{\max,N}$. Moreover, the values of those correspond well. On the other hand, the values of $a_{\max,N}$ under the corresponding MI are larger than those under PMI and the corresponding MSW, although the values of $d_{\max}$ under the corresponding MI correspond well to those under PMI and the corresponding MSW. It is noted that the value of $a_{\max,N}$ under MSW are maximized at around $t_0 = 0.4$ [s] ($\approx T_2/2$). PMI cannot capture such peak values because this corresponds to the critical case for the second mode. The critical case for the second mode will be investigated in the next section. It can be observed from Figures 11c, d and 12c, d that the distributions of the maximum

interstory drifts and the distributions of the maximum floor accelerations under the critical PMI corresponds well to those under the corresponding MSW. On the other hand, the floor accelerations under MI are much larger than those under PMI and MSW.

From the above results, the critical PMI with $\mathbf{u} = \alpha_1 \boldsymbol{\phi}_1$ can efficiently and accurately evaluate the maximum interstory drifts, the maximum floor accelerations and the critical input period of MSW for elastic-plastic MDOF models. It is noted again that, to directly find the critical input period of MSW, the repetition of the time-history response analyses under MSW with various input periods is required. However, such repetition is not required in the case of the critical PMI as stated in Section 2.2.

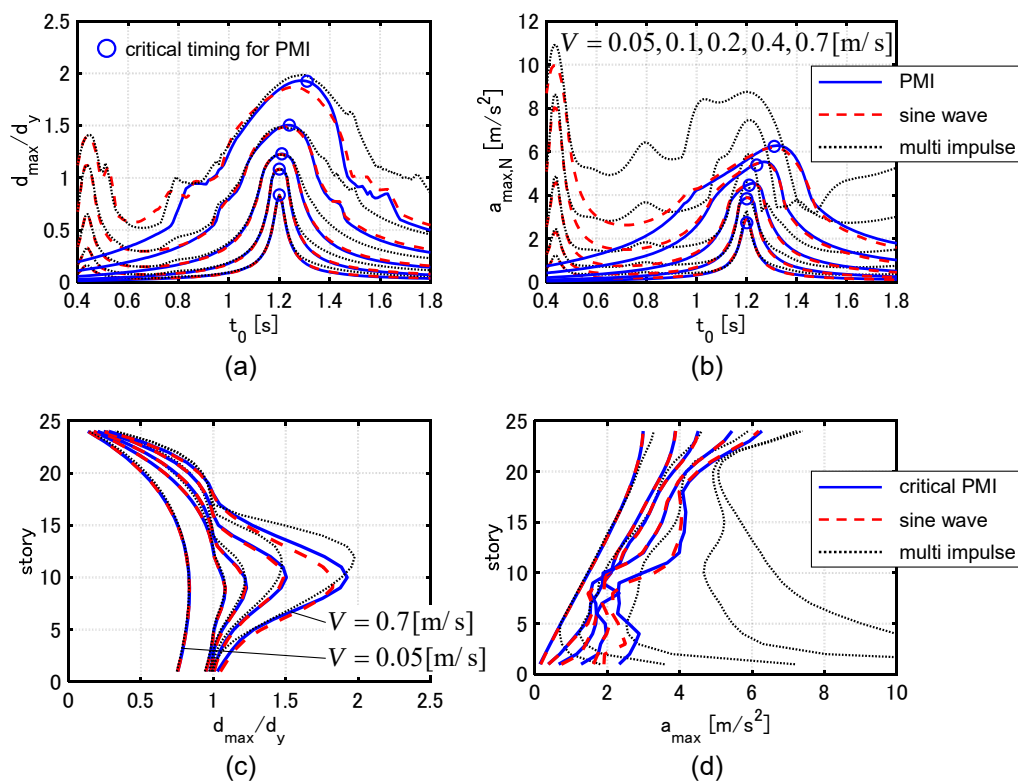


Figure 11 Comparisons of maximum responses for P-Model under PMI, MSW and MI, (a) $d_{\max}$ with respect to t_0 , (b) $a_{\max,N}$ with respect to t_0 , (c) distributions of maximum interstory drifts, (d) distributions of maximum floor accelerations.

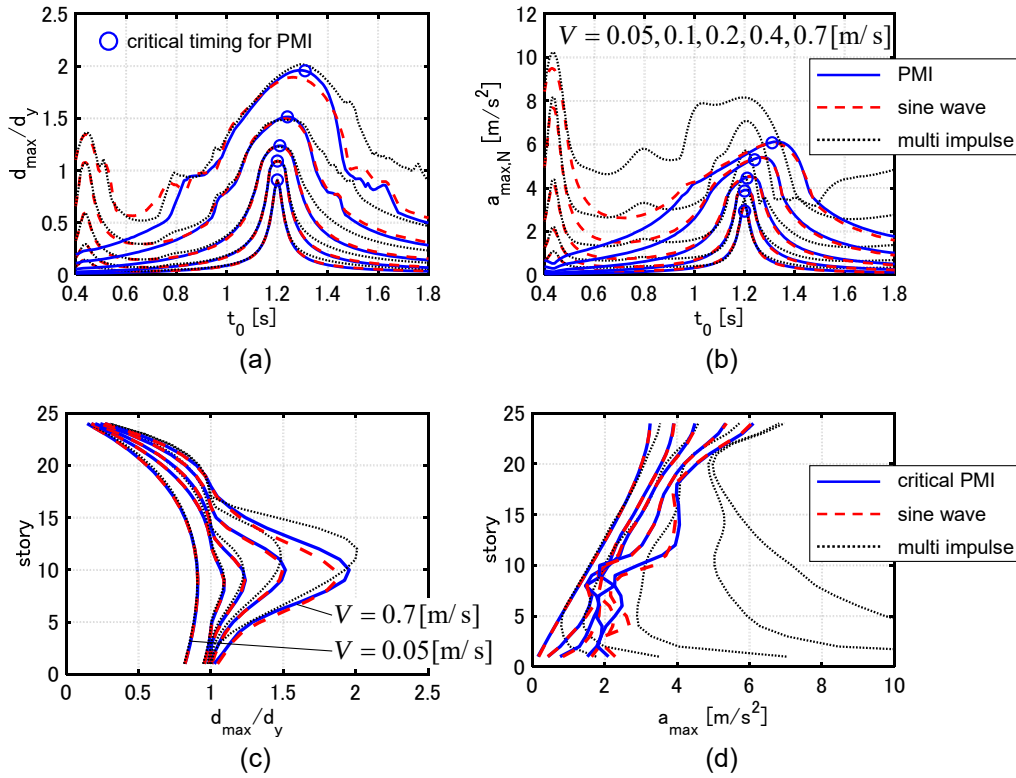


Figure 12 Comparisons of maximum responses for NP-Model under PMI, MSW and MI, (a) $d_{\max}$ with respect to t_0 , (b) $a_{\max,N}$ with respect to t_0 , (c) distributions of maximum interstory drifts, (d) distributions of maximum floor accelerations.

4.3 Critical case for second eigenmode

In this section, the maximum responses under PMI with two cases $\mathbf{u} = \alpha_2 \boldsymbol{\phi}_2, \alpha_1 \boldsymbol{\phi}_1 + \alpha_2 \boldsymbol{\phi}_2$ are investigated to treat the critical case for the second mode. The critical timings of PMI with $\mathbf{u} = \alpha_2 \boldsymbol{\phi}_2, \alpha_1 \boldsymbol{\phi}_1 + \alpha_2 \boldsymbol{\phi}_2$ are defined as the timings when the sum $(\alpha_2 \boldsymbol{\phi}_2^T) \mathbf{M} \ddot{\mathbf{u}}$ of the elastic second-mode inertial forces attains zero. This corresponds to the maximization of the input energy by PMI with $\mathbf{u} = \alpha_2 \boldsymbol{\phi}_2$. It is noted that the ordinary MI as the ground motion is not treated in this section.

Figures 13a, b and 14a, b show the maximum interstory drifts $d_{\max}$ and the maximum top accelerations $a_{\max,N}$ under PMI with $\mathbf{u} = \alpha_2 \boldsymbol{\phi}_2, \alpha_1 \boldsymbol{\phi}_1 + \alpha_2 \boldsymbol{\phi}_2$, and MSW with respect to t_0 . The critical timing for PMI with $\mathbf{u} = \alpha_1 \boldsymbol{\phi}_1 + \alpha_2 \boldsymbol{\phi}_2$ at each input level is also plotted (blue circle). It is noted that the critical timing for PMI with $\mathbf{u} = \alpha_2 \boldsymbol{\phi}_2$ almost coincides with that for PMI with $\mathbf{u} = \alpha_1 \boldsymbol{\phi}_1 + \alpha_2 \boldsymbol{\phi}_2$. Figures 13c, d, and 14c, d present the distributions of the maximum interstory drifts and the distributions of the maximum floor accelerations under the critical PMI with $\mathbf{u} = \alpha_1 \boldsymbol{\phi}_1 + \alpha_2 \boldsymbol{\phi}_2$, and those under the corresponding MSW and PMI with $\mathbf{u} = \alpha_2 \boldsymbol{\phi}_2$. It can be observed from Figures 13a, b and 14a, b that the critical PMI with $\mathbf{u} = \alpha_1 \boldsymbol{\phi}_1 + \alpha_2 \boldsymbol{\phi}_2$ and the corresponding MSW almost maximize $a_{\max,N}$ and locally maximize $d_{\max}$. Moreover, the values of those correspond well. It can be observed from Figures 13c, d and 14c, d that the distributions of the maximum interstory

drifts and the distributions of the maximum floor accelerations under the critical PMI with $\mathbf{u} = \alpha_1 \boldsymbol{\phi}_1 + \alpha_2 \boldsymbol{\phi}_2$ corresponds well to those under the corresponding MSW, except for the floor accelerations in lower stories. It is pointed out that the interstory drifts in the 10-th story under PMI with $\mathbf{u} = \alpha_2 \boldsymbol{\phi}_2$ are quite small because the interstory drift of the 10-th story in the second-mode vector is almost zero (see Figure 3). Since the critical PMI with $\mathbf{u} = \alpha_1 \boldsymbol{\phi}_1 + \alpha_2 \boldsymbol{\phi}_2$ and the corresponding MSW excite the fundamental-mode responses, the maximum interstory drifts under the critical PMI with $\mathbf{u} = \alpha_1 \boldsymbol{\phi}_1 + \alpha_2 \boldsymbol{\phi}_2$ correspond to those under the corresponding MSW better than PMI with $\mathbf{u} = \alpha_2 \boldsymbol{\phi}_2$.

From the above results, the critical PMI with $\mathbf{u} = \alpha_1 \boldsymbol{\phi}_1 + \alpha_2 \boldsymbol{\phi}_2$ can effectively simulate the critical case for the second mode.

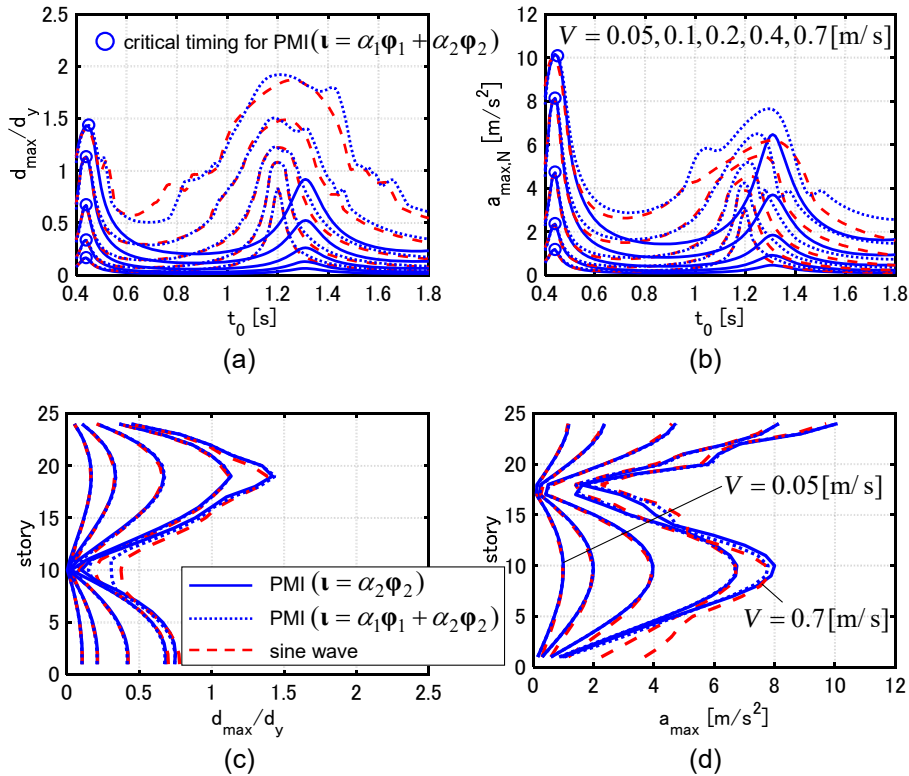


Figure 13 Comparisons of maximum responses for P-Model under PMI ($\mathbf{u} = \alpha_2 \boldsymbol{\phi}_2, \alpha_1 \boldsymbol{\phi}_1 + \alpha_2 \boldsymbol{\phi}_2$) and MSW, (a) $d_{\max}$ with respect to t_0 , (b) $a_{\max,N}$ with respect to t_0 , (c) distributions of maximum interstory drifts, (d) distributions of maximum floor accelerations.

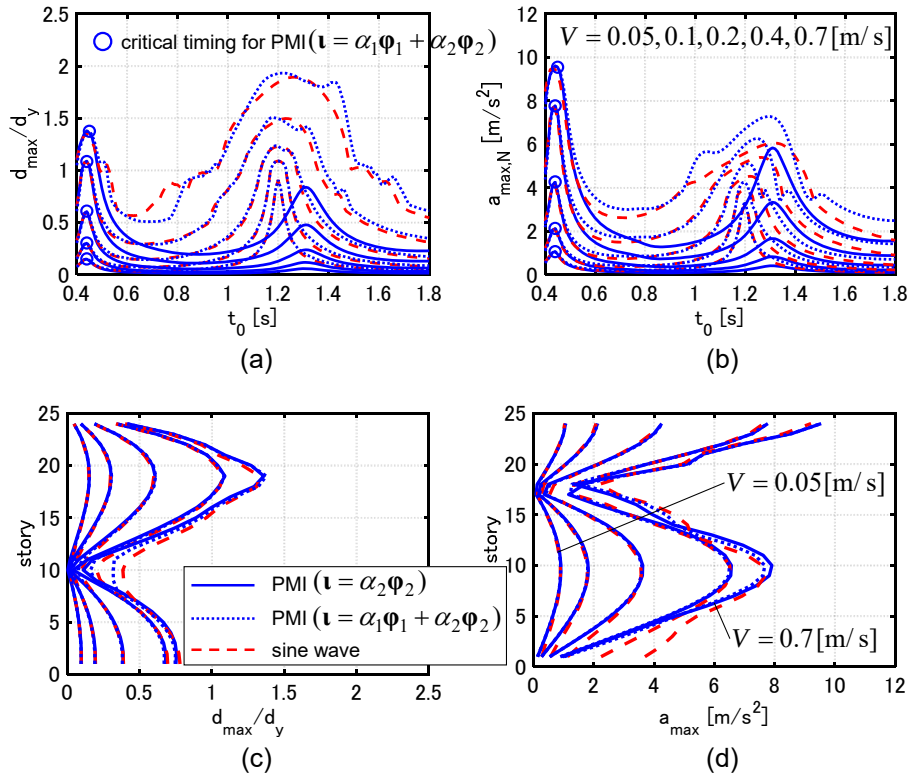


Figure 14 Comparisons of maximum responses for NP-Model under PMI ($\mathbf{t} = \alpha_2 \boldsymbol{\phi}_2, \alpha_1 \boldsymbol{\phi}_1 + \alpha_2 \boldsymbol{\phi}_2$) and MSW, (a) $d_{\max}$ with respect to t_0 , (b) $a_{\max,N}$ with respect to t_0 , (c) distributions of maximum interstory drifts, (d) distributions of maximum floor accelerations.

5. Realistic application of critical PMI to high-rise buildings for investigation into input energy and cumulative plastic deformation ductility demand

In Section 5.1, the input energies and the maximum interstory drifts are investigated for elastic MDOF models and elastic-plastic MDOF models under the critical PMI. In Section 5.2, the time-history response analysis is conducted for high-rise building models with different numbers of stories to investigate the influence of building heights on the cumulative plastic deformation ductility demand.

5.1 Input energy to elastic/elastic-plastic MDOF models under critical PMI

The increment E of the input energy by each impulse input is calculated by subtracting the kinetic energy just before the impulse input from that just after the impulse input. When the model exhibits a stationary response under the critical PMI, E can be expressed as

$$\begin{aligned}
 E &= (1/2) \{ \mathbf{v}_c + V(\alpha_1 \boldsymbol{\phi}_1) \}^T \mathbf{M} \{ \mathbf{v}_c + V(\alpha_1 \boldsymbol{\phi}_1) \} - (1/2) \mathbf{v}_c^T \mathbf{M} \mathbf{v}_c, \\
 &= V(\alpha_1 \boldsymbol{\phi}_1)^T \mathbf{M} \mathbf{v}_c + (1/2) V^2 (\alpha_1 \boldsymbol{\phi}_1)^T \mathbf{M} V (\alpha_1 \boldsymbol{\phi}_1)
 \end{aligned}
 \tag{35}$$

where $\mathbf{v}_c$ is the velocity response just before the impulse input. It can be understood that E depends on the elastic fundamental-mode component of $\mathbf{v}_c$ because the higher modes are orthogonal to $\alpha_1 \boldsymbol{\phi}_1$ with respect to $\mathbf{M}$.

Figure 15 shows the increment of the input energy by each impulse input and the maximum interstory drifts of the P-model under the critical PMI. It is noted that the elastic response case and the elastic-plastic response case are treated. It can be observed that the hysteretic energy dissipation decreases both of E (or the elastic fundamental-mode component of $\mathbf{v}_c$) and the maximum interstory drifts.

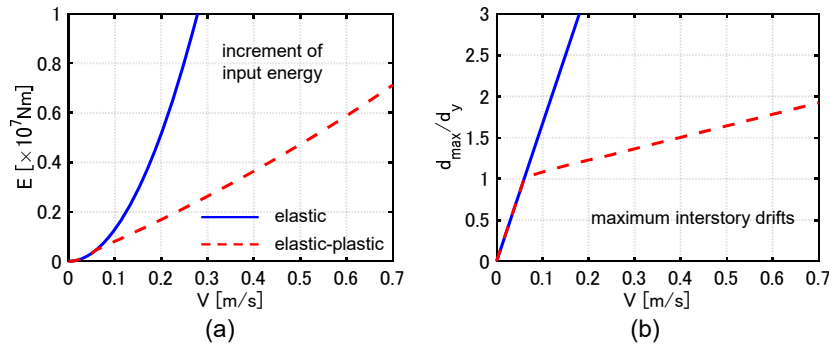


Figure 15 Increment of input energy and maximum interstory drift of P-model, (a) increment of input energy, (b) maximum interstory drift.

5.2 Cumulative plastic deformation ductility demand of high-rise buildings under long-duration, long-period ground motion

In this section, time-history response analysis is conducted for high-rise building models with different numbers of stories to investigate the cumulative plastic deformation ductility demand. It should be noted that, while the analysis for finding the resonant frequency and response to a sinusoidal excitation requires a tedious repetitive procedure to change the input frequency, the proposed analysis using the critical PMI does not need such repetition. This property is to be emphasized.

Consider four shear building models of $N=24, 36, 48, 60$ stories. All the models have a trapezoidal distribution of story stiffnesses ($k_1 / k_N = 2.5$). The undamped fundamental natural periods are 2.4, 3.6, 4.8, 6.0 [s], respectively. All the floor masses have the same value ($=400 \times 10^3$ [kg]). The common story height is 4 [m]. The structural damping ratio is set to 0.02 (stiffness proportional type) for all the models. The common yield interstory drift is $d_y = 4 / 150$ [m] and the bilinear hysteresis in the story shear-interstory drift relation is assumed. The post-yield stiffness ratio is set to 0.1.

The time interval t_0 of PMI is adjusted to the critical one for each model. The durations of PMI are set to 120 [s], and the input velocities of PMI are set to $V = 0.01, 0.02, \dots, 0.7$ [m/s].

Figure 16 illustrates the maximum interstory drifts $d_{\max}$ and the cumulative plastic deformation ductility $\eta_{\max} = \max\{\eta_i\}$, where η_i is the cumulative plastic deformation ductility in the i -th story. It can be observed that $\eta_{\max}$ becomes smaller with the increase of the number of stories although the values of $d_{\max}$ are almost common in all the models. This is because the durations of the inputs are common in all the cases and the (equivalent) fundamental period becomes long with the increase of the number of stories. Assuming that the maximum interstory drifts are almost identical for models with different numbers of stories and the story ductility has a correlation with the frame member ductility, the damage of frame members becomes small with the increase of the number of stories. The present analysis is for the story ductility. Once the data on the relation between the story ductility and the member ductility are provided, the limit on the input velocity can be obtained based on the limit on the member ductility (e.g. see the reference [28]). In general, it may be said that a certain kind of passive control is necessary to satisfy the limit condition on the member ductility (accumulated plastic deformation ductility).

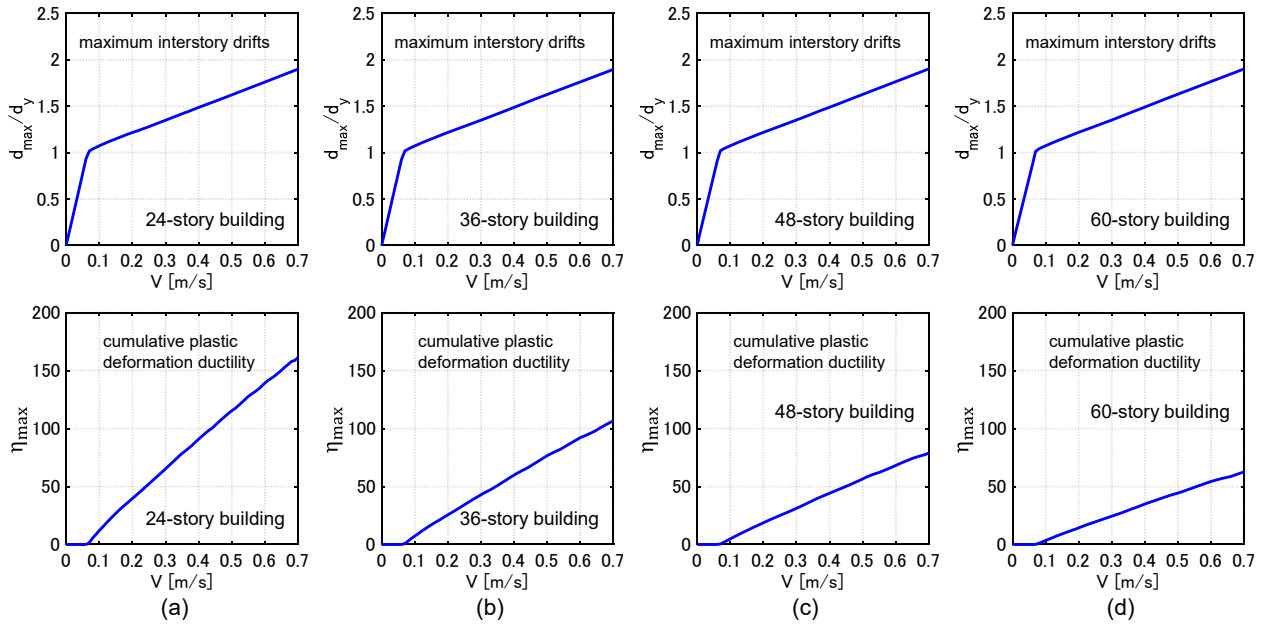


Figure 16 Maximum interstory drifts and cumulative plastic deformation ductility, (a) 24-story building, (b) 36-story building, (c) 48-story building, (d) 60-story building.

6. Conclusions

In this paper, a pseudo-multi impulse (PMI) consisting of impulsive horizontal loads was proposed as an extension of the ordinary multi impulse (MI) as the base input to better simulate the critical response of elastic/elastic-plastic MDOF models under long-period, long-duration ground motions. The main conclusions can be summarized as follows.

- (1) MI and a multi-cycle sine wave (MSW) work well as representatives for long-duration, long-period ground motions for SDOF models. However, the use of MI may lead to the overestimation of the

responses under the corresponding MSW for MDOF models due to its impulsive nature. PMI was introduced to precisely capture the critical responses for elastic-plastic MDOF models under long-period, long-duration ground motions. PMI consisting of a multitude of impulsive lateral forces hardly excites the higher-mode responses of elastic MDOF models because the undamped fundamental participation vector $\alpha_1 \phi_1$ is adopted as the influence coefficient vector of PMI.

- (2) The critical time interval of PMI can be obtained without any repetition. In other words, the time intervals are not required to be set beforehand in the time-history response analysis under the critical PMI. In the analysis, the critical interval condition of each impulse input is easily (automatically) judged, and accordingly the change of the response (velocity and acceleration) by each impulse input can be provided at each input timing.
- (3) The transfer functions with respect to ground accelerations and the transfer functions with respect to lateral forces with $\mathbf{u} = \alpha_1 \phi_1$ were derived. In the case of proportionally-damped MDOF models, PMI excites only the fundamental-mode responses. On the other hand, PMI excites all the mode responses for non-proportionally damped MDOF models. However, the contribution of higher-mode responses to the total responses is extremely small in such cases.
- (4) It was demonstrated through the numerical examples that the critical PMI with $\mathbf{u} = \alpha_1 \phi_1$ can efficiently and accurately evaluate the maximum interstory drifts, the maximum floor accelerations and the critical input period of MSW for elastic-plastic MDOF models.
- (5) PMI with $\mathbf{u} = \alpha_1 \phi_1 + \alpha_2 \phi_2$ was presented to treat the critical response for the second mode. It was demonstrated through the numerical examples that PMI can also effectively simulate such case.
- (6) The input energy by each impulse input and the maximum interstory drift were investigated for elastic MDOF models and elastic-plastic MDOF models under the critical PMI. It was demonstrated that the hysteretic energy dissipation decreases both of the input energy and the maximum interstory drift.
- (7) It was demonstrated through the time-history response analysis that the cumulative plastic deformation ductility demand becomes small with the increase of the number of stories of high-rise buildings under the critical PMI with the constant input velocity and the constant duration.

6. Acknowledgement

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Appendix 1

Equation (29) can be obtained as follows.

$$\begin{aligned}
 & (\lambda_m \Psi_m^T \Psi_m^T)(i\omega \mathbf{a} + \mathbf{b}) \begin{pmatrix} \lambda_m \Psi_m \\ \Psi_m \end{pmatrix} \\
 &= (\lambda_m \Psi_m^T \Psi_m^T) \begin{pmatrix} -\mathbf{M} & i\omega \mathbf{M} \\ i\omega \mathbf{M} & \mathbf{K} + i\omega \mathbf{C} \end{pmatrix} \begin{pmatrix} \lambda_m \Psi_m \\ \Psi_m \end{pmatrix} \\
 &= (-\lambda_m \Psi_m^T \mathbf{M} + i\omega \Psi_m^T \mathbf{M} \quad i\omega \lambda_m \Psi_m^T \mathbf{M} + \Psi_m^T \mathbf{K} + i\omega \Psi_m^T \mathbf{C}) \begin{pmatrix} \lambda_m \Psi_m \\ \Psi_m \end{pmatrix} \quad (A1) \\
 &= (-\lambda_m^2 + 2i\omega \lambda_m) \Psi_m^T \mathbf{M} \Psi_m + i\omega \Psi_m^T \mathbf{C} \Psi_m + \Psi_m^T \mathbf{K} \Psi_m \\
 &= -\{\omega^2 - (i\lambda_m + \omega)^2\} \Psi_m^T \mathbf{M} \Psi_m + i\omega \Psi_m^T \mathbf{C} \Psi_m + \Psi_m^T \mathbf{K} \Psi_m
 \end{aligned}$$

682 **Appendix 2**

683 Equation (33a) can be derived as follows.

$$\begin{aligned}
\mathbf{H}_n^g &= \int_0^\infty \left\{ \frac{-e^{-h_n^* \omega_n^* t}}{\omega_{Dn}^*} \sum_{j=1}^N \mu_{n,j}^g (\alpha_j \Phi_j) \sin(\omega_{Dn}^* t - \theta_{n,j}^g) \right\} e^{-i\omega t} dt \\
&= \sum_{j=1}^N \left\{ \int_0^\infty \frac{-e^{-h_n^* \omega_n^* t}}{\omega_{Dn}^*} \mu_{n,j}^g (\alpha_j \Phi_j) \sin(\omega_{Dn}^* t - \theta_{n,j}^g) e^{-i\omega t} dt \right\} \\
&= \sum_{j=1}^N \left\{ \frac{-\mu_{n,j}^g (\alpha_j \Phi_j)}{2i\omega_{Dn}^*} \int_0^\infty e^{-h_n^* \omega_n^* t} \left(e^{i(\omega_{Dn}^* t - \theta_{n,j}^g)} - e^{-i(\omega_{Dn}^* t - \theta_{n,j}^g)} \right) e^{-i\omega t} dt \right\} \\
&= \sum_{j=1}^N \left\{ \frac{-\mu_{n,j}^g (\alpha_j \Phi_j)}{2i\omega_{Dn}^*} \int_0^\infty \left(e^{-i\theta_{n,j}^g} e^{(-h_n^* \omega_n^* - i\omega + i\omega_{Dn}^*)t} - e^{i\theta_{n,j}^g} e^{(-h_n^* \omega_n^* - i\omega - i\omega_{Dn}^*)t} \right) dt \right\} \\
&= \sum_{j=1}^N \left\{ \frac{-\mu_{n,j}^g (\alpha_j \Phi_j)}{2i\omega_{Dn}^*} \left(\frac{e^{-i\theta_{n,j}^g}}{h_n^* \omega_n^* + i\omega - i\omega_{Dn}^*} - \frac{e^{i\theta_{n,j}^g}}{h_n^* \omega_n^* + i\omega + i\omega_{Dn}^*} \right) \right\} \\
&= \sum_{j=1}^N \left\{ \frac{-\mu_{n,j}^g (\alpha_j \Phi_j)}{2i\omega_{Dn}^*} \left(\frac{e^{-i\theta_{n,j}^g} [h_n^* \omega_n^* + i\omega + i\omega_{Dn}^*]}{(h_n^* \omega_n^* + i\omega)^2 + (\omega_{Dn}^*)^2} - \frac{e^{i\theta_{n,j}^g} [h_n^* \omega_n^* + i\omega - i\omega_{Dn}^*]}{(h_n^* \omega_n^* + i\omega)^2 + (\omega_{Dn}^*)^2} \right) \right\} \\
&= \sum_{j=1}^N \left\{ \frac{-\mu_{n,j}^g (\alpha_j \Phi_j)}{2i\omega_{Dn}^*} \cdot \frac{-2i(h_n^* \omega_n^* + i\omega) \sin(\theta_{n,j}^g) + 2i\omega_{Dn}^* \cos(\theta_{n,j}^g)}{(h_n^* \omega_n^* + i\omega)^2 + (\omega_{Dn}^*)^2} \right\} \quad (A2) \\
&= \sum_{j=1}^N \left\{ -\mu_{n,j}^g (\alpha_j \Phi_j) \frac{-[(h_n^* \omega_n^* + i\omega) / \omega_{Dn}^*] \sin(\theta_{n,j}^g) + \cos(\theta_{n,j}^g)}{(h_n^* \omega_n^* + i\omega)^2 + (\omega_{Dn}^*)^2} \right\} \\
&= \sum_{j=1}^N \left\{ -\mu_{n,j}^g (\alpha_j \Phi_j) \frac{-[(h_n^* \omega_n^* + i\omega) / \omega_{Dn}^*] \sin(\theta_{n,j}^g) + \cos(\theta_{n,j}^g)}{(\omega_n^*)^2 - \omega^2 + 2ih_n^* \omega_n^* \omega} \right\}
\end{aligned}$$