THESIS

Chiral Primordial Gravitational Waves Sourced by Axion-Gauge Couplings

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Abstract

In this thesis, we study the cosmological mechanism about the generation of primordial gravitational waves sourced by matter sector from cosmic inflation, especially provided by the vector field coupled to the string axion during an inflationary period. We suggest several models of axionic inflations which predict a sizable amount of primordial gravitational waves sourced by gauge fields, which are potentially testable with future gravitational wave detectors.

In the conventional inflationary scenario, primordial gravitational waves are produced by vacuum fluctuations of space-time being stretched out and getting classicalized across the Hubble horizon during inflation. This mechanism is common on the relevant scales discussed at inflationary age, so its amplitude has almost statistically scale-invariant and parity symmetric features. However, cosmic microwave background (CMB) observations so far show no evidence of primordial gravitational waves through the imprint of polarization of B-mode in CMB photons. Based on this fact, its amplitude should be tiny even on smaller scales where gravitational wave interferometers are going to probe, if the dominant contribution of gravitational waves are vacuum fluctuations. In addition to that, we might not be able to obtain sufficient information to search for fundamental physics in the early universe only by means of the amplitude and scale dependence of tensor modes.

After giving a short review of above standard inflationary pictures, we suggest another possibility to alter those pessimistic situations by developing an axionic inflation with gauge fields, in the case of both Abelian and non-Abelian gauge groups. The key phenomenon to understand is that the rolling axion amplifies one helicity mode of gauge particles via its parity-violating interaction, and hence particle production of gauge field occurs. Remarkably, the enhanced gauge field creates the parity-violating tensor power spectrum during inflation.

Firstly, we study the mechanism of generating chiral primordial gravitational waves sourced by U(1) gauge field at its quantum loop effect, and explore the possibility of their detection with future laser interferometers or pulsar timing array missions. There, we show that particle production of the gauge field also amplifies scalar modes and then leads to an overproduction of primordial black holes in our present universe. We propose a model which avoids such observational constraints by introducing many U(1) gauge sectors coupled to an axion. Next, we also develop an axionic inflation with SU(2) gauge field called "Chromo-Natural inflation", and find that this scenario predicts chiral gravitational waves sourced by gauge field at linear level, while they are produced too much to satisfy the current CMB data. In order to mend this conflict, we finally extend a model of Chromo-Natural inflation by introducing the dilatonic field which is kinematically coupled with gauge field. We discuss the possibility of Chromo-Natural inflation which provides testable chiral primordial gravitational waves at intermediate scales planned to observe by gravitational wave detectors, with satisfying the consistency with large scale CMB observations.

Contents

Chapter 1

Introduction

Recently, the first detection of gravitational waves (GWs) emitted by binary black hole mergers has been achieved by the LIGO/VIRGO teams, opening up the gravitational wave astronomy [1]. From this time forward, it is no doubt to go into an age when gravitational waves originating not only from astrophysical sources, but also from primordial signatures predicted by scenarios of the very early universe are developed, especially primordial gravitational waves from inflation [2–5]. As a first step, it is extremely important to understand how much the fundamental physics can be clarified through the information of primordial gravitational waves, exploring their detectability with future gravitational wave experiments.

The primordial inflation quantum-mechanically accounts for the origin of Cosmic Microwave Background radiation (CMB) temperature anisotropy and large scale structure in our present universe. Through these observations, it allows us to explore the high energy physics by means of inflation. One of the most important prediction is that it also provides vacuum fluctuations of gravitons, called primordial gravitational waves, whose amplitude is directly related to the energy scale of inflation in the standard scenario. We can search for the imprint of primordial gravitational waves on the polarization of CMB B-mode, conventionally parameterized by the tensor-to-scalar ratio r (defined in (2.71)). Unfortunately, however, current CMB observations tell us that its amplitude is tiny and the BICEP2/Keck Array collaboration has reported the upper bound $r \lesssim 0.07$ [6, 7]. Moreover, due to the decreasing of Hubble parameter during inflation, its scale dependence is more power on larger scales. Therefore, it is also getting harder to detect it in future gravitational wave experiments. In fact, practically testable abundance of gravitational wave with space-based interferometers such as DECIGO (not its ultimate version) [8] and BBO [9] or the pulsar timing array mission such as SKA [10] require $r = \mathcal{O}(0.01)$ in the case of a standard inflationary model.

From the theoretical point of view, however, there is a room to suppose another source of metric tensor modes caused by the existence of matter sector at the inflationary era. In supergravity or superstring theory as an example, there exists a complex scalar field whose real part and imaginary part are called “dilaton” and “axion” (pseudo scalar field), irrespectively. These two scalar fields are candidates for an inflaton and their couplings to the gauge sector are of our interest. Namely, what we emphasize is that we should develop the understanding of not only the scalar sector but also the vector sector in the inflationary period.

There are generally two ways how an inflaton field is coupled to the gauge field. One is the dilaton coupling to the gauge field through a gauge kinetic function. The other is the axion coupling to the gauge field through a topological Chern-Simons term. Once we introduce the dilaton field and its gauge kinetic function, there are interesting phenomenologies such as the generation of primordial magnetic fields [11–19]. In particular, recently, it has been found that a non-trivial gauge-kinetic function could trigger the growth of the gauge field which stops at the saturating point and the amplitude of the gauge field is sustained during inflation [20, 21]. The survived gauge field results in the statistical anisotropy in the primordial fluctuations [22, 23]. Thus, we can expect the cross correlation between gravitational waves and curvature perturbations on top of the statistical anisotropy in auto correlation of gravitational waves [23]. It is also possible to generate gravitational waves from the particle production of gauge fields by the dilaton [24, 25].

Axions presented here are suggested by string theory through the flux compactification of higher dimension space-time, and they are coupled to gauge fields through the Chern-Simons term introduced in order to cancel a gauge anomaly [26, 27]. It is known that axions are among the best-motivated inflaton candidates since they have a nearly flat potential protected by its shift symmetry, and so historically several axionic inflation models have been studied [28–36]. Remarkably, in the presence of the axion-gauge coupling at an inflationary stage, there occurs the transient tachyonic instability in one of helicity modes of the gauge field near the horizon-crossing [37]. As a result, amplified gauge particles source other coupled scalar and tensor fluctuations during inflation and can lead to the production of cosmologically well-motivated objects such as primordial black holes (PBHs) [38–42], observable CMB non-gaussianities or spectral distortions [15,38,43–51], present intergalactic magnetic fields [15, 19, 37, 52, 53], baryogenesis [54–57] and a sizable amount of primordial gravitational waves [24, 25, 40, 42, 44, 46, 48, 51, 58–75]. Intriguingly, the property of tensor power spectrum could be scale-dependent and parity-violating, which are totally different signatures from that of vacuum fluctuations. We can test this interesting outcome by observing the correlation between CMB temperature anisotropy and B-mode polarization or several types of CMB bispectra [64, 75–80], the residual signal of pulsar-timing arrays [81] and the cross correlation of several gravitational wave interferometers [82]. The important thing is that the spectrum of sourced primordial gravitational waves can be scale-variant depending on the microscopic dynamics of gauge fields and the coupled axion, so that we have a chance to detect it even if there is no indirect signatures of gravitational waves on the large-scale CMB observations.

In this thesis, we study the possibility of testing chiral gravitational waves sourced by the axion-gauge interaction with future gravitational wave detectors or the pulsar timing array mission. We discuss the model of axionic inflation coupled with U(1) gauge group and SU(2) gauge group, respectively. When the axion is coupled to the U(1) gauge field, particle production of gauge field occurs due to the rolling axion and it enhances both tensor and scalar fluctuations at second order effect during inflation. We show that there is a correlation between the amplification of gravitational waves and that of curvature perturbation and hence it leads to an overproduction of PBHs if sizable amount of chiral gravitational waves are provided at the intermediate scales where gravitational wave experiments are going to probe. Based on this fact, we suggest a possible inflationary scenario of axion coupled with many Abelian gauge field, which suppresses the amount of PBHs and produces detectable gravitational waves at the frequency higher than nHz.

On the other hand, in the case of SU(2) gauge field, it is known that the classical energy density of gauge field can give rise to an effective Hubble friction via the coupling of axion and generate a slow-roll inflationary solution for a wide range of parameters called "Chromo-Natural inflation" [36]. Due to the presence of the background gauge field, fluctuations of gauge field can possesses tensor perturbations and one of circular polarization states experiences a tachyonic instability near the horizon crossing, which produces chiral gravitational waves. We show that, however, this inflation model is in conflict with CMB observations because of either too red scalar spectral index or too much amount of gravitational waves provided. We revisited this problem and found the possibility to avoid it, by allowing for the existence of gauge kinetic function of dilatonic field coupled to SU(2) gauge group. Due to the background motion of dilaton, SU(2) gauge field experiences scale-dependent particle production through the gauge kinetic coupling. Based on this property, we invented an inflationary scenario where the amount of chiral gravitational waves is suppressed on CMB scales, but becomes larger on intermediate scales where forthcoming gravitational wave experiments can observe.

The plan of this thesis is as follows.

In chapter 2

We give a short review about the conventional standard big bang cosmology and the theory of inflation.

In chapter 3

We discuss the generation of chiral gravitational waves from axionic inflation with U(1) gauge field. Also, we present the dynamics of scalar perturbations and introduce a possibility to avoid the overprodution of PBHs based on our work [72].

In chapter 4

We present the model of axionic inflation with SU(2) gauge field dubbed Chromo-Natural inflation and present the mechanism to provide chiral gravitational waves. We also clarify that this model is inconsistent with CMB data due to the overproduction of chiral gravitational waves.

In chapter 5

We extend the model of Chromo-Natural inflation by introducing a dilatonic field coupled with gauge field, and suggest our scenario [65] which suppresses the production of chiral gravitational waves on CMB scales but enhances its amplitude on the intermediate scales.

In chapter 6

We summarize this thesis.

Throughout this thesis, we set the unit as $\hbar = c = 1$ unless otherwise stated.

Chapter 2

An Overview of Inflationary Universe

In this chapter, we present a short review about the theory of inflation.

2.1 Homogeneous and isotropic universe

First of all, we describe background dynamics of the homogeneous and isotropic spacetime. The motion of spacetime is described by Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{M_p^2} T_{\mu\nu} \ . \tag{2.1}$$

The background metric of our universe is given by the following one called "Friedmann-Lemaitre-Robertson-Walker (FLRW or FRW) metric"

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - Kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right) \ , \tag{2.2}$$

where $a(t)$ is called "scale factor" which characterizes the size of our universe. An important quantity characterizing the expansion of FLRW universe is the following function

$$H \equiv \frac{\dot{a}}{a} \tag{2.3}$$

called "Hubble parameter" . This function gives the concept of "horizon" as we will explain later. And K is the curvature of space whose signature represents the topology of our universe. We can understand it by the following coordinate transformation

$$ds^2 = -dt^2 + a(t)^2 \left(d\chi^2 + \sigma_K(\chi)(d\theta^2 + \sin^2\theta d\phi^2) \right) \ , \tag{2.4}$$

where

$$\sigma_K(\chi) = \begin{cases} \sinh^2\chi \quad (K = -1) \\ \chi^2 \quad (K = 0) \\ \sin^2\chi \quad (K = 1) \ . \end{cases} \tag{2.5}$$

From the symmetry, the energy momentum tensor $T_{\mu\nu}$ can be assumed to take the form of a perfect fluid

$$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu} \ , \tag{2.6}$$

where ρ is the energy density, P is the pressure and u^μ is the four-velocity of matter, which satisfies $u_\alpha u^\alpha = -1$. Substituting it into Einstein equation, from $(0,0)$ component we get the constraint equation

$$H^2 = \frac{1}{3M_p^2}\rho - \frac{K}{a^2} . \tag{2.7}$$

This equation is called "Friedmann equation", which represents that the expansion of the universe is determined by the energy density of matter. As to the equation of motion for scale factor, from (i,i) component we get

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6M_p^2}(\rho + 3P) , \tag{2.8}$$

or equivalently it leads to

$$\dot{\rho} + 3H(\rho + P) = 0 . \tag{2.9}$$

In addition to the above equations, we need an equation of state of matter field

$$P = w\rho , \tag{2.10}$$

in order to solve the equations: The standard Big Bang scenario tells us $w \geq 0$: $w = 1/3$ for relativistic (radiation) matter and $w = 0$ for non-relativistic (dust) matter. Integrating (2.9), we obtain

$$\rho \propto a^{-3(1+w)} . \tag{2.11}$$

and we can see that the big bang universe experiences a decelerating expansion due to the minus sign in the right hand side of (2.8). As we show later, this result leads to the cosmological fine-tuning problems in our universe.

2.2 The problem in the standard Big Bang cosmology

Before we address the main question, we need to state the concept of "horizon" in cosmology. The causal structure of the universe is characterized by the propagation of light. The photon trajectory is given by a null geodesic in the FLRW spacetime

$$0 = -dt^2 + a(t)^2 d\chi^2 . \tag{2.12}$$

Here we take the radial direction as the direction of photon propagation. Then rewriting dt as

$$dt = a(\tau)d\tau , \tag{2.13}$$

(2.12) is reduced to

$$0 = -d\tau^2 + d\chi^2 , \tag{2.14}$$

so that photons travel along the light cone on the Minkowski spacetime conformally related to the original spacetime. The above variable

$$\tau(t) = \int^t \frac{dt'}{a(t')} = \int d\ln a \,(aH)^{-1} \tag{2.15}$$

is called "conformal time" and convenient to represent the causal structure of the spacetime. If particles are separated by distances greater than $\tau(\infty)$, they never could have communicated with each other. On the other hand, the comoving Hubble radius $(aH)^{-1}$ in the integration (2.15) is also called horizon: if they are separated by distances greater than $(aH)^{-1}$, they cannot talk to each other at that point of time. The evolution of this quantity is crucially important to understand the idea of inflation. In the universe dominated by a fluid with $P = w\rho$, the comoving Hubble radius scales as

$$(aH)^{-1} \propto a^{\frac{1}{2}(1+3w)} . \tag{2.16}$$

Hence the comoving horizon shrinks as we go back to the past of the universe for $w \geq 0$, which means that the comoving scale entering the horizon today has been far outside the horizon at the age of CMB decoupling. Current CMB observation tells us, however, our universe was extremely homogeneous at the time of last scattering. It is therefore quite strange that such causally separated regions in our universe finally settle in almost the same equilibrium state. This is called the "horizon problem of our universe".

In addition to that, the flatness of our universe also raises the following question. Defining the dimensionless energy fraction of curvature,

$$\Omega_K \equiv -\frac{K}{a^2H^2} \tag{2.17}$$

is constrained by current CMB observations as [83]

$$\Omega_K(t_0) = -0.052^{+0.049}_{-0.055} , \tag{2.18}$$

where t_0 is the current cosmic time. In terms of this value, we can naively estimate the initial value of $\Omega_K(t_p)$ at the Planck time, $t_p \sim 10^{-44}$sec , as

$$|\Omega_K(t_p)| = \frac{a(t_0)^2H(t_0)^2}{a(t_p)^2H(t_p)^2}|\Omega_K(t_0)| \sim 10^{-61} . \tag{2.19}$$

Note that in the above estimation we approximate that our universe is radiation dominated from the initial time to the present time. From (2.7), the above result means that the energy fraction of matter sector had to be extremely close to unity with 10^{-61} accuracy at that time, which is called as the "flatness problem of our universe".

2.3 Theory of inflation

Is there a way to solve the horizon and flatness problems of our universe? Reconsidering the reason that causes these problems, we find that the problem is in that the comoving Hubble horizon is monotonically increasing in time. Conversely, we can expect that these problems can be solved if the comoving horizon has been decreasing at a certain time since particles far outside the horizon at the last scattering could have been communicated with each other. Moreover, shrinking comoving horizon makes the spatial curvature small, so that the flatness problem will be also solved. According to (2.16), in such an era the inequality $w < -1/3$ hold and the universe was in the accelerated expansion phase before Big Bang occurs. We call this era "inflation".

Let us construct a simple model that realizes such kind of situation. Rewriting the equation of motion for the scale factor as

$$\frac{\ddot{a}}{a} = H^2\left(1 + \frac{\dot{H}}{H^2}\right) \equiv H^2(1 - \epsilon_H) , \tag{2.20}$$

we find the expansion of the universe is accelerated if $\epsilon_H < 1$ is realized. We can realize this situation by introducing a scalar field φ called "inflaton". The action of this system becomes:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)\right] . \tag{2.21}$$

We can also find the energy momentum tensor from here

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \partial_\mu\varphi\partial_\nu\varphi - g_{\mu\nu}\left(\frac{1}{2}(\partial_\alpha\varphi)^2 + V\right) \tag{2.22}$$

and get the energy density and the pressure of inflaton

$$\rho = -T^0_0 = \frac{1}{2}\dot{\varphi}^2 + V , \qquad P = T^i_i = \frac{1}{2}\dot{\varphi}^2 - V , \tag{2.23}$$

assuming the spatial homogeneity. If the kinetic energy is sufficiently smaller than the potential energy

$$\frac{1}{2}\dot{\varphi}^2 \ll V , \tag{2.24}$$

$P \simeq -\rho$ $(w \simeq -1 < -1/3)$ is satisfied and accelerated expansion occurs.

We analyze the background motion of this system. As to the metric, we adapt the ADM parametrization

$$ds^2 = -N^2dt^2 + q_{ij}(dx^i + N^i dt)(dx^j + N^j dt) , \tag{2.25}$$

where N is a lapse function and N^i is a shift function, which appear as non-dynamical Lagrange multipliers in the action. An induced metric q_{ij} on the three dimensional spatial hypersurface is used to raise or lower the index as

$$q^{ik}q_{kj} = \delta^i_j , \qquad N^i = q^{ij}N_j . \tag{2.26}$$

As the background, we consider the flat FLRW metric

$$ds^2 = -N^2dt^2 + a(t)^2\delta_{ij}dx^idx^j \qquad (N_i = 0,\ q_{ij} = a(t)^2\delta_{ij}) . \tag{2.27}$$

Here, we keep the variable N in order to get the constraint equation. Substituting these variables into the action leads to

$$S = \int dx^4\frac{a^3}{N}\left[-3M_p^2\frac{\dot{a}^2}{a^2} + \frac{1}{2}\dot{\chi}^2 - N^2V\right] . \tag{2.28}$$

After the variation with respect to each variable and setting $N = 1$, we get Friedmann equation and equations of motion for spacetime

$$H^2 = \frac{1}{3M_p^2}\left(\frac{1}{2}\dot{\varphi}^2 + V\right) , \qquad \dot{H} = -\frac{1}{2M_p^2}\dot{\varphi}^2 \tag{2.29}$$

and inflaton

$$\ddot{\varphi} + 3H\dot{\varphi} + V_\varphi = 0 . \tag{2.30}$$

In order for accelerated expansion to occur, the quantity ϵ_H called the "Hubble slow-roll parameter" must be less than unity. Usually we consider the case with $\epsilon_H \ll 1$ and then equivalently it reads

$$3M_p^2 H^2 \simeq V \,. \tag{2.31}$$

Moreover, in order to keep accelerated expansion sufficiently long, the time variation of ϵ_H parameter

$$\eta_H \equiv \frac{1}{2}\frac{\dot{\epsilon}_H}{\epsilon_H H} \tag{2.32}$$

also has to be sufficiently small $\eta_H \ll 1$. From this condition, (2.30) is reduced to

$$3H\dot{\varphi} \simeq -V_\varphi \,. \tag{2.33}$$

(2.31) and (2.33) are called the "slow-roll equations" and they characterize the inflationary period. The time interval of inflationary period is expressed by the number of e-foldings

$$N(t) \equiv -\int_{t_{\rm end}}^{t} H dt = \ln\left(\frac{a(t_{\rm end})}{a(t)}\right) \,. \tag{2.34}$$

After the inflation ends, the inflaton starts to decay at the bottom of potential and the energy transfer between the inflaton and the coupled matter sector happens. As a result, our universe gets reheated and Big Bang cosmology is successfully realized. This phase transition is called the "reheating of our universe" and radiation energy dominates our universe. On the other hand, the period before reheating is called the "pre-heating" and the universe is matter-dominated during the oscillations of inflaton.

Finally, let us estimate the number of e-foldings necessary to explain the present observable universe. The primordial density fluctuations at a given scale with a wave number k exit the horizon at $k = aH \equiv a_k H_k$ and re-enter the horizon after inflation ends. Comparing this scale with the present horizon size, it follows

$$\frac{k}{a_0 H_0} = \frac{a_k}{a_{\rm end}}\frac{a_{\rm end}}{a_{\rm reh}}\frac{a_{\rm end}}{a_0}\frac{H_k}{H_0} \,, \tag{2.35}$$

$$\simeq e^{-N_k}\left(\frac{\rho_{\rm reh}}{\rho_{\rm end}}\right)^{1/3}\left(\frac{\rho_{r0}}{\rho_{\rm reh}}\right)^{1/4}\left(\frac{\rho_k}{\rho_0}\right)^{1/2} \,, \tag{2.36}$$

where $N_k \equiv \ln(a_{\rm end}/a_k)$ and we labelled the energy density of universe at each period: $\rho_{\rm end}$ (at the end of inflation), $\rho_{\rm reh}$ (at the reheating), ρ_0 (at present), and ρ_k (at the horizon crossing during inflation). Also we note that $\rho_{r0} \equiv (a_{\rm eq}/a_0)^4 \rho_{r\rm eq}$ is the present energy density of radiation. Thus, in order for the observed CMB photons to be inside the horizon at the beginning of inflation, the required number of e-foldings $N_{\rm CMB} \equiv N_{k=k_{\rm CMB}}$ is approximately given by [84]

$$N_{\rm CMB} \simeq 62 - \ln\left(10^{16}{\rm GeV}/V_{\rm end}^{1/4}\right) - \frac{1}{3}\ln\left(\rho_{\rm reh}^{1/4}/V_{\rm end}^{1/4}\right) \,, \tag{2.37}$$

where $V_{\rm end}$ is the potential energy of inflaton at the end of inflation. Assuming the instant reheating and large field inflation models for example, the number of e-folding corresponding to the CMB scales is therefore around 60.

2.4 Cosmological perturbations

In this subsection, we briefly explain basic facts of cosmological perturbation theory and derive the two-point functions of curvature perturbation and primordial gravitational waves from inflation. We introduce perturbations around the background as

$$\varphi(t,\boldsymbol{x}) = \bar{\varphi}(t) + \delta\varphi(t,\boldsymbol{x}) \,, \tag{2.38}$$

$$N = 1 + \Phi \,, \qquad N_i = \partial_i \beta + \beta_i^V \,, \tag{2.39}$$

$$q_{ij} = a(t)^2 \Big((1-2\Psi)\delta_{ij} + 2\partial_i\partial_j E + 2\partial_{(j}W_{i)} + h_{ij} \Big) \,. \tag{2.40}$$

Perturbations are classified by the transformation properties into scalars, vectors and tensors on spatial hypersurfaces: vector fluctuations β_i^V, W_i satisfy the transverse condition $\partial_i \beta_i^V = \partial_i W_i = 0$ and tensor fluctuations h_{ij} obey the transverse and traceless conditions $\partial_i h_{ij} = h_{ii} = 0$. From now on, we focus on scalar and tensor fluctuations.

In the cosmological perturbation theory, the above split into background and perturbations is not unique but depends on the choice of coordinates dubbed as "gauge choice". Under the following gauge transformation

$$x^\mu \quad \longrightarrow \quad x'^\mu = x^\mu + \xi^\mu \,, \qquad \xi^\mu = (\xi^0 \,,\, \delta^{ij}\partial_j \xi + \xi_V^i) \,, \tag{2.41}$$

tensor flucutations are invariant but scalar fluctuations transform as

$$\Phi \to \Phi - \dot{\xi}^0 \,, \tag{2.42}$$

$$\beta \to \beta + a^{-1}\xi^0 - a\dot{\xi} \,, \tag{2.43}$$

$$\Psi \to \Psi + H\xi^0 \,, \tag{2.44}$$

$$E \to E - \xi \,. \tag{2.45}$$

Similar to metric fluctuations, we can also define perturbations of energy momentum tensor

$$T^0_0 = -(\bar{\rho} + \delta\rho) \,, \tag{2.46}$$

$$T^0_i = (\bar{\rho} + \bar{P}) a v_i \,, \tag{2.47}$$

$$T^i_0 = -a^{-1}(\bar{\rho} + \bar{P})(v^i - B^i) \tag{2.48}$$

$$T^i_j = \delta^i_j(\bar{P} + \delta P) + \Sigma^i_j \tag{2.49}$$

with scalar $\{\delta\rho,\ \delta P\}$, vector B_i, and tensor Σ_{ij} variables. Here, we neglect the effect of anisotropic stress, i.e., $\Sigma_{ij} = 0$, and consider only scalar fluctuations. The scalar fluctuations are transformed as

$$\delta\rho \quad \longrightarrow \quad \delta\rho - \dot{\bar{\rho}}\xi^0 \,, \qquad \delta P \quad \longrightarrow \quad \delta P - \dot{\bar{P}}\xi^0 \,. \tag{2.50}$$

Using these gauge transformations, we can compose the gauge invariant curvature perturbation on uniform density hypersurfaces

$$\zeta \equiv -\Psi - \frac{H}{\dot{\bar{\rho}}}\delta\rho \,. \tag{2.51}$$

This quantity is cosmologically important because it remains constant outside the horizon for adiabatic matter perturbations and gives primordial initial conditions for density perturbations after inflation. Here, inflaton dominates the energy of our universe so that on the flat-slicing, $\Psi = 0$, ζ can be written by

$$\zeta = -\frac{H}{\dot{\varphi}}\delta\varphi \; . \tag{2.52}$$

Let us derive the two-point function of curvature perturbation. We expand scalar fluctuations of inflaton by quantum operators in Fourier space

$$\delta\varphi = \int \frac{d\boldsymbol{k}}{(2\pi)^3}\delta\hat{\varphi}_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \; , \tag{2.53}$$

$$\delta\hat{\varphi}_{\boldsymbol{k}} = \delta\varphi_k a_{\boldsymbol{k}} + \delta\varphi^*{}_k a^\dagger{}_{\boldsymbol{k}} \; , \qquad [a_{\boldsymbol{k}}, a^\dagger{}_{-\boldsymbol{k}'}] = (2\pi)^3\delta(\boldsymbol{k}+\boldsymbol{k}') \; , \tag{2.54}$$

where $a_{\boldsymbol{k}}$ and $a^\dagger{}_{\boldsymbol{k}}$ are, respectively, an annihilation and a creation operator which obey an ordinal commutation relation. By using the canonically normalized variable $\delta\tilde{\varphi} \equiv a\delta\varphi$, we get the following second order action

$$\delta S = \frac{1}{2}\int d\tau \frac{d\boldsymbol{k}}{(2\pi)^3}\left[|\delta\tilde{\varphi}'_k|^2 - \left(k^2 - \frac{a''}{a}\right)|\delta\tilde{\varphi}_k|^2\right] \tag{2.55}$$

which leads to the equation of motion for $\delta\tilde{\varphi}$:

$$\delta\tilde{\varphi}''_k + \left(k^2 - \frac{2}{\tau^2}\right)\delta\tilde{\varphi}_k = 0 \; . \tag{2.56}$$

Note that we ignored the mass term of inflaton and the gravity coupling to inflaton in the above equation since they are slow-roll suppressed. Solving (2.56), we get two independent solutions

$$\delta\tilde{\varphi}_k = \frac{\alpha}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)e^{-ik\tau} + \frac{\beta}{\sqrt{2k}}\left(1 + \frac{i}{k\tau}\right)e^{ik\tau} \; , \tag{2.57}$$

where α, β are integration constants. From the following quantum commutation relation

$$\left[\delta\hat{\tilde{\varphi}}_{\boldsymbol{k}}, \hat{\pi}_{\tilde{\varphi}-\boldsymbol{k}'}\right] = i(2\pi)^3\delta(\boldsymbol{k}+\boldsymbol{k}') \; , \qquad \hat{\pi}_{\tilde{\varphi}} = \frac{\partial\mathcal{L}}{\partial\delta\hat{\tilde{\varphi}}'} = \delta\hat{\tilde{\varphi}}' \; , \tag{2.58}$$

we get $|\alpha|^2 - |\beta|^2 = 1$. In order to completely fix these parameters, we must choose a vacuum state at infinite past, namely at sufficiently sub-horizon scale $-k\tau \ll 1$. At that time, the effect of gravitation is negligible so that we can expect that the choise of Minkowski vacuum is standard :

$$\delta\tilde{\varphi}_k \xrightarrow{x\to\infty} \frac{1}{\sqrt{2k}}e^{-ik\tau} \; . \tag{2.59}$$

This vacuum is called “Bunch-Davies vacuum”. Then we get $\alpha = 1, \beta = 0$ and curvature perturbation reads

$$\zeta_k = -\frac{H}{\dot{\varphi}}\frac{H}{\sqrt{2k^3}}(i - k\tau)\, e^{-ik\tau} \; . \tag{2.60}$$

At super-horizon limit $-k\tau \to 0$, it freezes out and surely becomes the constant value. Hence, defining the two point function of curvature perturbation as

$$\langle \hat{\zeta}_{\boldsymbol{k}} \hat{\zeta}_{\boldsymbol{k}'} \rangle \equiv (2\pi)^3 \delta(\boldsymbol{k} + \boldsymbol{k}') \frac{2\pi^2}{k^3} \Delta_\zeta(k) , \tag{2.61}$$

we obtain the following dimensionless power spectrum

$$\Delta_\zeta = \left. \frac{H^2}{8\pi^2 M_p^2 \epsilon_H} \right|_{k=aH} \equiv A_s \left(\frac{k}{k_*} \right)^{n_s - 1} , \tag{2.62}$$

where A_s is its amplitude and n_s is its spectral tilt at a pivot scale k_*. Using slow-roll conditions, spectral index is approximated as

$$n_s - 1 \simeq -2\epsilon_H - 2\eta_H . \tag{2.63}$$

As to the tensor perturbation, we can also derive its two-point function analogous to the approach of curvature perturbation. We expand tensor fluctuations as two helicity operator functions in Fourier space

$$h_{ij} = \sum_{\lambda=L,R} \int \frac{d\boldsymbol{k}}{(2\pi)^3} \hat{h}^\lambda_{\boldsymbol{k}} e^\lambda_{ij}(\hat{\boldsymbol{k}}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} , \tag{2.64}$$

$$\hat{h}^\lambda_{\boldsymbol{k}} = h^\lambda_k b^\lambda_{\boldsymbol{k}} + h^{\lambda *}{}_k b^{\lambda\dagger}{}_{\boldsymbol{k}} , \qquad [b^\lambda_{\boldsymbol{k}}, b^{\lambda'\dagger}{}_{\boldsymbol{k}'}] = \delta^{\lambda\lambda'} (2\pi)^3 \delta(\boldsymbol{k} + \boldsymbol{k}') , \tag{2.65}$$

where $e_{ij}(\hat{\boldsymbol{k}})$ is the polarization tensor which obeys the following unitary and orthonormal conditions $e^{\lambda *}_{ij}(\hat{\boldsymbol{k}}) = e^\lambda_{ij}(-\hat{\boldsymbol{k}})$, $e^{\lambda *}_{ij}(\hat{\boldsymbol{k}}) e^{\lambda'}_{ij}(\hat{\boldsymbol{k}}) = \delta^{\lambda\lambda'}$. Then defining canonically normalized field $\psi^\lambda_k \equiv M_p a h^\lambda_k / 2$, one can find the following second order action for tensor modes

$$\delta S = \sum_{\lambda=L,R} \frac{1}{2} \int d\tau \frac{d\boldsymbol{k}}{(2\pi)^3} \left[|\psi^{\lambda\prime}{}_k|^2 - \left(k^2 - \frac{a''}{a} \right) |\psi^\lambda_k|^2 \right] \tag{2.66}$$

and straightforwardly lead to the equation of motion for the wave function

$$\psi^{\lambda\prime\prime}{}_k + \left(k^2 - \frac{2}{\tau^2} \right) \psi^\lambda_k = 0 . \tag{2.67}$$

This is the same form as the equation of motion for scalar mode (2.56). Hence the two-point function of gravitational waves

$$\langle \hat{h}^\lambda_{\boldsymbol{k}} \hat{h}^\lambda_{\boldsymbol{k}'} \rangle \equiv (2\pi)^3 \delta(\boldsymbol{k} + \boldsymbol{k}') \frac{2\pi^2}{k^3} \Delta^\lambda_h(k) \tag{2.68}$$

are given by

$$\Delta^L_h = \Delta^R_h = \left. \frac{H^2}{\pi^2 M_p^2} \right|_{k=aH} \equiv A_t \left(\frac{k}{k_*} \right)^{n_t} , \tag{2.69}$$

where A_t is its amplitude and n_t is its spectral tilt at a pivot scale

$$n_t \equiv \frac{d \ln \Delta_h}{d \ln k} \simeq -2\epsilon_H . \tag{2.70}$$

The amplitude A_t is characterized by the following dimensionless parameter

$$r \equiv \frac{\Delta_h^L + \Delta_h^R}{\Delta_\zeta} = 16\epsilon_H \tag{2.71}$$

called "tensor-to-scalar ratio". This value is quite important to estimate the abundance of primordial gravitational waves. So far, Planck satellite and BICEP2/Keck Array collaboration has constrained the amount of r due to the B-mode polarization and told us the following upper bound [7]

$$r < 0.07\ . \tag{2.72}$$

2.5 Observation of primordial GWs with GW detectors

Now, let us consider the possibility of testing primordial GWs with not only CMB observation but gravitational wave detectors or pulsar timing arrays, which probe with the frequency higher than nHz. The present energy intensity of stochastic gravitational waves at $t = t_0$ can be characterized by the following dimensionless quantity [85]

$$\Omega_{\rm GW}(f, t_0) \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm GW}(t_0)}{d\ln f}\ , \tag{2.73}$$

where $\rho_{\rm GW}$ is the energy density of gravitational waves, $f = k/2\pi$ is its frequency and $\rho_c = 3M_p^2H_0^2$ is the present critical energy density of our universe. The energy density of primordial gravitational waves $\rho_{\rm GW}$ are expressed by the $(0,0)$ component of the energy momentum tensor. Because of the equivalence principle, however, it cannot be defined locally but can be defined with a spatial averaging over several wavelengths

$$\rho_{\rm GW} = \frac{M_p^2}{4}\langle \dot{h}_{ij}\dot{h}_{ij}\rangle\ . \tag{2.74}$$

So the intensity is roughly estimated as $\Omega_{\rm GW}(2\pi f = k) \sim k^2\langle |h_k|^2(t_0)\rangle$ which is redshifted as a^{-4} like a radiation particle. The amplitude of gravitational waves today reads $h_k(t_0) = h_k(t_k)(a_k/a_0)$, where t_k $(a_k = a(t_k))$ is the time of horizon entry. We are interested in the scale observed by the planned gravitational wave interferometers or pulsar timing arrays near the frequencies around nHz $\sim$ dHz, which correspond to scales around $k = 10^5 - 10^{15}{\rm Mpc}^{-1}$. They are sufficiently smaller than the scale of entering at matter-radiation equality $k_{\rm eq} \simeq 0.05h^2{\rm Mpc}^{-1}$. Therefore the primordial gravitational waves observed by interferometers entered the horizon during radiation dominated era. We can relate its power spectrum with present one by using the transfer function $\mathcal{T}(k) \equiv a_k/a_0$:

$$\langle |h_k|^2(t_0)\rangle^{1/2} = \mathcal{T}(k)\Delta_h(k)^{1/2}\ . \tag{2.75}$$

Due to the conservation of the entropy density $g_{*S}(T)a^3T^3$ (g_{*S} is the effective number of relativistic degrees of freedom contributing to the entropy density), the transfer function reads

$$\mathcal{T}(k) \simeq 1.66 \times \frac{T_0^2}{M_pk} g_{*S}(T_0)^{2/3} g_{*S}(T_k)^{-1/6}\ . \tag{2.76}$$

Taking only the standard model particles, $g_{*S}(T_k) \simeq 100$ hold and we get today's number

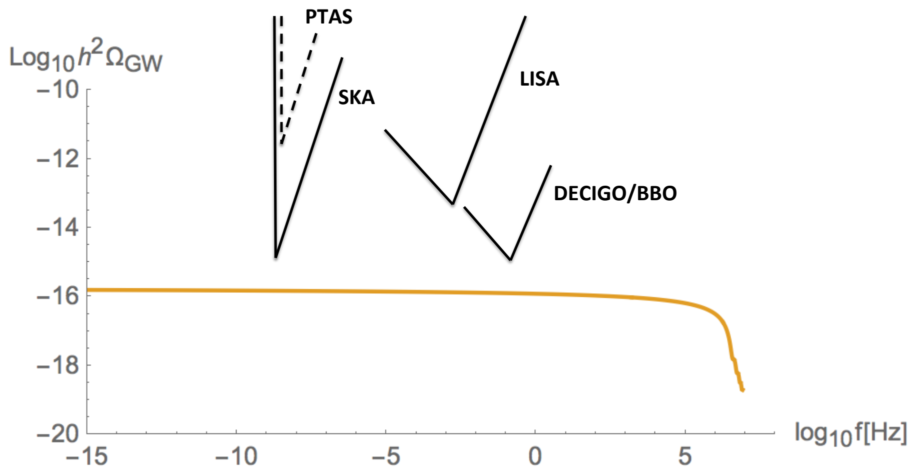

Figure 2.1: The schematic plot of present energy density of primordial GWs as a function of its frequency (orange line). We assume that tensor to scalar ratio is around $O(10^{-2})$ which is consistent with CMB data. Its amplitude is below each sensitivity curve which is expected by planned space-based interferometers (DECIGO/BBO) or a pulsar timing array experiment (SKA). Note that the dotted black line shows the analytical noise curve of current pulsar timing array experiments.

of degrees of freedom $g_{*S}(T_0) \simeq 3.91$ for photons and three species of neutrinos. Thus, the present energy density of GWs is estimated by [86]

$$h^2\Omega_{\rm GW}(k) = \frac{k^2h^2}{6H_0^2}\langle |h_k|^2(t_0)\rangle \equiv A_{\rm GW}\Delta_h(k)\,, \qquad A_{\rm GW} \simeq 2.74\times 10^{-6}\left(\frac{g_{*S}(T_k)}{100}\right)^{-1/3}\,. \tag{2.77}$$

Converting Δ_h with tensor-to-scalar ratio $r_{0.01} \equiv r/10^{-2}$, it is roughly approximated by $h^2\Omega_{\rm GW} \sim 10^{-16} r_{0.01}$. We depict its amplitude in Fig.2.1 by rewriting the number of e-foldings as a function of frequency

$$N(k = 2\pi f) = \ln\left(\frac{a_{\rm end}}{a_k}\right) = N_{\rm CMB} + \ln\left(\frac{k_{\rm CMB}}{0.002{\rm Mpc}^{-1}}\right) - \ln\left(\frac{f}{3\times 10^{-18}{\rm Hz}}\right)\,. \tag{2.78}$$

In this plot, we used Starobinsky inflation model and tensor-to-scalar ratio is around 0.01. We can see that it is challenging to detect this tiny amplitude even with future gravitational wave detectors or pulsar timing array missions.

Chapter 3

Primordial GWs Sourced by U(1) Gauge Field

In the previous chapter, we showed that the cosmic inflation provides vacuum fluctuations of metric tensor modes (primordial gravitational waves) and its amplitude is directly related to the energy scale of inflation in the standard scenario. Unfortunately, however, we also clarified that it is getting harder to detect this signal even in future gravitational wave experiments due to the non-detection of the imprint of B-mode polarization in CMB data.

From this chapter, we present another scenario which can predict large amplitude of gravitational waves compared with vacuum modes, namely generation of the metric tensor mode sourced by the amplification of gauge particle coupled with the rolling axion. To begin with, we discuss the model of axionic inflation coupled with U(1) gauge field and develop the mechanism of providing chiral gravitational waves from the particle production of gauge field.

3.1 Review of the particle production of U(1) gauge field

In this section, we present the mechanism of the particle production of U(1) gauge field A_μ coupled to the axion φ which plays the role of inflaton. We consider the following action

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}(\partial\varphi)^2 - V(\varphi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}\frac{\varphi}{f}F_{\mu\nu}\tilde{F}^{\mu\nu}\right], \qquad (3.1)$$

where R is the Ricci scalar ($g = \det g_{\mu\nu}$ is the determinant of the metric), $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of U(1) gauge field. As to the gauge fixing, we chose temporal gauge $A_0 = 0$ and Coulomb gauge $\partial_i A_i = 0$. The Hodge dual of the field strength is defined by $\tilde{F}^{\mu\nu} = \sqrt{-g}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}/2!$, where $\epsilon^{\mu\nu\rho\sigma}$ is the antisymmetric Levi-Civita tensor which satisfies $\epsilon^{0123} = g^{0\mu}g^{1\nu}g^{2\rho}g^{3\sigma}\epsilon_{\mu\nu\rho\sigma} = 1/g$. As to the metric, we assume a spatially flat FLRW metric $ds^2 = -dt^2 + a(t)^2 d\boldsymbol{x}^2 = a(\tau)^2[-d\tau^2 + d\boldsymbol{x}^2]$ where a is the scale factor as a function of the cosmic time t or the conformal time τ .

We firstly solve the equation of motion for the gauge field in a homogeneous background, assuming that back-reaction on the inhomogeneous perturbation is negligible. It is given by

$$A_i'' - \nabla^2 A_i + \frac{\bar{\varphi}'}{f}\epsilon_{ijk}\partial_j A_k = 0\,, \tag{3.2}$$

where the prime means the derivative with respect to τ and $\bar{\varphi} = \bar{\varphi}(t)$ is a background inflaton field. In order to describe the production of quanta of gauge fields, we promote the classical field A_i to an operator

$$A_i(\tau, \boldsymbol{x}) = \sum_{\lambda=\pm}\int \frac{d\boldsymbol{k}}{(2\pi)^3}\hat{A}_\lambda(\boldsymbol{k},\tau)e^\lambda{}_i(\hat{\boldsymbol{k}})e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,, \tag{3.3}$$

$$\hat{A}_\lambda(\boldsymbol{k},\tau) = a_{\lambda\boldsymbol{k}}A_\lambda(k,\tau) + a^\dagger{}_{\lambda-\boldsymbol{k}}A^*_\lambda(k,\tau)\,, \tag{3.4}$$

where $a_{\lambda\boldsymbol{k}}$, $a^\dagger{}_{\lambda-\boldsymbol{k}}$ are annihilation and creation quantum operators which obey the following commutation relation $[a_{\lambda\boldsymbol{k}}, a^\dagger{}_{\lambda'-\boldsymbol{k}'}] = \delta_{\lambda\lambda'}(2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k}')$. The polarization vector $e^\pm{}_i(\hat{\boldsymbol{k}}) = e^{\mp *}{}_i(\hat{\boldsymbol{k}})$ satisfies the normalization-orthogonal condition $e^{i\lambda}(\hat{\boldsymbol{k}})e^{\lambda' *}{}_i(\hat{\boldsymbol{k}}) = \delta_{\lambda\lambda'}$, the transverse property $k^i e^\pm{}_i(\hat{\boldsymbol{k}}) = 0$ and the circular polarization state defined by $i\epsilon_{ijl}\,k^j e^{l\pm}(\hat{\boldsymbol{k}}) = \pm k e^\pm{}_i(\hat{\boldsymbol{k}})$. Substituting these relations into (3.2), we have the equation of motion for each mode function

$$\left[\partial_\tau^2 + k^2 \pm \frac{2k\xi}{\tau}\right]A_\pm = 0\,, \qquad \xi \equiv -\frac{\dot{\varphi}}{2fH} > 0 \quad \left(\text{“}\cdot\text{”} = \frac{d}{dt}\right). \tag{3.5}$$

Here, we defined the new parameter ξ which characterizes the production rate of the gauge field as we will show later. We can approximate it as a constant value as long as the time variation of ξ is slow-roll suppressed. For our convenience, we choose ξ as a positive value.

Let us analyze (3.5). The exact solution is given by the Whittaker function

$$A_\pm = e^{\pm\pi\xi/2}W_{\mp i\xi,1/2}(2ik\tau)/\sqrt{2k}\,, \tag{3.6}$$

where we assume that the gauge field is initially in the adiabatic vacuum $A_\pm \to e^{-ik\tau}/\sqrt{2k}$ $(-k\tau \to \infty)$. While the minus mode A_- is not amplified, we can see that the plus mode A_+ experiences a tachyonic instability for $-k\tau < 2\xi$. We are interested in the parameter region $\xi \gtrsim 1$, so that around the horizon-crossing the gauge field can be substantially approximated as the following real value

$$A_+ \simeq \frac{1}{\sqrt{2k}}\left(\frac{-k\tau}{2\xi}\right)^{1/4} e^{\pi\xi - 2\sqrt{-2\xi k\tau}} \equiv \frac{1}{\sqrt{2k}}\mathcal{A}(\xi, -k\tau)\,, \tag{3.7}$$

$$A_+' \simeq \sqrt{\frac{\xi}{-\tau}}\mathcal{A}(\xi, -k\tau) \tag{3.8}$$

in the following time interval $(8\xi)^{-1} \lesssim -k\tau \lesssim 2\xi$. It is enhanced by a factor $e^{\pi\xi}$ and this amplification affects the production of tensor and scalar perturbations. In the super-horizon regime $-k\tau \to 0$, the enhanced mode A_+ settles in a constant value $e^{\pi\xi}/(2\sqrt{\pi k\xi})$, so the energy density of gauge fields dilutes due to the expansion of the universe. Thus, in the region $(8\xi)^{-1} \lesssim -k\tau \lesssim 2\xi$ the gauge field makes the most contribution to the physical enhancement of fluctuations. We plot the time evolution of electrical energy density from the modes with a momentum k in Fig.3.1. Going backward to the

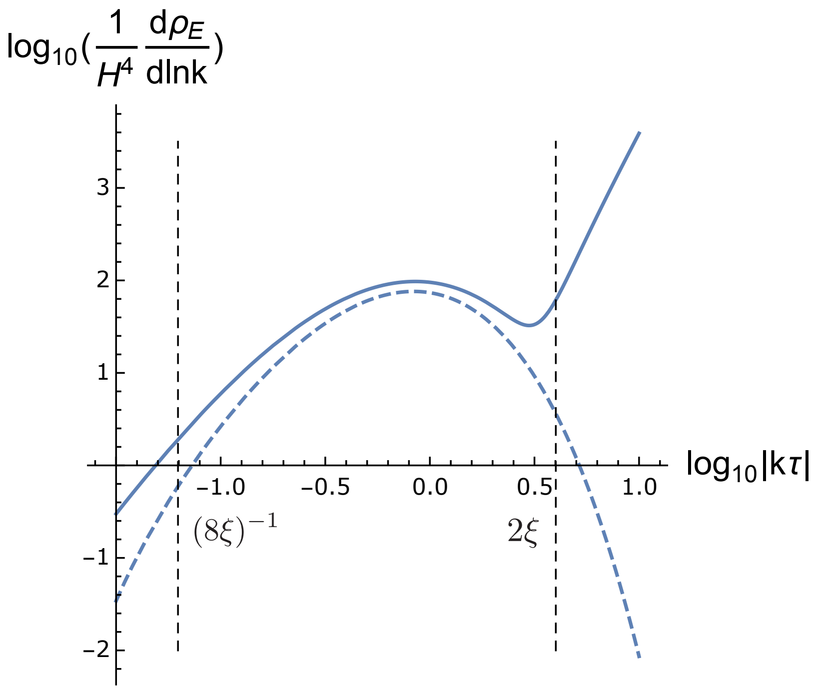

Figure 3.1: Time evolution of the mode function of electrical energy density with $\xi = 2$. The solid (dashed) blue line is represented by the solution of one helicity mode function (3.6) ((3.7)). These lines show three different curves from the early time to the late time (from right to left). Particle production of gauge field occurs around horizon crossing.

infinite past (deep sub-horizon regime), it is UV divergent so that we should renormalize it in the loop calculation. Near horizon crossing, however, it experiences a physical amplification due to the coupling of rolling axion and finally dilutes away due to the expansion of the universe in super-horizon regime.

The enhancement of gauge field could also affect the background dynamics. The Friedmann equation and the equation of motion for the axion $\varphi = \varphi(t)$ read

$$3M_p^2H^2 = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) + \frac{1}{2}\langle E_i^2 + B_i^2\rangle \,, \tag{3.9}$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V_\varphi = -\frac{1}{f}\langle E_iB_i\rangle \,, \tag{3.10}$$

where we defined the electric and magnetic components of the gauge field as

$$E_i \equiv -\frac{1}{a^2}A_i' \,, \qquad B_i \equiv \frac{1}{a^2}\epsilon_{ijk}\partial_j A_k \,. \tag{3.11}$$

In the above equations, we introduced the back-reaction effect of the amplification of gauge field, which are given by

$$\langle E_iB_i\rangle = -\frac{1}{a^4}\sum_{\lambda=\pm1}\int\frac{d^3k}{(2\pi)^3}\frac{k}{2}\lambda|A_\lambda|^{2\prime} \simeq -H^4\int_0^{2\xi}\frac{x^3dx}{4\pi^2}\sqrt{\frac{2\xi}{x}}|\mathcal{A}|^2 \,, \tag{3.12}$$

$$\frac{1}{2}\langle E_i^2 + B_i^2\rangle = \frac{1}{a^4}\sum_{\lambda=\pm1}\int\frac{d^3k}{2(2\pi)^3}\left(|A_\lambda{}'|^2 + k^2|A_\lambda|^2\right) \simeq H^4\int_0^{2\xi}\frac{x^3dx}{4\pi^2}\left(\frac{\xi}{x}+\frac{1}{2}\right)|\mathcal{A}|^2 \,, \tag{3.13}$$

where $x \equiv -k\tau$ and we assume that the UV-divergence of the vacuum mode at $x \gg 2\xi$ is renormalized away. Note that we can extend the integration of $\mathcal{A}$ to the super-horizon limit since it is actually a good approximation of the exact solution. At large ξ, these expectation values are approximately given by [46]

$$\langle E_i B_i \rangle \simeq -2.4 \times 10^{-4} \frac{H^4}{\xi^4} e^{2\pi\xi} , \quad \frac{1}{2}\langle E_i^2 + B_i^2 \rangle \simeq 1.4 \times 10^{-4} \frac{H^4}{\xi^3} e^{2\pi\xi} . \tag{3.14}$$

Therefore, if the following conditions

$$\frac{\langle E_i B_i \rangle}{3 f H \dot{\varphi}} \ll 1 , \qquad \frac{\langle E_i^2 + B_i^2 \rangle}{6H^2} \ll 1 , \tag{3.15}$$

hold, we can neglect back-reaction on the evolution of inflaton. Among these conditions the first one is always stronger than the second one in our parameter region.

3.2 Tensor mode amplification

Here, we show the mechanism of providing the metric tensor modes sourced by the gauge field. We use the inverse decay method in this calculation[1]. The tensor modes h_{ij} are included in the spatial components of the metric as $g_{ij} = a^2(\delta_{ij} + h_{ij})$ with $\partial_i h_{ij} = h_{ii} = 0$. Defining a new canonical variable $\psi_{ij} = a M_p h_{ij}/2$, the equation of motion for ψ_{ij} is obtained from the Einstein equation

$$\left[\partial_\tau^2 - \nabla^2 - \frac{2}{\tau^2}\right] \psi_{ij} = -\frac{a^3}{M_p} \Pi_{ij}^{lm} (E_l E_m + B_l B_m) , \tag{3.16}$$

where Π_{ij}^{lm} is the transverse-traceless projector given by

$$\Pi_{ij}^{lm} \equiv \Pi_i^l \Pi_j^m - \frac{1}{2} \Pi_{ij} \Pi^{lm} , \qquad \Pi_{ij} \equiv \delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2} . \tag{3.17}$$

In order to get the equation in Fourier modes, we go to the momentum space of ψ_{ij} :

$$\begin{aligned} \psi_{ij} &= \int \frac{d\boldsymbol{k}}{(2\pi)^3} \hat{\psi}_{ij}(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \\ &= \sum_{\lambda=\pm} \int \frac{d\boldsymbol{k}}{(2\pi)^3} e_{ij}^{\lambda}(\hat{\boldsymbol{k}}) \hat{\psi}_\lambda(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} , \end{aligned} \tag{3.18}$$

where we defined the polarization tensor $e_{ij}^{\pm}(\hat{\boldsymbol{k}}) \equiv e_i^{\pm}(\hat{\boldsymbol{k}}) e_j^{\pm}(\hat{\boldsymbol{k}})$ so that the following relations $\hat{\psi}_\pm(\boldsymbol{k}) = e_\mp^{ij}(\hat{\boldsymbol{k}}) \hat{\psi}_{ij}(\boldsymbol{k})$, $e_\pm^{ij} \Pi_{ij}^{lm} = e_\pm^{lm}$ hold. Substituting these relations, we get

$$\left[\partial_\tau^2 + k^2 - \frac{2}{\tau^2}\right] \hat{\psi}_\pm(\boldsymbol{k}) = -\frac{a^3}{M_p} e_\mp^{lm}(\hat{\boldsymbol{k}}) \int \frac{d\boldsymbol{p}}{(2\pi)^3} \left(\hat{E}_l(\boldsymbol{p}) \hat{E}_m(\boldsymbol{k}-\boldsymbol{p}) + \hat{B}_l(\boldsymbol{p}) \hat{B}_m(\boldsymbol{k}-\boldsymbol{p}) \right) , \tag{3.19}$$

[1] It is known that this method derives almost the same result when we use the in-in method [45].

where

$$\hat{E}_i(\boldsymbol{p}) = -\sum_{\lambda=\pm} \frac{1}{a^2}\hat{A}_\lambda{}' e^\lambda{}_i(\hat{\boldsymbol{p}}) \,, \qquad \hat{B}_i(\boldsymbol{p}) = \sum_{\lambda=\pm} \frac{\lambda k}{a^2}\hat{A}_\lambda e^\lambda{}_i(\hat{\boldsymbol{p}}) \,. \tag{3.20}$$

In (3.63), we can separate the solution of $\hat{\psi}_\pm$ into two parts $\hat{\psi}_\pm = \hat{\psi}_V + \hat{\psi}_{S\pm}$: the first term $\hat{\psi}_V$ is a homogeneous solution which comes from vacuum fluctuations of quasi de Sitter space-time, so its mode function obeys the following Bunch-Davies solution

$$\psi_V = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)e^{-ik\tau} \,. \tag{3.21}$$

On the other hand, the second term $\hat{\psi}_{S\pm}$ is a peculiar solution sourced by the gauge field in the r.h.s. of (3.63), so that it is given by

$$\begin{aligned}
\hat{\psi}_{S\pm}(k,\tau) = &-\frac{1}{M_p}\int_{-\infty}^0 d\tau_1 \frac{G_k(\tau,\tau_1)}{a(\tau_1)}\int\frac{d\boldsymbol{p}}{(2\pi)^3}e^{lm}_\mp(\hat{\boldsymbol{k}}) \\
&\times\Big[\Big(\hat{A}_+{}'(\boldsymbol{p})\hat{A}_+{}'(\boldsymbol{k}-\boldsymbol{p}) + p|\boldsymbol{k}-\boldsymbol{p}|\hat{A}_+(\boldsymbol{p})\hat{A}_+(\boldsymbol{k}-\boldsymbol{p})\Big)e^+_l(\hat{\boldsymbol{p}})e^+_m(\widehat{\boldsymbol{k}-\boldsymbol{p}}) \\
&+\Big(\hat{A}_-{}'(\boldsymbol{p})\hat{A}_-{}'(\boldsymbol{k}-\boldsymbol{p}) + p|\boldsymbol{k}-\boldsymbol{p}|\hat{A}_-(\boldsymbol{p})\hat{A}_-(\boldsymbol{k}-\boldsymbol{p})\Big)e^-_l(\hat{\boldsymbol{p}})e^-_m(\widehat{\boldsymbol{k}-\boldsymbol{p}})\Big] \,,
\end{aligned} \tag{3.22}$$

where

$$\begin{aligned}
G_k(\tau,\tau_i) &= i\,\Theta(\tau-\tau_i)\left(\psi_V(k,\tau)\psi_V^*(k,\tau_i) - \psi_V^*(k,\tau)\psi_V(k,\tau_i)\right) \\
&= i\,\Theta(\tau-\tau_i)\left[\frac{\tau-\tau_i}{k^2\tau\tau_i}\cos[k(\tau-\tau_i)] - \frac{k^2\tau\tau_1+1}{k^3\tau\tau_1}\sin[k(\tau-\tau_i)]\right]
\end{aligned} \tag{3.23}$$

is the retarded Green function associated with the homogeneous solution of (3.63).

Let us analyze the two-point correlation function of $\hat{\psi}_{S\pm}$. Firstly, in (3.22) we can approximate $\hat{A}_- = 0$ in the integration since the most contributing part of the integration comes from the enhancement of $\hat{A}_+$ around horizon-crossing, so we ignore the terms in the third line of (3.22). Then after renormalization of UV part, we can reduce the integration of time interval to the region where the particle production of the gauge field takes place. Therefore, rewriting $\hat{\psi}_{S\pm}$ in $\hat{h}_{S\pm} = 2\hat{\psi}_{S\pm}/(aM_p)$ we can find

$$\langle\hat{h}_{S\pm}(\boldsymbol{k})\hat{h}_{S\pm}(\boldsymbol{k}')\rangle \simeq (2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k}')\frac{4H^4}{k^3M_p^4}f_h^\pm(\xi)e^{4\pi\xi} \,, \tag{3.24}$$

where we introduced a numerical factor

$$\begin{aligned}
f_h^\pm(\xi) \equiv &\int\frac{d\boldsymbol{p}_*}{(2\pi)^3}|e^l_\mp(\hat{\boldsymbol{k}})e^+_l(\hat{\boldsymbol{p}})|^2|e^m_\mp(\hat{\boldsymbol{k}})e^+_m(\widehat{\boldsymbol{k}-\boldsymbol{p}})|^2\frac{\sqrt{p_*|\hat{\boldsymbol{k}}-\boldsymbol{p}_*|}}{4\xi} \\
&\times\left[\int_\infty^0 dx_1\, x_1^{3/2}\left(\cos x_1 - \frac{\sin x_1}{x_1}\right)\left(\frac{2\xi}{x_1} + \sqrt{p_*|\hat{\boldsymbol{k}}-\boldsymbol{p}_*|}\right)e^{-2\sqrt{2\xi}\left(\sqrt{p_*}+\sqrt{|\hat{\boldsymbol{k}}-\boldsymbol{p}_*|}\right)\sqrt{x_1}}\right]^2 \,.
\end{aligned} \tag{3.25}$$

Here $x_1 = -k\tau$, $\boldsymbol{p}_* \equiv \boldsymbol{p}/|\boldsymbol{k}|$ and we used the following relation $|e^l_\mp(\hat{\boldsymbol{k}})e^\lambda_l(\hat{\boldsymbol{p}})|^2 = \left(1 \pm \lambda\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{p}}\right)^2/4$ in the above expressions. Although we used the approximate solution (3.7) to the whole integrand of the correlation function, this simplification can be justified because the region where both amplifications of gauge fields $\hat{A}_+(\boldsymbol{p})$, $\hat{A}_+(\boldsymbol{k}-\boldsymbol{p})$ are maximal gives the most dominant contribution to the integration and furthermore we can extend x_1 from 0 to ∞ since (3.7) has most of its support at $(8\xi)^{-1} \lesssim x_1 \lesssim 2\xi$ [44]. In this calculation, we set $\hat{\boldsymbol{k}} = (0,0,1)$ and $\boldsymbol{p} = p(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ which corresponds to the following polarization vector

$$\boldsymbol{e}^{\pm}(\hat{\boldsymbol{p}}) = \frac{1}{\sqrt{2}}(\cos\theta\cos\phi \mp i\sin\phi,\ \cos\theta\sin\phi \pm i\cos\phi,\ -\sin\theta)\,. \tag{3.26}$$

Then for large ξ, we can write

$$f_h^+(\xi) \simeq \frac{4.3\times 10^{-7}}{\xi^6}\,, \qquad f_h^-(\xi) \simeq \frac{9.2\times 10^{-10}}{\xi^6}\,. \tag{3.27}$$

We can see that they are good approximations from Fig.3.2 and $f_h^+(\xi)$ is larger than $f_h^-(\xi)$ by a factor 10^{-3}. This numerical discrepancy comes from the suppression factors $|e^l_{\mp}(\hat{\boldsymbol{k}})e^+_l(\hat{\boldsymbol{p}})|^2|e^m_{\mp}(\hat{\boldsymbol{k}})e^+_m(\widehat{\boldsymbol{k}-\boldsymbol{p}})|^2$ in the integration. Therefore, the total power spectrum of tensor modes is given by

$$\langle\hat{h}_{\pm}(\boldsymbol{k})\hat{h}_{\pm}(\boldsymbol{k}')\rangle = \langle\hat{h}_{V\pm}(\boldsymbol{k})\hat{h}_{V\pm}(\boldsymbol{k}')\rangle + \langle\hat{h}_{S\pm}(\boldsymbol{k})\hat{h}_{S\pm}(\boldsymbol{k}')\rangle \tag{3.28}$$

$$\equiv (2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k}')\frac{2\pi^2}{k^3}\Delta_h^{\pm}(k)\,. \tag{3.29}$$

In the super-horizon limit, we get

$$\Delta_h^+(k) \simeq \frac{H^2}{\pi^2 M_p^2}\left(1+\frac{2H^2}{M_p^2}f_h^+(\xi)e^{4\pi\xi}\right)\,, \tag{3.30}$$

$$\Delta_h^-(k) \simeq \frac{H^2}{\pi^2 M_p^2}\left(1+\frac{2H^2}{M_p^2}f_h^-(\xi)e^{4\pi\xi}\right) \tag{3.31}$$

and the following chirality parameter of tensor spectra

$$\chi \equiv \frac{\Delta_h^+(k)-\Delta_h^-(k)}{\Delta_h^+(k)+\Delta_h^-(k)}\,. \tag{3.32}$$

At large ξ, the chirality gets extremely close to unity and the provided spectrum is therefore totally parity-violating.

3.3 Scalar mode amplification

We also estimate the two-point correlation function of the curvature perturbation on uniform density hypersurfaces $\zeta = -H\delta\varphi/\dot{\varphi}$. Not only tensor modes, but the fluctuation of inflaton $\delta\varphi$ is also produced by the intrinsic inhomogeneities of Chern-Simons gauge interactions $\delta_{EB} \equiv (E_iB_i - \langle E_iB_i\rangle)|_{\delta\varphi=0}$. The equation of motion for $\delta\varphi$ reads[2]

$$\ddot{\delta\varphi} + 3\beta H\dot{\delta\varphi} - a^{-2}\nabla^2\delta\varphi = -\frac{1}{f}\delta_{EB}\,, \tag{3.33}$$

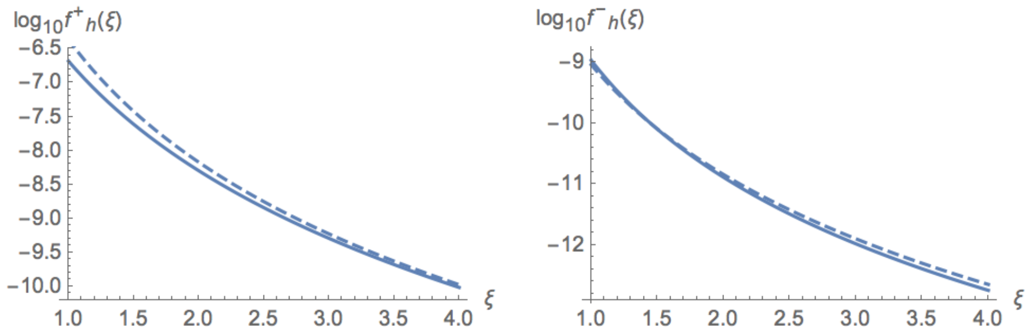

Figure 3.2: Plot of numerical factors $f_h^+(\xi)$ (left figure) and $f_h^-(\xi)$ (right figure). The dotted lines show (3.27) and both lines are in good agreements with (3.25) (solid lines).

where the additional friction term

$$\beta \equiv 1 + \frac{1}{3fH}\frac{\partial\langle E_i B_i\rangle}{\partial\dot{\varphi}} \simeq 1 + \frac{2\pi\xi}{3fH\dot{\varphi}}\langle E_i B_i\rangle \tag{3.34}$$

comes from the dependence of back-reaction $\langle E_i B_i\rangle$ on $\dot{\varphi}$ via ξ. Note that β is always positive in the parameter region where we are interested in. Here again, for $\delta\varphi$ we have two solutions $\delta\varphi = \delta\varphi_V + \delta\varphi_S$ from (3.33). The homogeneous solution $\delta\varphi_V$ is an ordinal vacuum fluctuation so that $\langle\delta\varphi_{V\boldsymbol{k}}^2\rangle^{1/2} = H/(2\pi)$ is satisfied. When the effect of backreaction is negligible, the system follows

$$\ddot{\delta\varphi} + 3H\dot{\delta\varphi} - a^{-2}\nabla^2\delta\varphi \simeq \frac{1}{f}E_i B_i\,, \tag{3.35}$$

hence in Fourier modes it is rewritten as

$$\left[\frac{\partial^2}{\partial\tau^2} + k^2 - \frac{2}{\tau^2}\right](a\delta\hat{\varphi}_{\boldsymbol{k}}) = \frac{a^3}{f}\int\frac{d\boldsymbol{p}}{(2\pi)^3}e_i^+(\hat{\boldsymbol{p}})e_i^+(\widehat{\boldsymbol{k}-\boldsymbol{p}})\hat{E}^+_{\boldsymbol{p}}\hat{B}^+_{\boldsymbol{k}-\boldsymbol{p}}\,. \tag{3.36}$$

As to the mode function sourced by gauge field $\delta\hat{\varphi}_{S\boldsymbol{k}}$, one can find

$$a\delta\hat{\varphi}_{S\boldsymbol{k}} \simeq \frac{1}{f}\int_{-\infty}^{0}d\tau_1\frac{G_k(\tau,\tau_1)}{a(\tau_1)}\int\frac{d\boldsymbol{p}}{(2\pi)^3}\hat{A}_+{}'(\boldsymbol{p})|\boldsymbol{k}-\boldsymbol{p}|\hat{A}_+(\boldsymbol{k}-\boldsymbol{p})e_l^+(\hat{\boldsymbol{p}})e_m^+(\widehat{\boldsymbol{k}-\boldsymbol{p}})\,, \tag{3.37}$$

As a result, defining the dimensionlsss power spectrum of the curvature perturbation

$$\langle\hat{\zeta}(\boldsymbol{k})\hat{\zeta}(\boldsymbol{k}')\rangle \equiv (2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k}')\frac{2\pi^2}{k^3}\Delta_\zeta(k)\,, \tag{3.38}$$

it is approximated as

$$\Delta_\zeta = \Delta_{\zeta v} + \Delta_{\zeta s} \simeq \Delta_{\zeta v}\left(1 + \Delta_{\zeta v}f_s(\xi)e^{4\pi\xi}\right)\,, \qquad \Delta_{\zeta v} = \left(\frac{H^2}{2\pi\dot{\varphi}}\right)^2\,, \tag{3.39}$$

[2]Note that we do not include the mass term of the fluctuation of inflaton and other cubic interactions via the scalar metric fluctuation because they are sub-dominant in the slow-roll expansion.

where a numerical factor $f_s(\xi)$ reads

$$f_s(\xi) \equiv 4\pi^2\xi^2 \int \frac{d\boldsymbol{p}_*}{(2\pi)^3} |e^m_\mp(\hat{\boldsymbol{p}}) e^+_m(\widehat{\boldsymbol{k}-\boldsymbol{p}})|^2 \sqrt{p_*|\hat{\boldsymbol{k}}-\boldsymbol{p}_*|} \times \left[\int_\infty^0 dx_1\, x_1 \left(\cos x_1 - \frac{\sin x_1}{x_1} \right) \left(\sqrt{p_*} + \sqrt{p_*|\hat{\boldsymbol{k}}-\boldsymbol{p}_*|} \right) e^{-2\sqrt{2\xi}\left(\sqrt{p_*}+\sqrt{|\hat{\boldsymbol{k}}-\boldsymbol{p}_*|}\right)\sqrt{x_1}} \right]^2 . \tag{3.40}$$

For $\xi \geq 1$, it is approximated by [44]

$$f_s(\xi) \simeq \frac{3\times 10^{-5}}{\xi^{5.4}} \qquad (2 \leq \xi \leq 3) . \tag{3.41}$$

On the other hand, when backreaction effect to the background motion of scalar field is relevant, we use a following heuristic estimation to the two-point function [45]. Since the particle production occurs near the horizon crossing, we can evaluate the derivative as $\partial \sim H$. If the backreaction becomes relevant, β gets much greater value than $\mathcal{O}(1)$. Therefore the friction term in (3.33) becomes dominant and $\delta\varphi_S$ is roughly estimated as

$$\delta\varphi_S \sim \frac{\delta_{EB}}{3f\beta H^2} . \tag{3.42}$$

Then, defining the dimensionlsss power spectrum of the curvature perturbation

$$\langle \hat{\zeta}(\boldsymbol{k})\hat{\zeta}(\boldsymbol{k}')\rangle \equiv (2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k}')\frac{2\pi^2}{k^3}\Delta_\zeta(k) , \tag{3.43}$$

we find

$$\Delta_\zeta = \Delta_{\zeta v} + \Delta_{\zeta s} \simeq \left(\frac{H^2}{2\pi\dot{\varphi}}\right)^2 + \left(\frac{\sigma}{3f\beta H\dot{\varphi}}\right)^2 \tag{3.44}$$

in terms of the variance

$$\sigma^2 \equiv \langle(\delta_{EB})^2\rangle = \langle E_iE_j\rangle\langle B_iB_j\rangle + \langle E_iB_j\rangle\langle B_iE_j\rangle \tag{3.45}$$

with

$$\langle E_iE_j\rangle\langle B_iB_j\rangle \simeq \frac{1}{a^8}\int \frac{d\boldsymbol{p}}{(2\pi)^3}\frac{d\boldsymbol{q}}{(2\pi)^3} e^+_i(\boldsymbol{p})e^+_i(\boldsymbol{q})e^{+*}_j(\boldsymbol{p})e^{+*}_j(\boldsymbol{q})|A'_+(p)|^2q^2|A_+(q)|^2 , \tag{3.46}$$

$$\langle E_iB_j\rangle\langle B_iE_j\rangle \simeq \frac{1}{a^8}\int \frac{d\boldsymbol{p}}{(2\pi)^3}\frac{d\boldsymbol{q}}{(2\pi)^3} e^+_i(\boldsymbol{p})e^+_i(\boldsymbol{q})e^{+*}_j(\boldsymbol{p})e^{+*}_j(\boldsymbol{q})pA'_+(p)A^*_+(p)A^{*\prime}{}_+(q)qA_+(q) . \tag{3.47}$$

Then, using the following quantities

$$I_1 \equiv \frac{1}{a^4}\int \frac{d\boldsymbol{k}}{(2\pi)^3}|A'_+(k)|^2k^2 \simeq 2.2\times 10^{-5}\frac{H^4}{\xi^3}e^{2\pi\xi} , \tag{3.48}$$

$$I_2 \equiv \frac{1}{a^4}\int \frac{d\boldsymbol{k}}{(2\pi)^3}\frac{\partial_\tau}{2}|A_+(k)|^2k^3 \simeq 1.9\times 10^{-5}\frac{H^4}{\xi^4}e^{2\pi\xi} , \tag{3.49}$$

$$I_3 \equiv \frac{1}{a^4}\int \frac{d\boldsymbol{k}}{(2\pi)^3}|A_+(k)|^2k^4 \simeq 1.9\times 10^{-5}\frac{H^4}{\xi^5}e^{2\pi\xi} , \tag{3.50}$$

this variance is approximately given by

$$\begin{aligned}\sigma &= \sqrt{\frac{(4\pi)^2}{3}(I_2^2 + I_1 I_3)} \\ &= 2.0\times 10^{-4}\frac{H^4}{\xi^4}e^{2\pi\xi} \simeq |\langle E_i B_i\rangle| \qquad (\xi \gtrsim 1)\ . \end{aligned} \tag{3.51}$$

Remarkably, (3.44) is in excellent agreement with (3.39) at small ξ and is therefore good approximation even in the region where backreaction of gauge field is negligible. At large ξ, two-point function settles in a constant value

$$\Delta_\zeta \simeq \frac{1}{(2\pi\xi)^2}\ . \tag{3.52}$$

3.4 Numerical predictions

Now, let us show the blue-tilted power spectrum sourced by axion-gauge coupling. For the inflaton's potential, we adopt the Starobinsky type

$$V = V_0(1 - e^{-\gamma\varphi})^2 \tag{3.53}$$

as an example[3]. In order to agree with the constraint from the CMB observation at pivot scale $k_* = 0.05\mathrm{Mpc}^{-1}$, we keep the following conditions [6]

$$\Delta_\zeta(k_*) \simeq 2\times 10^{-9}\ , \qquad n_s = 0.968 \pm 0.06\ , \qquad r \lesssim 0.07\ . \tag{3.54}$$

In addition to these, we have to take care of the contribution of gauge field quanta to the scalar perturbation. This can be highly non-gaussian since it comes schematically from the product of two gaussian gauge fields, so that we have the following constraint on the small non-gaussianity in CMB scales [6,43]

$$\xi_{\mathrm{CMB}} \equiv \xi(N_{\mathrm{CMB}}) \lesssim 2.5\ . \tag{3.55}$$

Keeping above constraints in mind, we depict the time evolution of inflaton and its velocity in Fig.3.3 and the corresponding power spectrum in Fig.3.4. At late times, the motion of inflaton slows down owing to the backreaction of gauge field compared to the motion without that.

In Fig.3.4, we used the relation (2.78) and converted the dependence on the number of e-foldings into that on frequency. Here, we rewrite the tensor spectra as the present energy intensity of gravitational waves $\Omega_{\mathrm{GW}}(f)$ expressed by (2.77). We can see that these amplitudes are growing on small scales due to the increase of ξ and we have a chance to test this parity-violating gravitational waves with future space-based interferometers such as LISA, DECIGO and BBO, and also with the pulsar timing array observation such as SKA. As to the ground-based interferometers, although the expected amplitude is a few orders of magnitude lower than the design sensitivity of the advanced LIGO and KAGRA missions, we have a chance to test the scenario with the third generation of experiments such as Einstein Telescope.

[3]As is discussed in [87], such a Starobinsky-like potential of axion can be realized from the flux compactifications of string theory.

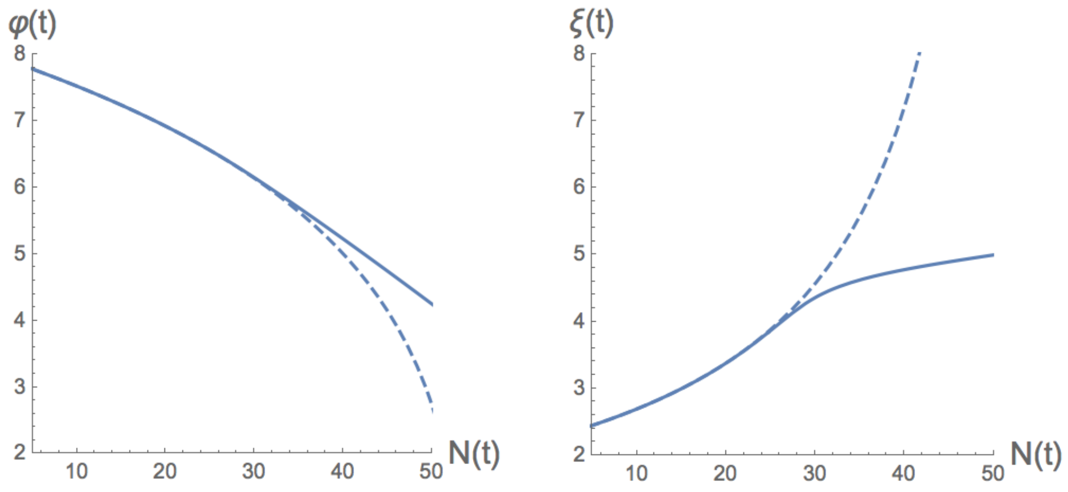

Figure 3.3: Time evolution of the inflaton and its velocity parameterized by ξ as a function of the number of e-folds $N(t)$ with (solid blue line) and without (dashed blue line) including back-reaction of the gauge field with $\xi_{\rm CMB} \simeq 2.4$. At the late stage the effect of gauge field slows down the variation of φ and prolongs the inflationary phase. In this figure, we set $V_0 = 10^{-9} M_p^4$, $\gamma = 0.35$, $f = 10^{-2} M_p$.

Simultaneously, however, we should also take care of the amplification of curvature perturbation, leading to the overproduction of primordial black holes (PBHs) in small scales. It is known that when the curvature perturbation exceeds a critical value $\zeta \gtrsim \zeta_c \sim 1$, at horizon re-entry after inflation it leads to the black hole formation. We can represent this probability in terms of the variance $\langle \zeta_c^2 \rangle$ and constrain the amplitude of the power-spectrum of curvature perturbation. In our case, the curvature perturbation sourced by gauge fields is non-gaussian and is schematically written by $\zeta = g^2 - \langle g^2 \rangle$, where g is the Gaussian distribution field. So its variance is roughly estimated as $\Delta_\zeta \sim \langle g^2 \rangle^2$. The bound on the distribution of non-gaussian ζ has already been studied and estimated by [38, 88]

$$\Delta_{\zeta_c} = O(10^{-4}) - O(10^{-3}) \,. \tag{3.56}$$

In the figure, the amount of scalar power spectrum is enhanced on small scales and exceed the upper bound on the non-detection of PBHs. According to [38], we have $\xi_{\rm CMB} \lesssim 1.5$ for the bound at CMB scales in order to avoid an overproduction of PBHs.

It is known that, however, the above negative situation can be circumvented by considering another mechanism of providing curvature perturbation such as the curvaton scenario [89], or introducing many Abelian gauge fields [58]. In the next section, we suggest the model of axionic inflation coupled to many U(1) gauge fields, which predicts sizable chiral gravitational waves with the amount of curvature perturbation suppressed.

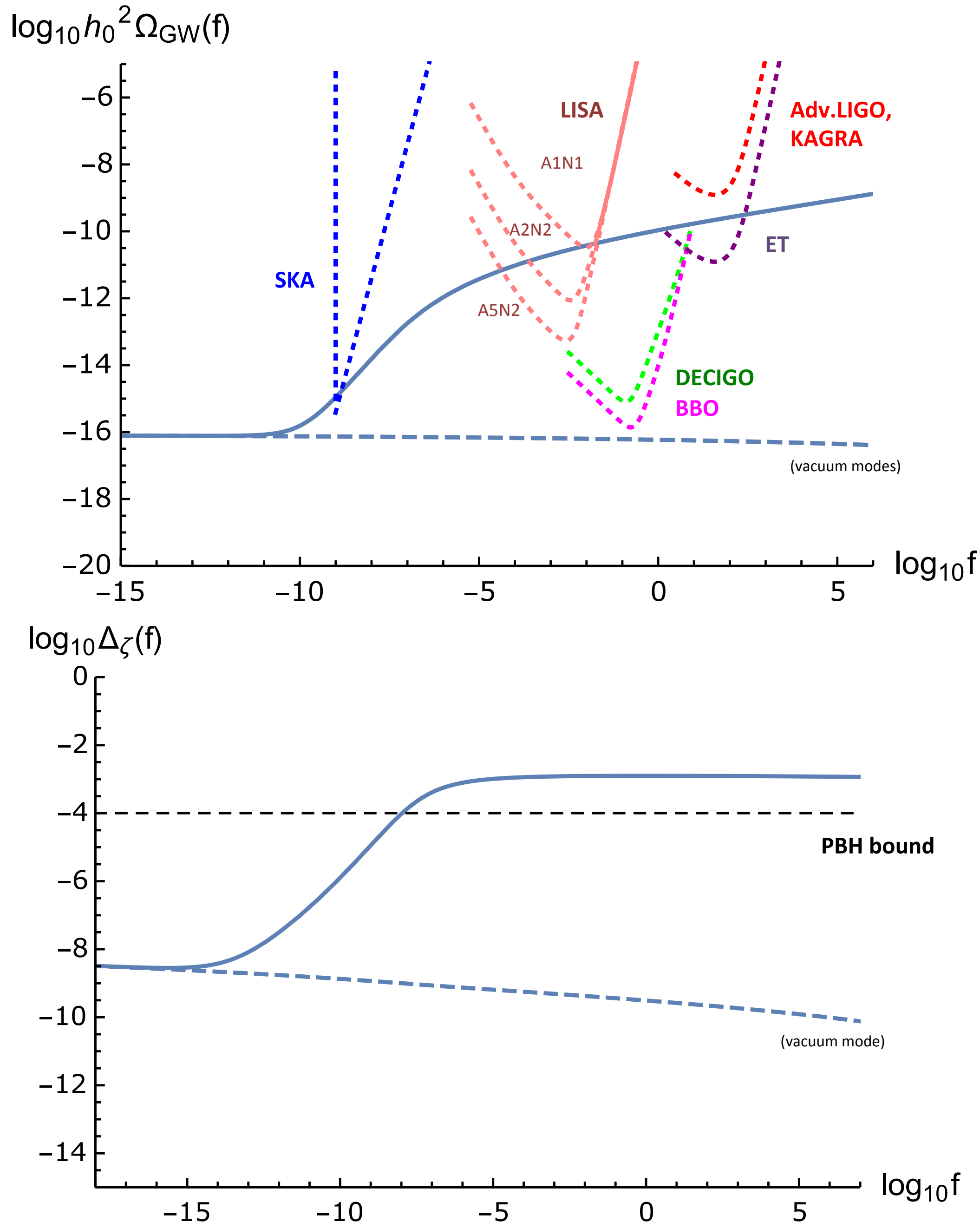

Figure 3.4: (Top panel) : Plot of the tensor power spectrum and several sensitivity curves of gravitational wave experiments. We describe the present energy intensity of primordial gravitational waves in this model with $\xi_{\rm CMB} \simeq 2.4$ (solid blue line). Compared with the spectrum of vacuum fluctuations (dashed blue line), they are naturally blue-tilted due to the increase of velocity of axion. The provided amplitude is beyond the analytical plots of sensitivity curves of LISA [90], DECIGO / BBO [91], Einstein Telescope [92] and SKA mission [10], while poorer than that of advanced LIGO [93] and KAGRA [94]. (Bottom panel) : Plot the power spectrum of curvature perturbation with $\xi_{\rm CMB} \simeq 2.4$ (solid blue line). The dashed blue line represents the power spectrum of vacuum fluctuations. We can see that it has an excess of the upper bound on the non-detection of PBHs (black line). In both figures, we set $V_0 = 10^{-9} M_p^4,\ \gamma = 0.35,\ f = 10^{-2} M_p$.

3.5 Axionic inflation with many U(1) sectors

In this section, we present the way to avoid the overproduction of PBHs by introducing many U(1) gauge fields or higher-ranked gauge groups coupled to an axion inspired by the dimensionally reduced string theory [72]. In this work, however, we focus on the dynamics of Abelian sector $F^{(i)}_{\mu\nu} = \partial_\mu A^{(i)}_\nu - \partial_\nu A^{(i)}_\mu$ $(i = 1, 2, ...)$ for an analytical reason and do not lead to something specific gauge groups from string compactifications. The detailed string theory embeddings of our scenarios and their phenomenological features are beyond the scope of this paper and we leave such an analysis to future work.

To begin with, we consider the axionic inflaton φ coupled with two gauge groups. For simplicity, we neglect the non-linear interaction of the non-Abelian gauge sector and we treat Abelian gauge fields embedded in the original gauge groups. So, we work with the following action

$$\int d^4x\sqrt{-g_4}\left[\frac{M_p^2}{2}R - \frac{1}{2}(\partial\varphi)^2 - V(\varphi) - \sum_{n=1}^{\mathcal{N}}\frac{1}{4}F^{(n)}_{\mu\nu}F^{\mu\nu}_{(n)} - \sum_{m=1}^{\mathcal{M}}\frac{1}{4}F^{(m)}_{\mu\nu}F^{\mu\nu}_{(m)} \right. \\ \left. +\frac{1}{4}\frac{\varphi}{f}\left(\sum_{n=1}^{\mathcal{N}}F^{(n)}_{\mu\nu}\tilde{F}^{\mu\nu}_{(n)} - \sum_{m=1}^{\mathcal{M}}F^{(m)}_{\mu\nu}\tilde{F}^{\mu\nu}_{(m)}\right)\right] . \tag{3.57}$$

Note that we introduced the topological Chern-Simons terms with different sign for two gauge groups. It is known that this form can be derived from a string compactification (see the Appendix). As to the metric, we assume a flat FLRW metric $ds^2 = -dt^2 + a(t)^2 d\boldsymbol{x}^2 = a(\tau)^2[-d\tau^2 + d\boldsymbol{x}^2]$. The gauge field $A^{(i)}(\tau, \boldsymbol{x})$ interacts with the homogeneous axion field $\bar{\varphi}(t)$ via their Chern-Simons terms. In the gauge conditions $A^{(i)}_0 = \partial_j A^{(i)}_j = 0$, the equations of motion for each gauge field are given by

$$\partial_\tau^2 A^{(n)}_i - \nabla^2 A^{(n)}_i - \frac{\bar{\varphi}'}{f}\epsilon_{ijk}\partial_j A^{(n)}_k = 0 , \tag{3.58}$$

$$\partial_\tau^2 A^{(m)}_i - \nabla^2 A^{(m)}_i + \frac{\bar{\varphi}'}{f}\epsilon_{ijk}\partial_j A^{(m)}_k = 0 \tag{3.59}$$

(the prime means the derivative with respect to τ). In order to describe the generation of gauge field induced by the rolling axion, we quantize $A^{(i)}_i$ in the basis of the circular polarization vectors $e^\pm_i(\hat{\boldsymbol{k}})$, where the helicity operators $\hat{A}^{(i)}_\pm(\boldsymbol{k}, \tau)$ decompose into annihilation and creation operators $\hat{A}^{(i)}_\pm(\boldsymbol{k}, \tau) = a^{(i)}{}_\pm(\boldsymbol{k})A^{(i)}_\pm(k, \tau) + a^{(i)\dagger}{}_\pm(-\boldsymbol{k})A^{(i)*}_\pm(k, \tau)$, obeying $[a^{(i)}{}_{\lambda\boldsymbol{k}}, a^{(j)\dagger}{}_{\lambda'-\boldsymbol{k}'}] = \delta_{ij}\delta_{\lambda\lambda'}(2\pi)^3\delta^3(\boldsymbol{k} + \boldsymbol{k}')$. Hence the equations of motion for the mode functions $A^{(i)}_\pm(k, \tau)$ read

$$\left[\partial_\tau^2 + k^2 \pm \frac{2k\xi}{\tau}\right]A^{(n)}_\pm = 0 , \tag{3.60}$$

$$\left[\partial_\tau^2 + k^2 \mp \frac{2k\xi}{\tau}\right]A^{(m)}_\pm = 0 , \tag{3.61}$$

where $\xi \equiv \dot{\varphi}/(2fH)$ (the dot means the derivative with respect to t) relates the velocity of the axion and hereafter we treat it as a positive value. We can see that $A^{(n)}_+$ and $A^{(m)}_-$ experience a tachyonic instability in the same time interval $|k\tau| \lesssim 2\xi$ and grow up by a factor $e^{\pi\xi}$ (see (3.7), (3.8)), especially the polarization of the enhanced mode is opposite.

Hence, as we show later in total both polarization modes of the gauge fields contribute to the generation of gravitational waves. On the other hand, $A_-^{(n)}$ and $A_+^{(m)}$ are not produced during inflation and can therefore be ignored in the amplification of perturbations.

The transverse traceless tensor modes $h_{ij}(\tau, \boldsymbol{x})$ are given by the spatial component of the metric $g_{ij} = a^2(\delta_{ij} + h_{ij})$. In terms of the canonically normalized variable $\psi_{ij} = aM_p h_{ij}/2$, from the Einstein equations we get the equation of motion for tensor modes

$$\left[\partial_\tau^2 - \nabla^2 - \frac{2}{\tau^2}\right]\psi_{ij} = -\frac{a^3}{M_p}\sum_{i=1}^{\mathcal{N}+\mathcal{M}}(E_i^{(i)}E_j^{(i)} + B_i^{(i)}B_j^{(i)})^{TT} , \tag{3.62}$$

where $E_i^{(i)}$, $B_i^{(i)}$ are the electric and magnetic component of the gauge field and TT denotes the transverse traceless projection of the spatial component of energy-momentum tensor of gauge field. The two helicities of tensor modes $\hat{\psi}_\pm(\boldsymbol{k})$ are obtained by multiplying ψ_{ij} by the circular polarization tensors $e_\mp^{ij}(\hat{\boldsymbol{k}}) = e_\mp^i(\hat{\boldsymbol{k}})e_\mp^j(\hat{\boldsymbol{k}})$. Ignoring the contribution of $\hat{A}_-^{(n)}$ and $\hat{A}_+^{(m)}$, one can then find

$$\begin{aligned}\left[\partial_\tau^2 + k^2 - \frac{2}{\tau^2}\right]\hat{\psi}_\pm(\boldsymbol{k}) \simeq -\frac{1}{aM_p}\int\frac{d\boldsymbol{p}}{(2\pi)^3}e_\mp^i(\hat{\boldsymbol{k}})e_\mp^j(\hat{\boldsymbol{k}}) \\ \times\left[e_i^+(\hat{\boldsymbol{p}})e_j^+(\widehat{\boldsymbol{k}-\boldsymbol{p}})\sum_{n=1}^{\mathcal{N}}\hat{J}_+^{(n)}(\boldsymbol{p}, \boldsymbol{k}-\boldsymbol{p}) + e_i^-(\hat{\boldsymbol{p}})e_j^-(\widehat{\boldsymbol{k}-\boldsymbol{p}})\sum_{m=1}^{\mathcal{M}}\hat{J}_-^{(m)}(\boldsymbol{p}, \boldsymbol{k}-\boldsymbol{p})\right] ,\end{aligned} \tag{3.63}$$

where

$$\hat{J}_\pm^{(i)}(\boldsymbol{p}, \boldsymbol{k}-\boldsymbol{p}, \tau) \equiv \hat{A}_\pm^{(i)\prime}(\boldsymbol{p}, \tau)\hat{A}_\pm^{(i)\prime}(\boldsymbol{k}-\boldsymbol{p}, \tau) + p|\boldsymbol{k}-\boldsymbol{p}|\hat{A}_\pm^{(i)}(\boldsymbol{p}, \tau)\hat{A}_\pm^{(i)}(\boldsymbol{k}-\boldsymbol{p}, \tau) . \tag{3.64}$$

Therefore, the tensor modes are composed of two solutions $\hat{\psi}_\pm = \hat{\psi}_V + \hat{\psi}_{S\pm}$: $\hat{\psi}_V$ is the homogeneous solution which corresponds to the vacuum fluctuations and $\hat{\psi}_{S\pm}$ is the peculiar solution sourced by two gauge quanta $\hat{A}_+^{(n)}$, $\hat{A}_-^{(m)}$.

Let us calculate the two-point correlation function of the graviton $\hat{h}_\pm = 2\hat{\psi}_\pm/(aM_p) = \hat{h}_V + \hat{h}_{S\pm}$. Since $\hat{h}_V$, $\hat{h}_{S\pm}$ are statistically independent, the correlation function is calculated as $\langle\hat{h}_\pm(\boldsymbol{k})\hat{h}_\pm(\boldsymbol{k}')\rangle = \langle\hat{h}_{V\pm}(\boldsymbol{k})\hat{h}_{V\pm}(\boldsymbol{k}')\rangle + \langle\hat{h}_{S\pm}(\boldsymbol{k})\hat{h}_{S\pm}(\boldsymbol{k}')\rangle$. We are interested in the correlation function due to the sourced component. In view of the conservation of the angular momentum, one positive-helicity graviton is easy to be produced by two positive-helicity photons, and vice versa[4]. Therefore we can approximately get

$$\begin{aligned}\left((2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k}')\right)^{-1}\langle\hat{h}_{S+}(\boldsymbol{k})\hat{h}_{S+}(\boldsymbol{k}')\rangle \simeq \frac{4H^2\tau^2}{M_p^2}\int\frac{d\boldsymbol{p}}{(2\pi)^3}|e_-^i(\hat{\boldsymbol{k}})e_i^+(\hat{\boldsymbol{p}})|^2|e_-^j(\hat{\boldsymbol{k}})e_j^+(\widehat{\boldsymbol{k}-\boldsymbol{p}})|^2 \\ \times\sum_{n=1}^{\mathcal{N}}\left|\int_{-\infty}^0 d\tau_1\frac{G_k(\tau, \tau_1)}{a(\tau_1)}J_+^{(n)}(\boldsymbol{p}, \boldsymbol{k}-\boldsymbol{p}, \tau_1)\right|^2 ,\end{aligned} \tag{3.65}$$

$$\begin{aligned}\left((2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k}')\right)^{-1}\langle\hat{h}_{S-}(\boldsymbol{k})\hat{h}_{S-}(\boldsymbol{k}')\rangle \simeq \frac{4H^2\tau^2}{M_p^2}\int\frac{d\boldsymbol{p}}{(2\pi)^3}|e_+^i(\hat{\boldsymbol{k}})e_i^-(\hat{\boldsymbol{p}})|^2|e_+^j(\hat{\boldsymbol{k}})e_j^-(\widehat{\boldsymbol{k}-\boldsymbol{p}})|^2 \\ \times\sum_{m=1}^{\mathcal{M}}\left|\int_{-\infty}^0 d\tau_1\frac{G_k(\tau, \tau_1)}{a(\tau_1)}J_-^{(m)}(\boldsymbol{p}, \boldsymbol{k}-\boldsymbol{p}, \tau_1)\right|^2 ,\end{aligned} \tag{3.66}$$

[4]One can actually check that this numerical discrepancy comes from the orthogonal relation of polarization vectors $e^{i\lambda}(\hat{\boldsymbol{k}})e^{\lambda'*}{}_i(\hat{\boldsymbol{k}}) = \delta_{\lambda\lambda'}$.

where $G_k(\tau, \tau_1)$ is the retarded Green function associated with the homogeneous solution of $\hat{\psi}_\pm$ (denoted as (3.23)) and $J_\pm^{(n,m)}$ is a classical function of $\hat{J}_\pm^{(n,m)}$. Since different gauge fields are statistically uncorrelated with each other, $\langle \hat{h}_{S\pm}(\boldsymbol{k})\hat{h}_{S\pm}(\boldsymbol{k}')\rangle$ increase by the number of gauge fields $\mathcal{N}$, $\mathcal{M}$. Note that in the above calculations we do not include the commutation part of the source terms $\hat{J}_\pm^{(i)}$. However, these simplifications are justified since mode functions of the produced gauge fluctuations are substantially real-valued, up to an irrelevant constant phase. Thus, defining the dimensionless power spectrum

$$\langle \hat{h}_\pm(\boldsymbol{k})\hat{h}_\pm(\boldsymbol{k}')\rangle \equiv (2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k}')\frac{2\pi^2}{k^3}\Delta_h^\pm(k) , \tag{3.67}$$

we have the following tensor power spectra

$$\Delta_h^+(k) = \frac{H^2}{\pi^2 M_p^2}\left(1 + \mathcal{N}\frac{2H^2}{M_p^2} f_h^+(\xi) e^{4\pi\xi}\right) , \tag{3.68}$$

$$\Delta_h^-(k) = \frac{H^2}{\pi^2 M_p^2}\left(1 + \mathcal{M}\frac{2H^2}{M_p^2} f_h^+(\xi) e^{4\pi\xi}\right) , \tag{3.69}$$

$$f_h^+(\xi) \simeq \frac{4.3\times 10^{-7}}{\xi^6} \qquad (\xi \gtrsim 1) . \tag{3.70}$$

Thus, both polarization modes of gravitons are amplified by the gauge fields and its chirality is characterized by the difference of $\mathcal{N}$ and $\mathcal{M}$. So the amount of chirality χ is given by

$$\chi \equiv \frac{\Delta_h^+(k) - \Delta_h^-(k)}{\Delta_h^+(k) + \Delta_h^-(k)} \simeq \frac{\mathcal{N}-\mathcal{M}}{\mathcal{N}+\mathcal{M}} \qquad (\xi \gtrsim 1) . \tag{3.71}$$

In the case of $\mathcal{N} = \mathcal{M}$ the provided tensor spectra are completely parity symmetric. By contrast to the predictions in the conventional model discussed in the preceding subsection, these spectra could have the same amplitude and therefore predict no parity-violating phenomena through CMB and gravitational wave observations.

At the end of this subsection, we also estimate the two-point correlation function of the curvature perturbation on uniform density hypersurfaces $\zeta = -H\delta\varphi/\dot{\varphi}$. Not only tensor modes, but also the perturbation of inflaton $\delta\varphi$ is produced by the intrinsic inhomogeneities of Chern-Simons gauge interactions $\delta_{EB}^{(i)} \equiv \left(E_i^{(i)}B_i^{(i)} - \langle E_i^{(i)}B_i^{(i)}\rangle\right)|_{\delta\varphi=0}$. The equation of motion for $\delta\varphi$ reads[5]

$$\delta\ddot{\varphi} + 3\beta H\delta\dot{\varphi} - a^{-2}\nabla^2\delta\varphi = \frac{1}{f}\left(\sum_{n=1}^{\mathcal{N}}\delta_{EB}^{(n)} - \sum_{m=1}^{\mathcal{M}}\delta_{EB}^{(m)}\right) , \tag{3.72}$$

where the additional friction term

$$\beta \equiv 1 - \frac{1}{3fH}\left(\sum_{n=1}^{\mathcal{N}}\frac{\partial\langle E_i^{(n)}B_i^{(n)}\rangle}{\partial\dot{\varphi}} - \sum_{m=1}^{\mathcal{M}}\frac{\partial\langle E_i^{(m)}B_i^{(m)}\rangle}{\partial\dot{\varphi}}\right) \tag{3.73}$$

[5]Note that we do not include the mass term of the fluctuation of inflaton and other cubic interactions via the scalar metric fluctuation because they are sub-dominant in the slow-roll expansion.

comes from the dependence of back-reaction $\langle E_i^{(i)} B_i^{(i)} \rangle$ on $\dot{\varphi}$ via ξ. Note that the back-reaction of the gauge field

$$\langle E_i^{(i)} B_i^{(i)} \rangle = -\frac{1}{a^4} \sum_{\lambda=\pm 1} \int \frac{d^3k}{(2\pi)^3} \frac{k}{2} \lambda |A_\lambda^{(i)}|^{2\prime} \simeq -2.4 \times 10^{-4} \lambda \frac{H^4}{\xi^4} e^{2\pi\xi} \quad (\xi \gtrsim 1) \tag{3.74}$$

depends on the sign of helicity of gauge field and therefore the contribution of each gauge field to the equation has the same sign. So β is always positive. Here again, for $\delta\varphi$ we have two solutions $\delta\varphi = \delta\varphi_V + \delta\varphi_S$ from (3.72). The homogeneous solution $\delta\varphi_V$ is the usual vacuum fluctuation so that $\langle \delta\varphi_{V\boldsymbol{k}}^2 \rangle^{1/2} = H/(2\pi)$. As to the peculiar solution $\delta\varphi_S$ sourced by the gauge field, we also use the following quick estimation [45]

$$\delta\varphi_S \sim \frac{\sum_{n=1}^{\mathcal{N}} \delta_{EB}^{(n)} - \sum_{m=1}^{\mathcal{M}} \delta_{EB}^{(m)}}{3 f \beta H^2} \, . \tag{3.75}$$

As a result, defining the dimensionlsss power spectrum of the curvature perturbation Δ_ζ by

$$\langle \hat{\zeta}(\boldsymbol{k}) \hat{\zeta}(\boldsymbol{k}') \rangle \equiv (2\pi)^3 \delta^3(\boldsymbol{k} + \boldsymbol{k}') \frac{2\pi^2}{k^3} \Delta_\zeta(k) \, , \tag{3.76}$$

Δ_ζ is approximated as

$$\Delta_\zeta = \Delta_{\zeta v} + \Delta_{\zeta s} \simeq \left(\frac{H^2}{2\pi \dot{\varphi}} \right)^2 + \left(\frac{\sigma}{3 f \beta H \dot{\varphi}} \right)^2 \, , \tag{3.77}$$

where $\sigma^2 \equiv \langle (\sum_{n=1}^{\mathcal{N}} \delta_{EB}^{(n)} - \sum_{m=1}^{\mathcal{M}} \delta_{EB}^{(m)})^2 \rangle$ is the variance of $E_i B_i$. Since the different contribution to the two point functions of $\delta_{EB}^{(i)}$, $\delta_{EB}^{(j)}$ $(i \neq j)$ adds incoherently $\langle \delta_{EB}^{(i)} \delta_{EB}^{(j)} \rangle = 0$, we have

$$\sigma^2 = \sum_{i=1}^{\mathcal{N}+\mathcal{M}} \langle (\delta_{EB}^{(i)})^2 \rangle \simeq \sum_{i=1}^{\mathcal{N}+\mathcal{M}} \langle E_i^{(i)} B_i^{(i)} \rangle^2 \qquad (\xi \gtrsim 1) \, . \tag{3.78}$$

Therefore, in contrast to the correlation function of tensor modes, that of scalar modes are suppressed by a factor $\mathcal{N}$, $\mathcal{M}$ at large ξ.

Now, let us numerically show the blue tensor spectra generated by the inflationary scenario presented in this subsection. For an inflaton's potential, we use the Starobinsky type (3.53) and the same parameter values as in the previous Sec.3.4. Here again, we keep the conditions (3.54) and (3.55) in order to satisfy the constraint from the CMB observation. The background equation of motion for the axion reads

$$\ddot{\varphi} + 3H\dot{\varphi} + V_\varphi = \frac{1}{f} \left(\sum_{n=1}^{\mathcal{N}} \langle E_i^{(n)} B_i^{(n)} \rangle - \sum_{m=1}^{\mathcal{M}} \langle E_i^{(m)} B_i^{(m)} \rangle \right) \, , \tag{3.79}$$

where we introduce the back-reaction effect of gauge fields given by (3.74). We numerically plot the time evolution of ξ in Fig.3.5. We can see that in the late time of inflationary stage the axion moves more slowly due to the dissipation of gauge quanta. Then we depict the evolution of the tensor spectra (3.68) (3.69) in the top panel of Fig.3.6 with $\mathcal{N} = 25$, $\mathcal{M} = 5$. Here, we rewrite the tensor spectra as the present energy intensity

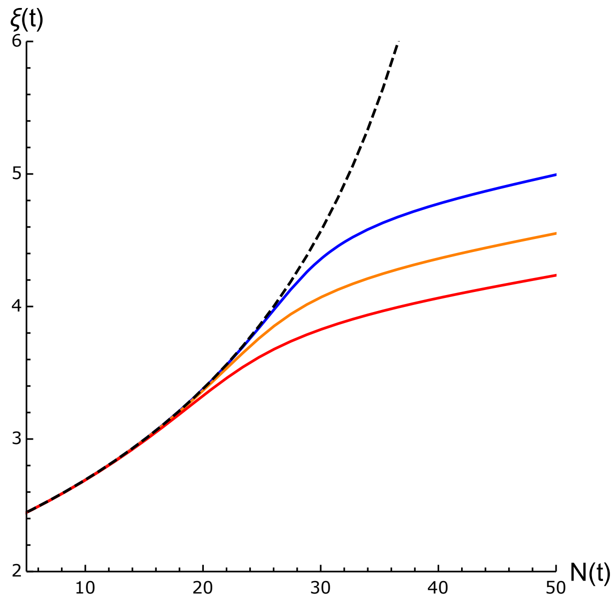

Figure 3.5: We plotted the time evolution of ξ as a function of the number of e-folds $N(t)$ with (solid lines) and without (dashed black line) including back-reaction of the gauge field on the motion of $\bar{\varphi}$ when $\xi_{\rm CMB} \simeq 2.4$. The friction force of gauge field becomes stronger as the number of gauge sector is increasing with $\mathcal{N}+\mathcal{M} = 1$ (blue line), 10 (orange line) and 50 (red line). In this figure, we set $V_0 = 10^{-9}M_p^4$, $\gamma = 0.35$, $f = 10^{-2}M_p$.

of gravitational waves $\Omega^{\pm}_{\rm GW}(f)$ expressed by $\Omega^{\pm}_{\rm GW} \simeq 10^{-6}h_0^{-2}\Delta^{\pm}_h$ with the dimensionless Hubble parameter $h_0 \simeq 0.7$ [86]. We can see that these amplitudes are growing on small scales due to the increase of ξ and testable with LISA, DECIGO/BBO and SKA missions. In this case, from (3.71) the amount of chirality is about $\chi \sim 0.67$. If sizable blue tensor spectra are produced, we can test the parity-violation down to $\chi = 0.08$ by DECIGO or BBO missions [82]. Therefore, we have a chance to search for this small chirality of primordial tensor spectra in detail. Then, in the bottom panel of Fig.3.6, we plot the time evolution of the power-spectrum of curvature perturbation given by (3.77) and a rough upper bound on the constraint of PBHs. We can see that in this figure our model avoids the overproduction of curvature perturbation and satisfies the upper bound on the non-detection of PBHs during inflation by the contribution of many Abelian gauge components[6].

[6] In the case of a less number of Abelian gauge fields, back-reaction of gauge fields becomes smaller and ξ is easy to grow in small scales. Then the generating gravitational waves would be testable with more foreseeable experiments such as ground-based interferometers or LISA project, as is pointed out in the single gauge field case. At that time, however, the amount of scalar power spectrum is also enhanced and an overproduction of PBHs is unavoidable [38].

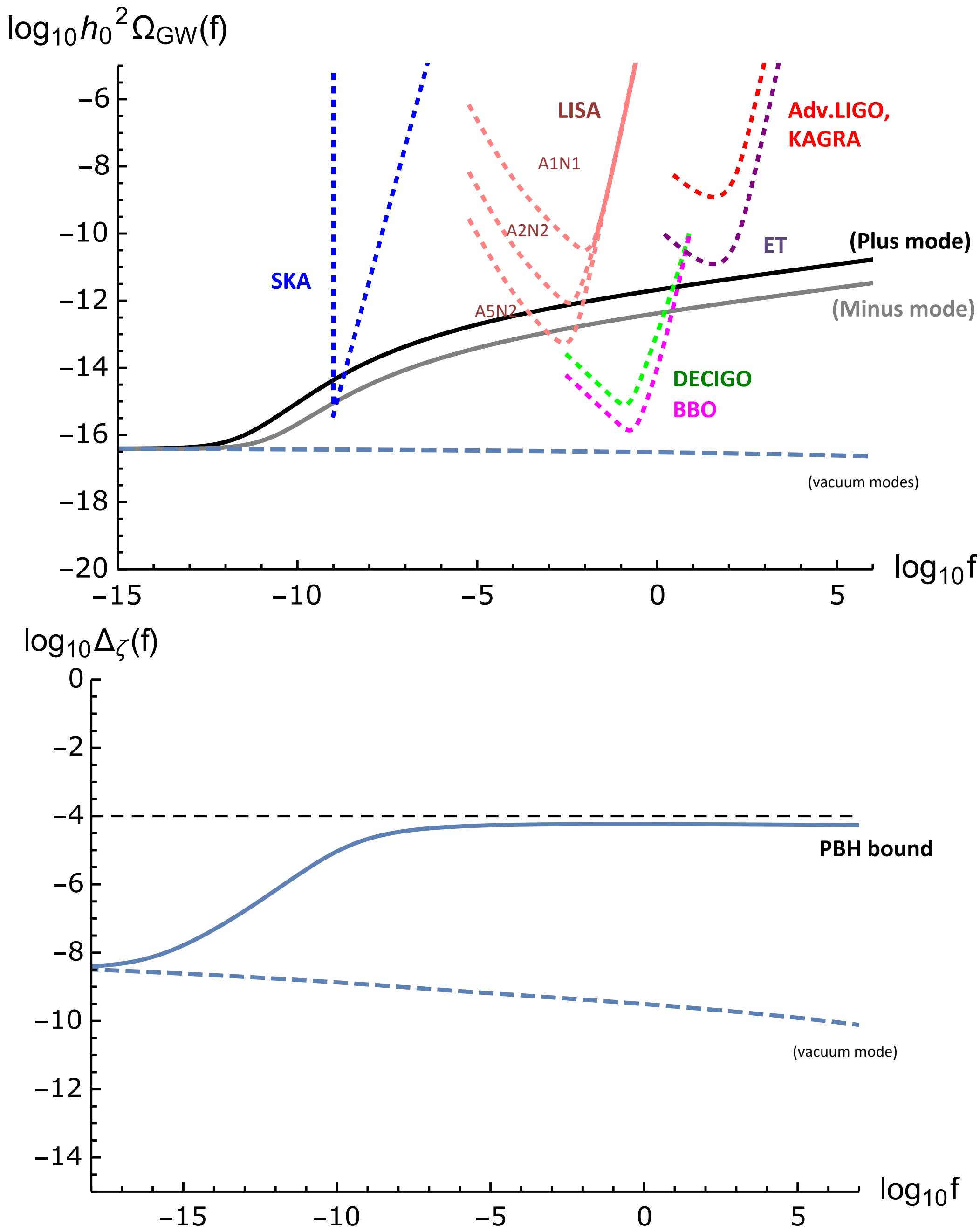

Figure 3.6: (Top panel) : Plot of the both left mode (black line) and right mode (gray line) of tensor power spectra with $\mathcal{N} = 25$, $\mathcal{M} = 5$. We describe the present energy intensity of primordial gravitational waves in this model with $\xi_{\rm CMB} \simeq 2.4$. Compared with the spectrum of vacuum fluctuations (dashed blue line), both helicity modes are amplified with blue-tilted and potentially testable with future space-based interferometers or the pulsar timing array experiment. (Bottom panel) : Plot the power spectrum of curvature perturbation with $\xi_{\rm CMB} \simeq 2.4$ (solid blue line). The dashed blue line represents the power spectrum of vacuum fluctuations. We can see that the amplitude satisfies the constraint on the non-detection of PBHs (black line). In both figures, we set $V_0 = 10^{-9} M_p^4$, $\gamma = 0.35$, $f = 10^{-2} M_p$.

Chapter 4

Primordial GWs Sourced by SU(2) Gauge Field

In the previous chapter, we studied particle production of U(1) gauge field coupled with an axionic inflaton and estimated the parity-violating gravitational waves sourced by gauge quanta at the second order. In this chapter, we present the mechanism of generating chiral tensor modes from an axionic inflation coupled with SU(2) gauge field, originally called Chromo-Natural inflation. This model was historically suggested in order to avoid the range of super-Planckian axion decay constant in the potential energy of successful natural inflation because its actual values are sub-Planckian [95]. Chromo-Natural inflation enables us to occur a long duration of inflationary period due to the friction force of axion-gauge coupling even with the sub-Planckian axion decay constant. Intriguingly, this model can cause an accelerated expansion with broader parameter regions which include apparently different models of gauge-flation or non-canonical single field inflation [96–100].

4.1 Background dynamics

In this section, we present an axionic inflationary model coupled with SU(2) gauge field

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu \tag{4.1}$$

and discuss the background motions. Here, g is its gauge coupling constant and ϵ^{abc} is the Levi-Civita symbol whose components are structure constants of SU(2) gauge field. The action reads

$$S = \int d^4x\sqrt{-\det g_{\mu\nu}}\left[\frac{M_p^2}{2}R - \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\lambda}{4}\frac{\varphi}{f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\right]. \tag{4.2}$$

The dual field strength $\tilde{F}^{a\mu\nu}$ is defined by

$$\tilde{F}^{a\mu\nu} = \frac{1}{2!}\sqrt{-\det g_{\mu\nu}}\epsilon^{\mu\nu\rho\sigma}F^a_{\rho\sigma}\,, \qquad \epsilon^{0123} = g^{0\alpha}g^{1\beta}g^{2\gamma}g^{3\delta}\epsilon_{\alpha\beta\gamma\delta} = \frac{1}{-\det g_{\mu\nu}}\,, \tag{4.3}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the antisymmetric Levi-Civita tensor. In this model, we assume that there exists a large dimensionless coupling constant $\lambda \gg 1$. This parameter is quite important in order to realize an attractor solution of chromo-natural inflation.

Let us consider homogeneous background dynamics in this set-up. As to the metric, we use a spatially flat isotropic metric

$$ds^2 = -N^2 dt^2 + a(t)^2 \delta_{ij} dx^i dx^j \,, \tag{4.4}$$

where N is a lapse function. As a gauge condition, we choose the temporal gauge $A^a_0 = 0$ and take an ansatz

$$A^a_i = a(t)Q(t)\delta^a_i \,, \tag{4.5}$$

which is rotationally invariant by identifying the global SU(2) symmetry with the spatial rotational symmetry. It is known that this configuration is dynamically stable [101]. The background field strength can be deduced as

$$F^a_{0i} = \frac{d(aQ)}{dt}\delta^a_i \,, \qquad F^a_{ij} = g\epsilon^{abc} A^b_i A^c_j = g a^2 Q^2 \epsilon^a_{ij} \,. \tag{4.6}$$

Substituting these configurations into the action, we obtain the Hamiltonian constraint and equation of motion for scale factor

$$3M_p^2 H^2 = \frac{1}{2}\dot{\varphi}^2 + V + \rho_E + \rho_B \,, \tag{4.7}$$

$$M_p^2 \dot{H} = -\left(\frac{1}{2}\dot{\varphi}^2 + \frac{2}{3}(\rho_E + \rho_B)\right) \,, \tag{4.8}$$

where we denote the electric and magnetic energy densities of gauge field by

$$\rho_E \equiv \frac{3}{2}\frac{1}{a^2}\left(\frac{d(aQ)}{dt}\right)^2 \,, \quad \rho_B \equiv \frac{3}{2} g^2 Q^4 \,. \tag{4.9}$$

φ and Q obey the following equations of motions

$$\ddot{\varphi} + 3H\dot{\varphi} + V_\varphi = 3\frac{\lambda}{f}(\dot{Q} + HQ)gQ^2 \,, \tag{4.10}$$

$$\ddot{Q} + 3H\dot{Q} + \left(2H^2 + \dot{H}\right)Q + 2g^2 Q^3 = -\frac{\lambda}{f}\dot{\varphi} g Q^2 \,. \tag{4.11}$$

We can see that in the right hand side of the above equations the effective "magnetic drift" terms appear. Chromo-natural inflation occurs when this friction force becomes relevant in the dynamics. In order to see this, we use slow-roll approximations and rewrite the above two equations of motions as

$$\left(1 + \frac{1}{3}\Lambda^2 m_Q^2\right) 3H\dot{\varphi} + V_\varphi - 3\frac{\lambda}{f} H g Q^3 + 2\left(1 + m_Q^2\right)\frac{\lambda}{f} H g Q^3 = 0 \,, \tag{4.12}$$

$$\left(1 + \frac{1}{3}\Lambda^2 m_Q^2\right) 3H\dot{Q} + \left(1 + \left(\frac{1}{2}\Lambda^2 + 1\right) m_Q^2\right) 2H^2 Q - \frac{1}{3}\Lambda m_Q V_\varphi = 0 \,, \tag{4.13}$$

where we defined two model parameters

$$\Lambda \equiv \frac{\lambda}{f} Q \,, \qquad m_Q \equiv \frac{gQ}{H} \,. \tag{4.14}$$

Here, we are interested in the parameter region of large λ. At this time, if $\Lambda^2 \gg 1$ and $m_Q^2 \gg \Lambda^{-2}$ hold, the equation of Q can be rewritten in terms of its effective potential as

$$3H\dot{Q} + \frac{\partial V_{\rm eff}(Q)}{\partial Q} \simeq 0\,, \qquad V_{\rm eff}(Q) \equiv 3H^2 Q - \frac{1}{\Lambda m_Q} V_\varphi\,. \tag{4.15}$$

We can see that the slow-roll motion of background gauge field $\dot{Q} \simeq 0$ is realized when

$$Q \simeq \left(\frac{f V_\varphi}{3\lambda g H}\right)^{1/3} \tag{4.16}$$

is satisfied. Note that above solution is a minimum value of $V_{\rm eff}$ and therefore it actually becomes an attractor value in this model. Using this relationship, one can also find the slow-roll equation for inflaton

$$\frac{1}{2}\lambda\frac{\dot{\varphi}}{fH} \simeq -\left(m_Q + \frac{1}{m_Q}\right)\,. \tag{4.17}$$

At this time, the energy fraction of electromagnetic gauge field is approximately given by

$$\epsilon_E \equiv \frac{2\rho_E}{3M_p^2 H^2} \simeq \frac{Q^2}{M_p^2} \ll 1\,, \quad \epsilon_B \equiv \frac{2\rho_B}{3M_p^2 H^2} \simeq \frac{g^2 Q^4}{M_p^2 H^2} = m_Q^2\,\epsilon_E \ll 1\,. \tag{4.18}$$

Therefore, the vev of gauge field is almost constant during inflation and it also supports the slow-roll motion of inflaton. Under these slow-roll conditions, Hubble slow-roll parameters are found to be modified as

$$\epsilon_H \simeq \frac{f}{\lambda}\frac{1+m_Q^2}{m_Q}\frac{V_\varphi}{V} \ll 1\,, \tag{4.19}$$

$$\eta_H \simeq \frac{f}{\lambda}\frac{1+m_Q^2}{m_Q}\left(\frac{V_\varphi}{V} - \frac{V_{\varphi\varphi}}{V_\varphi}\right) \ll 1\,. \tag{4.20}$$

Hence the inflationary expansion persists longer as λ increases.

Let us give a concrete background model of the chromo-natural inflation. As for the potential of inflaton, we adopt cosine-type

$$V = \mu^4\left[1 - \cos\left(\frac{\varphi}{f}\right)\right] \tag{4.21}$$

as an example. It is known that such a cosine-type potential is expected to arise from quantum non-perturbative effect such as instanton [102]. Then introducing the following dimensionless parameter

$$y \equiv \left(\frac{\lambda\mu^4}{3g^2 M_p^4}\right)^{2/3}\,, \tag{4.22}$$

slow-roll solutions are represented by

$$H \simeq \frac{\mu^2}{\sqrt{3}M_p}\sqrt{1-\cos\tilde{\varphi}}\,, \tag{4.23}$$

$$Q \simeq M_p\frac{y^{1/4}}{\lambda^{1/2}}\frac{(\sin\tilde{\varphi})^{1/3}}{(1-\cos\tilde{\varphi})^{1/6}}\,, \qquad m_Q \simeq \frac{(\sin\tilde{\varphi})^{1/3}}{\sqrt{y}\,(1-\cos\tilde{\varphi})^{2/3}}\,, \tag{4.24}$$

$$\dot{\varphi} \simeq -2\frac{fH}{\lambda}\,\frac{y\,(1-\cos\tilde{\varphi})^{4/3} + (\sin\tilde{\varphi})^{2/3}}{\sqrt{y}\,(1-\cos\tilde{\varphi})^{2/3}(\sin\tilde{\varphi})^{1/3}}\,, \tag{4.25}$$

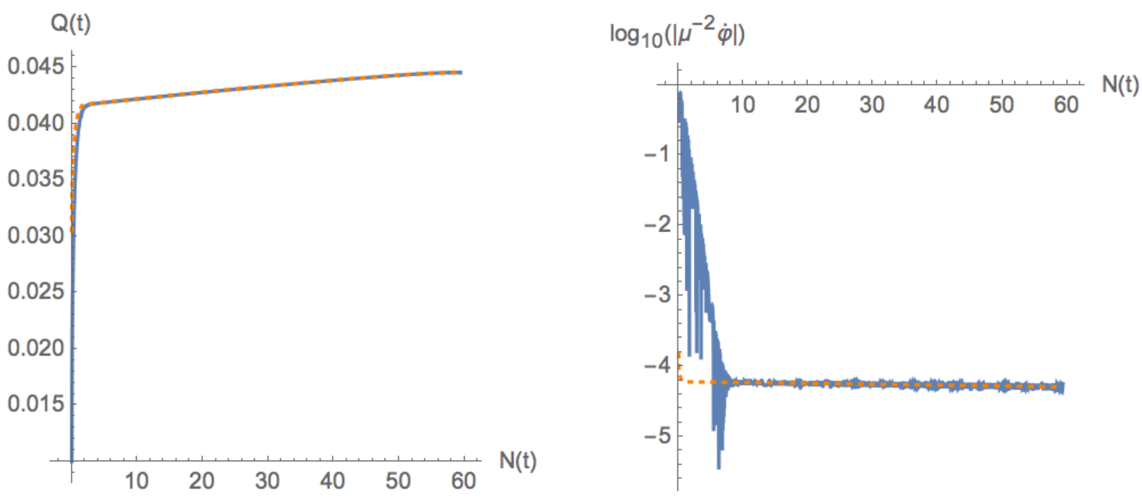

Figure 4.1: Time evolution of Q and $\dot{\varphi}$. Both of them (solid line) quickly settle in slow-roll attractor solutions (dotted lines). In this figure, we set $\lambda = 400$, $f = 10^{-2}M_p$, $y \simeq 0.4$, $\tilde{\varphi} \simeq 0.4$.

where $\tilde{\varphi} \equiv \varphi/f$. Then, the number of e-foldings can be represented by

$$N = -\frac{3\lambda}{2}\sqrt{y}\int \frac{z^4\, dz}{y\, z^4(\sin\tilde{\varphi})^{2/3} + (\sin\tilde{\varphi})^{4/3}}\,, \qquad z \equiv (1-\cos\tilde{\varphi})^{1/3}\ . \tag{4.26}$$

We describe the background motion of gauge field and velocity of inflaton In Fig.4.1. We can see that they actually obey the slow-roll approximate solution with great accuracy. Note that, however, $\dot{\varphi}$ shows small oscillation around its attractor solution, as is also discussed in [60]. We expect that this oscillation derives from the time evolution of Q which makes the effective potential of $\dot{\varphi}$ change. As a result, $\dot{\varphi}$ is tracking its time-dependent minimum value with oscillation during inflationary era.

4.2 Perturbation dynamics

In this section, we consider perturbations in this model. To analyze the dynamics, we need to define perturbed quantities. First, for the metric we use the following variables

$$N = 1 + 2\phi\,, \quad N_i = \partial_i\beta + \beta_i\,, \quad q_{ij} = a(t)^2(\delta_{ij} + \gamma_{ij})\,, \tag{4.27}$$

where the spatial metric is further decomposed as

$$\gamma_{ij} = -2\psi\delta_{ij} + 2E_{,ij} + 2W_{(i,j)} + h_{ij}\ . \tag{4.28}$$

Note that β_i and W_i are transverse vectors and h_{ij} is a transverse traceless tensor. We choose the flat-slicing gauge

$$\psi = E = W_i = 0\ . \tag{4.29}$$

Next, we decompose the non-Abelian gauge field as

$$A_0^a = a(t)\left[\partial_a Y + Y_a\right] \ , \tag{4.30}$$
$$A_i^a = a(t)\left[(Q(t) + \delta Q)\delta_{ai} + \epsilon_{iac}(\partial_c U + U_c) + \partial_i(\partial_a M + M_a) + T_{ai}\right] \ , \tag{4.31}$$

where Y_a , U_a and M_a are transverse vectors and T_{ai} is a transverse traceless tensor. Here, we treated the index a as if it were the spatial index. This is allowed because the simultaneous transformation of SU(2) and the rotation SO(3) keep the symmetry of the background. Remarkably, not only scalar and vector but also tensor perturbations are contained. As usual, scalar, vector, and tensor perturbations are decoupled from each other at the linear level. We treat that Y and Y_a as non-dynamical variables.

Due to SU(2) gauge symmetry, there is the following gauge transformation

$$A_\mu^a \longrightarrow A_\mu^a + g^{-1}\partial_\mu \alpha^a + \epsilon^{abc} A_\mu^b \alpha^c \ , \tag{4.32}$$

where α^a are gauge parameters. We can eliminate three valuables using this gauge freedom. We fix the gauge by setting

$$M = M_a = 0 \ . \tag{4.33}$$

Finally, we decompose the (pseudo) scalar field into the background and perturbation variable

$$\varphi = \bar{\varphi}(t) + \delta\varphi \ . \tag{4.34}$$

4.2.1 Tensor mode analysis

Let us analyze the dynamics of tensor perturbations in this model. Using a new variable $\psi_{ij} \equiv M_p a(\tau) h_{ij}/2$, the quadratic action $S_{\rm EH}$ for tensor perturbations is given by

$$\delta S_{\rm EH} = \frac{1}{2}\int d\boldsymbol{x} d\tau \left[\psi'^{ij}\psi'_{ij} - \psi^{ij,k}\psi_{ij,k} - 2\left(\frac{a''}{a} - \left(\frac{a'}{a}\right)^2\right)\psi^{ij}\psi_{ij}\right] \ , \tag{4.35}$$

where a prime represents a derivative with respect to the conformal time τ . We also have contributions to the quadratic action for tensor perturbations from the (pseudo) scalar actions as

$$\delta S_{\rm axion} = \int d\boldsymbol{x} d\tau \left(-a^2\psi^{ij}\psi_{ij}\right)\left[\frac{1}{2a^2}\bar{\varphi}' - V(\bar{\varphi})\right] \ . \tag{4.36}$$

Moreover, using a new variable $t_{ai} \equiv a(\tau) T_{ai}$, we obtain the quadratic actions for the tensor perturbations from the gauge sector

$$\begin{aligned}\delta S_{\rm gauge} \ = \ & -\frac{1}{2}\int d\boldsymbol{x} d\tau \Bigg[-\frac{(aQ)'^2}{M_p^2 a^2}\psi^{ij}\psi_{ij} + \frac{3a^2g^2Q^4}{M_p^2}\psi^{ij}\psi_{ij} - t_i'^a t_i'^a + t_{i,j}^a t_{i,j}^a - 8agQ\epsilon^{abi}t_j^b t_{j,i}^a \\ & + \frac{16}{M_p}\left(\frac{(aQ)'}{a}\psi^{ij}t'_{ij} - agQ^2\psi^{jm}\epsilon_{ij}^a t_{m,i}^a - a^2g^2Q^3\psi^{ij}t_{ij}\right)\Bigg]\end{aligned} \tag{4.37}$$

and that from the Chern-Simons sector

$$\delta S_{\rm CS} = -\int d\boldsymbol{x} d\tau \frac{1}{2}\frac{\lambda}{f}\bar{\varphi}'\left(\epsilon^{ijk}t_i^a t_{k,j}^a - agQt^{ij}t_{ij}\right) \ . \tag{4.38}$$

Using the slow-roll parameters and the following approximate scale factor

$$a(\tau) \simeq -\frac{1}{H\tau} \,, \tag{4.39}$$

we can write the total quadaratic action for tensor perturbations at the second order as

$$\begin{aligned}
\delta S_{\rm tensor} &\equiv \delta S_{\rm EH} + \delta S_{\rm scalar} + \delta S_{\rm gauge} + \delta S_{\rm CS} \\
&= \frac{1}{2}\int d^3\boldsymbol{x} d\tau \left[\psi'^{ij}\psi'_{ij} - \psi^{ij,k}\psi_{ij,k} + \frac{2}{\tau^2}\psi^{ij}\psi_{ij}\right] \\
&+ \frac{1}{2}\int d^3\boldsymbol{x} d\tau \left[t_i'^a t_i'^a - t^a_{i,j}t^a_{i,j} + \frac{2m_Q\xi}{\tau^2}t^a_i t^a_i - \frac{2}{\tau}\left(m_Q - \xi\right)\epsilon^{ijk}t^a_i t^a_{k,j}\right] \\
&+ \int d^3\boldsymbol{x} d\tau \left[\frac{2\sqrt{\epsilon_E}}{\tau}\psi^{ij}t'_{ij} - \frac{2\sqrt{\epsilon_B}}{\tau}\psi^{jm}\epsilon^a_{ij}t^a_{m,i} + \frac{2\sqrt{\epsilon_B}m_Q}{\tau^2}\psi^{ij}t_{ij} + \frac{\epsilon_E - \epsilon_B}{\tau^2}\psi^{ij}\psi_{ij}\right] \,,
\end{aligned} \tag{4.40}$$

where we defined a new variable

$$\xi \equiv \frac{\lambda}{2}\frac{\dot{\varphi}}{fH} \simeq -(m_Q + m_Q^{-1}) \,. \tag{4.41}$$

Next, we expand each canonical variable as Fourier mode

$$\psi_{ij}(\boldsymbol{x},\tau) = \sum_{A=\pm}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\hat{\psi}^A_k(\tau)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}e^A_{ij} \,, \tag{4.42}$$

$$t_{ij}(\boldsymbol{x},\tau) = \sum_{A=\pm}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\hat{t}^A_k(\tau)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}e^A_{ij} \,, \tag{4.43}$$

where $e^{\pm}_{ij}(\hat{\boldsymbol{k}})$ is the circular polarization tensor which satisfies the usual unitary conditions and a helicity eigen state defined by $ik^i\epsilon^a_{ij}e^{\pm}_{jm}(\hat{\boldsymbol{k}}) = \pm k e^{\pm}_{am}(\hat{\boldsymbol{k}})$. For our convenience, from now on we omit the symbol " ^ " in the representation of operators. Now, we introduce the following dimensionless time variable

$$x \equiv -k\tau \,. \tag{4.44}$$

Using the following background solutions

$$a(\tau) = \frac{k}{Hx} \,, \tag{4.45}$$

$$Q' = 0 \,, \tag{4.46}$$

$$\frac{\lambda}{f}\varphi' = -2\frac{1+m_Q^2}{m_Q}Ha \,, \tag{4.47}$$

we obtain the following equations of motions for tensor perturbations

$$\frac{d^2\psi^{\pm}_{\boldsymbol{k}}}{dx^2} + \left(1 - \frac{2}{x^2}\left(1 + \epsilon_E(1 - m_Q^2)\right)\right)\psi^{\pm}_{\boldsymbol{k}} = 2\sqrt{\epsilon_E}\left(\frac{1}{x}\frac{dt^{\pm}_{\boldsymbol{k}}}{dx} + \frac{m_Q(m_Q \mp x)}{x^2}t^{\pm}_{\boldsymbol{k}}\right) \,, \tag{4.48}$$

$$\frac{d^2t^{\pm}_{\boldsymbol{k}}}{dx^2} + \left(1 + \frac{m}{x^2} \mp \frac{m_t}{x}\right)t^{\pm}_{\boldsymbol{k}} = -2\sqrt{\epsilon_E}\left(\frac{d}{dx}\left(\frac{\psi^{\pm}_{\boldsymbol{k}}}{x}\right) - \frac{m_Q(m_Q \mp x)}{x^2}\psi^{\pm}_{\boldsymbol{k}}\right) \,, \tag{4.49}$$

where we defined

$$m \equiv 2(1 + m_Q^2) , \tag{4.50}$$

$$m_t \equiv 2\left(2m_Q + \frac{1}{m_Q}\right) . \tag{4.51}$$

From these equations, we notice that the dynamics of tensor perturbations depends on the helicity. For the metric perturbations, we can neglect the ϵ_E correction term stemmed from gauge interactions in the left hand side of (4.48). Thus, using the Green function method, the metric mode function is given by

$$\psi^{\pm}_{\boldsymbol{k}}(x) \simeq \psi^{\pm}_{\mathrm{vac}\boldsymbol{k}}(x) + \int_0^\infty dx' G(x, x') J^{\pm}_k(x') , \tag{4.52}$$

$$J^{\pm}_k(x') \equiv 2\sqrt{\epsilon_E}\left(\frac{1}{x'}\partial_{x'} + \frac{m_Q(m_Q \mp x')}{x'^2}\right) t^{\pm}_{\boldsymbol{k}}(x') . \tag{4.53}$$

where $\psi^{\pm}_{\mathrm{vac}\boldsymbol{k}}(x)$ is a vacuum mode fluctuation in de Sitter space,

$$\frac{d^2\psi^{\pm}_{\mathrm{vac}\boldsymbol{k}}}{dx^2} + \left(1 - \frac{2}{x^2}\right)\psi^{\pm}_{\mathrm{vac}\boldsymbol{k}} = 0 \tag{4.54}$$

and $G(x, x')$ is its retarded Green function. Thus, we can see that the contribution of gauge fields as well as vacuum metric fluctuations can produce gravitational waves. Hence, we need to know the dynamics of gauge field perturbations. For simplicity, we ignore the slow-roll suppressed interactions in the right hand side of equation

$$\frac{d^2 t^{\pm}_{\boldsymbol{k}}}{dx^2} + \left(1 + \frac{m}{x^2} \mp \frac{m_t}{x}\right) t^{\pm}_{\boldsymbol{k}} = 0 . \tag{4.55}$$

From this free equation, we can see that $t^{+}_{\boldsymbol{k}}$ has a tachyonic mass in the following time interval

$$\frac{1}{2}(m_t - \sqrt{m_t^2 - 4m}) < x < \frac{1}{2}(m_t + \sqrt{m_t^2 - 4m}) . \tag{4.56}$$

Since this instability occurs near the horizon-crossing and has a sufficiently large growth rate, we can safely neglect the backreaction of metric perturbations. It is easy to see that the interval (4.56) depends on m_Q and gets minimum value when $m_Q \approx 0.8$. On the other hand, $t^{-}_{\boldsymbol{k}}$ has no instability and its effect on the metric perturbation is negligible as we show later. Thus, we can expect that the dynamics of m_Q determines the growth rate of one helicity mode of gauge fields, and hence the amount of chiral gravitational waves is provided.

Let us focus on the t^{+}_{k} mode. It is known that solutions are given by Whittaker functions

$$t^{+}_k(x) = \frac{1}{\sqrt{2k}}\Big(C_1 M_{\kappa,\mu}(2ix) + C_2 W_{\kappa,\mu}(2ix)\Big) , \tag{4.57}$$

where C_1 and C_2 are constants and

$$\kappa \equiv i\frac{m_t}{2} , \qquad \mu^2 \equiv \frac{1}{4} - m . \tag{4.58}$$

In the sub-horizon limit $x \to \infty$, these functions have asymptotic expansion

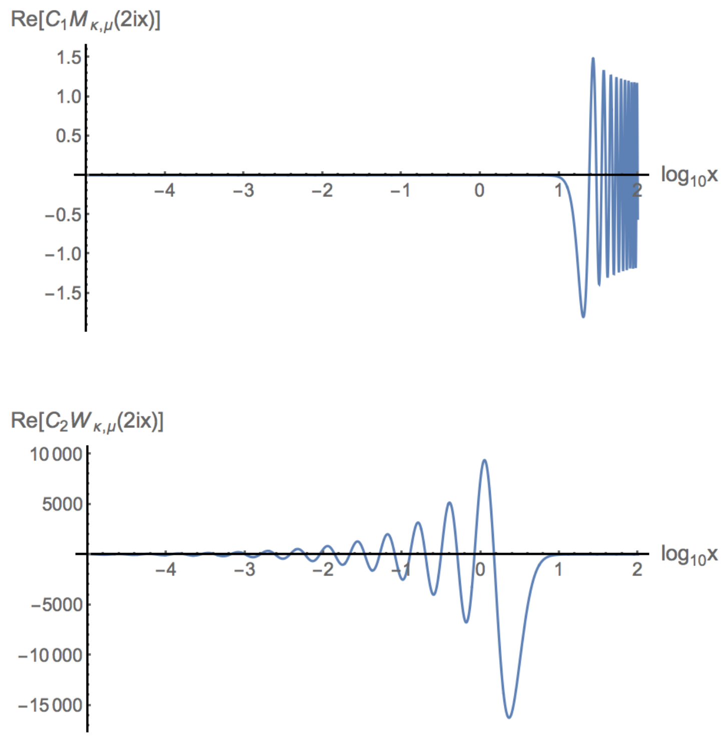

Figure 4.2: Time evolution of Whittaker functions. Here, $x = -k\tau$ is a dimensionless time variable and time flows from right to left. Compared with Whittaker M-function, Whittaker W-function experiences much amplifications around horizon crossing. In this figure, we set $m_Q = 5$.

$$M_{\kappa,\mu}(2ix) \simeq \frac{\Gamma(1+2\mu)}{\Gamma(\frac{1}{2}+\mu-\kappa)}(2i)^{-\kappa}e^{i(x+i\kappa\ln x)} + \frac{\Gamma(1+2\mu)}{\Gamma(\frac{1}{2}+\mu+\kappa)}(-1)^{\frac{1}{2}+\mu-\kappa}(2i)^{\kappa}e^{-i(x+i\kappa\ln x)}\,, \tag{4.59}$$

$$W_{\kappa,\mu}(2ix) \simeq (2i)^{\kappa}e^{-i(x+i\kappa\ln x)}\,. \tag{4.60}$$

In order for the mode function to describe the BD vacuum, the constants C_1 and C_2 should be

$$C_1 = \frac{\Gamma(\frac{1}{2}+\mu-\kappa)}{\Gamma(1+2\mu)}(2i)^{\kappa}\,, \qquad C_2 = -\frac{\Gamma(\frac{1}{2}+\mu-\kappa)}{\Gamma(\frac{1}{2}+\mu+\kappa)}(2i)^{\kappa}(-1)^{\frac{1}{2}+\mu-\kappa}\,. \tag{4.61}$$

In Fig.4.2, one can numerically check that near the horizon crossing Whittaker W-function grows exponentially and starts to decay in the super-horizon regime. Hence, it causes the tachyonic instability of gauge field around horizon-crossing. On the other hand, Whittaker M-function continues to decrease around and after the horizon crossing, so that it is irrelevant for the source of gravitational waves. As a result, we can approximate t_k^+ as

$$t_k^+(x) \simeq \frac{C_2}{\sqrt{2k}}W_{\kappa,\mu}(2ix) \tag{4.62}$$

in the integration at super-horizon regime (4.52) . In the computation of integration, we can perform it by using the following identities [61]

$$\int dx x^n e^{ix} W_{\kappa,\mu}(2ix) = \frac{x^{n+1} G^{2,2}_{2,3}\left(2ix \left| \begin{array}{ccc} -n, & 1+\kappa & \\ \frac{1}{2}-\mu, & \frac{1}{2}+\mu, & -1-n \end{array} \right.\right)}{\Gamma(\frac{1}{2}-\kappa-\mu)\Gamma(\frac{1}{2}-\kappa+\mu)} , \tag{4.63}$$

$$\int dx x^n e^{-ix} W_{\kappa,\mu}(2ix) = x^{n+1} G^{2,1}_{2,3}\left(2ix \left| \begin{array}{ccc} -n, & 1-\kappa & \\ \frac{1}{2}-\mu, & \frac{1}{2}+\mu, & -1-n \end{array} \right.\right) , \tag{4.64}$$

where G is the Meijer G-function. Thus, we get the two-point function of gravitational waves

$$\langle \hat{h}_\pm(\boldsymbol{k}) \hat{h}_\pm(\boldsymbol{k}') \rangle \equiv (2\pi)^3 \delta^3(\boldsymbol{k}+\boldsymbol{k}') \frac{2\pi^2}{k^3} \Delta^\pm_h(k) \tag{4.65}$$

with each helicity mode. For the stable circular polarization state, we obtain the conventional spectrum

$$\Delta^-_h(k) \simeq \frac{H^2}{\pi^2 M_p^2} . \tag{4.66}$$

For the other circular polarization state, due to the enhancement of the gauge field fluctuations, we have

$$\Delta^+_h(k) \simeq \frac{H^2}{\pi^2 M_p^2} \left[1 + 8|C_2|^2 \epsilon_E \left| \mathcal{I}_0(m_Q) - m_Q \mathcal{I}_1(m_Q) + m_Q^2 \mathcal{I}_2(m_Q) \right|^2 \right] , \tag{4.67}$$

where we defined

$$\begin{aligned} \mathcal{I}_0(m_Q) = \frac{i\,\Gamma(-\frac{3}{2}-\mu)\Gamma(-\frac{3}{2}+\mu)}{2} \Bigg(& \frac{(\frac{1}{4}-\mu^2-4\kappa)(\frac{9}{4}-\mu^2)+8\kappa(1+\kappa)}{\Gamma(1-\kappa)} \\ & - \frac{(\frac{1}{4}-\mu^2+4\kappa)(\frac{9}{4}-\mu^2)-8\kappa(1-\kappa)}{\Gamma(\frac{1}{2}-\mu-\kappa)\Gamma(\frac{1}{2}+\mu-\kappa)\Gamma(-\kappa)^{-1}} \Bigg) , \end{aligned} \tag{4.68}$$

$$\mathcal{I}_1(m_Q) = \frac{\Gamma(-\frac{1}{2}-\mu)\Gamma(-\frac{1}{2}+\mu)}{2} \left(\frac{\frac{1}{4}-\mu^2-2\kappa}{\Gamma(1-\kappa)} + \frac{\frac{1}{4}-\mu^2+2\kappa}{\Gamma(\frac{1}{2}-\mu-\kappa)\Gamma(\frac{1}{2}+\mu-\kappa)\Gamma(-\kappa)^{-1}} \right) , \tag{4.69}$$

$$\mathcal{I}_2(m_Q) = i\,\Gamma(-\tfrac{1}{2}-\mu)\Gamma(-\tfrac{1}{2}+\mu) \left(\frac{1-2(1+\kappa)(\frac{1}{4}-\mu^2)}{\Gamma(-\kappa)} + \frac{1-2(1-\kappa)(\frac{1}{4}-\mu^2)}{\Gamma(\frac{1}{2}-\mu-\kappa)\Gamma(\frac{1}{2}+\mu-\kappa)\Gamma(1-\kappa)^{-1}} \right) . \tag{4.70}$$

We plotted the ratio of spectra (4.66) and (4.67) in Fig. 4.3 . We can see that the chiral gravitational wave is exponentially enhanced as m_Q increases.

4.2.2 Scalar mode analysis

In this section, we present the dynamics of scalar perturbations and compute the two-point function of curvature perturbation. Let us derive the quadratic action for scalar perturbations. The Einstein-Hilbert action gives

$$\delta S_{\rm EH} = \int d\tau d\boldsymbol{x} \left[-12 a^4 H^2 \phi^2 - 4 a^2 H \phi \partial^2 \beta \right] , \tag{4.71}$$

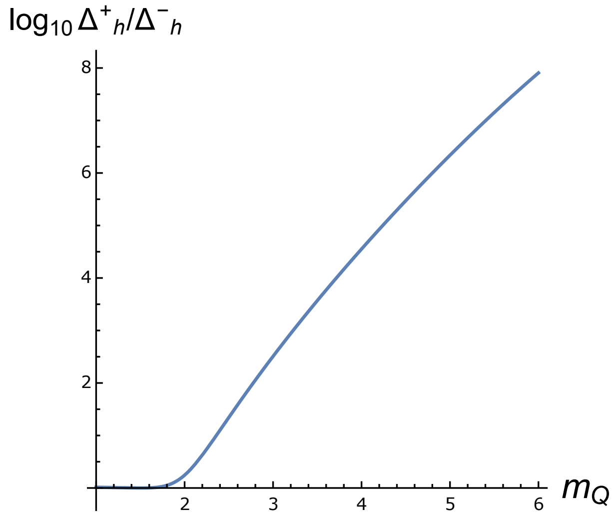

Figure 4.3: The behaviour of the ratio of the GW spectra as a function of m_Q with $Q_{\rm min} = 10^{-2}$. The amplitude grows exponentially as m_Q increases.

where ϕ and β are perturbation of the lapse and shift functions defined in (4.27) . The quadratic action for fluctuation of scalar field is given by

$$\delta S_{\rm axion} = \int d\tau d\boldsymbol{x} \Big[2a^4\dot{\bar\varphi}^2\phi^2 - 2\phi(a^3\dot{\bar\varphi}\delta\varphi' + a^4 V_{\bar\varphi}\delta\varphi) + a^2\dot{\bar\varphi}\beta\partial^2\delta\varphi \\ +\frac{1}{2}a^2\delta\varphi'^2 + \frac{1}{2}a^2\delta\varphi\partial^2\delta\varphi - \frac{1}{2}a^4 V_{\bar\varphi\bar\varphi}\delta\varphi^2 \Big] \,. \tag{4.72}$$

For the quadratic action $\delta S_{\rm gauge}$, we split it into two parts $\delta S^{\rm I}_{\rm gauge} + \delta S^{\rm II}_{\rm gauge}$: the former one includes only gauge field fluctuations

$$\delta S^{\rm I}_{\rm gauge} = \int d\tau d\boldsymbol{x} \Big[\frac{3}{2}(a\delta Q)'^2 - (aU)'\partial^2(aU)' + \frac{1}{2}a^4 Y\partial^4 Y - a^6 g^2 Q^2 Y\partial^2 Y \\ -a^2 Y\left(\partial^2(a\delta Q)' + 2agQ\partial^2(aU)' - 2a^3\frac{(a\dot{Q})}{a}g\partial^2 U\right) \\ +a^2\delta Q\partial^2\delta Q - a^2\delta U\partial^4\delta U - 6a^3 gQ\delta Q\partial^2 U - a^4 g^2 Q^2\left(9\delta Q^2 - 2U\partial^2 U\right)\Big] \,, \tag{4.73}$$

and the latter one has the interaction with other fields

$$\delta S^{\rm II}_{\rm gauge} = \int d\tau d\boldsymbol{x} \Big[4a^4\rho_E\phi^2 - \frac{2}{3}a^2\rho_B\beta\partial^2\beta \\ +2\beta a\left(\frac{(a\dot{Q})}{a}a\partial^2\delta Q + gQ^2(a\partial^2 U)' + a^3 g^2 Q^3\partial^2 Y - \frac{(a\dot{Q})}{a}a^2 gQ\partial^2 U\right) \\ -2a^2\phi\frac{(a\dot{Q})}{a}\left(3(a\delta Q)' - a^2\partial^2 Y\right) - 4a^3\phi gQ^2\left(3agQ\delta Q + \partial^2 U\right)\Big] \,. \tag{4.74}$$

Finally, the quadratic action $\delta S_{\rm CS}$ reads

$$\delta S_{\rm CS} = -\int a d\tau d\boldsymbol{x} \left[\frac{\lambda \dot{\varphi}}{f} a^2 \left(2\delta Q \partial^2 U + agQ(3\delta Q^2 - U\partial^2 U) \right) \right.$$
$$\left. + \frac{\lambda}{f} a \left(Y a^2 g Q^2 \partial^2 \delta\varphi + 3agQ^2 \delta\varphi' \delta Q - 2(a\dot{Q})\delta\varphi \partial^2 U \right) \right] . \tag{4.75}$$

We list up the full equations of motion used for numerical calculations. We can simply set $\phi = \beta = 0$ because these contributions are suppressed by slow-roll parameters. This truncation is known to be valid from the previous works on the stability of Chromo-Natural inflation [60,61]. Integrating out Y, in Fourier space we get

$$aY_{\boldsymbol{k}} = \frac{H}{2m_Q^2 + k^2\tau^2} \left(-\frac{\lambda}{f} Q m_Q \tau^2 a \delta\varphi_{\boldsymbol{k}} + \tau^3 (a\delta Q_{\boldsymbol{k}})' - 2m_Q (aU_{\boldsymbol{k}})'\tau^2 - 2m_Q \tau a U_{\boldsymbol{k}} \right) . \tag{4.76}$$

After the elimination of Y, in terms of the canonical fields Δ_i defined by

$$\delta\varphi_{\boldsymbol{k}} = \frac{\Delta_\varphi}{a} , \quad \delta Q_{\boldsymbol{k}} = \frac{\Delta_Q}{\sqrt{2}a} ,$$
$$U_{\boldsymbol{k}} = \frac{1}{\sqrt{2}ka} \left(\frac{agQ}{k} \Delta_Q + \sqrt{\frac{k^2 + 2a^2g^2Q^2}{k^2}} \Delta_U \right) , \tag{4.77}$$

we can derive the following equation of motion for scalar perturbations

$$\frac{d^2\Delta_i}{dx^2} + 2K_{ij}\frac{d\Delta_j}{dx} + \left(\Omega_{ij}^2 + \frac{dK_{ij}}{dx} \right) \Delta_j = 0 \tag{4.78}$$

with

$$i = (\varphi,\ Q,\ U) , \quad K_{\varphi\varphi} = K_{QQ} = K_{UU} = 0 , \quad K_{QU} = -K_{UQ} = 0 , \tag{4.79}$$

$$K_{\varphi Q} = -K_{Q\varphi} = \frac{\Lambda m_Q}{\sqrt{2}x} , \quad K_{U\varphi} = -K_{\varphi U} = \frac{\Lambda m_Q^2}{\sqrt{2}x\sqrt{x^2 + 2m_Q^2}} , \tag{4.80}$$

$$\Omega_{\varphi\varphi}^2 = 1 - \frac{2}{x^2}\left(1 - \frac{V_{\varphi\varphi}}{2H^2} \right) + \frac{\Lambda^2 m_Q^2}{x^2 + 2m_Q^2} , \quad \Omega_{QQ}^2 = 1 - \frac{2}{x^2}(1 - m_Q^2) , \tag{4.81}$$

$$\Omega_{UU}^2 = 1 + \frac{2m_Q^2}{x^2(x^2 + 2m_Q^2)} - 2\frac{x^2 - m_Q^2}{(x^2 + 2m_Q^2)^2} , \quad \Omega_{\varphi Q}^2 = \Omega_{Q\varphi}^2 = -\frac{3\Lambda m_Q}{\sqrt{2}x^2} , \tag{4.82}$$

$$\Omega_{U\varphi}^2 = \Omega_{\varphi U}^2 = \sqrt{2}\Lambda \frac{x^4 + 2m_Q^2 x^2 + 3m_Q^4}{x^2(x^2 + 2m_Q^2)^{3/2}} , \quad \Omega_{QU}^2 = \Omega_{UQ}^2 = \frac{2}{m_Q x^2}\sqrt{x^2 + 2m_Q^2} . \tag{4.83}$$

It should be noted that the coupled equations cannot be diagonalized straightforwardly because this system has kinetic mixing between inflaton and gauge field via topological Chern-Simons term. Due to this mixing effect, the quantization have to be done by promoting Δ_i into operators as

$$\Delta_i = D_{ij} a_j + \text{h.c.} \tag{4.84}$$

where $a_i(a_i^\dagger)$ are independent sets of annihilation (creation) operators. Since all modes are decoupled at infinite past, it is reasonable to assume that the diagonal parts of D_{ij} are in Bunch-Davies vacuum while the non-diagonal parts vanish at the initial time $x = x_{\rm in}$:

$$D_{i=j}(x_{\rm in}) = \frac{1}{\sqrt{2k}}\,, \qquad \frac{dD_{i=j}}{dx}(x_{\rm in}) = -i\sqrt{\frac{k}{2}}\,, \qquad D_{i\neq j}(x_{\rm in}) = \frac{dD_{i\neq j}}{dx}(x_{\rm in}) = 0\,. \tag{4.85}$$

To begin with, we have to check the stability of dynamics of scalar perturbations, especially around the region $m_Q \gtrsim \mathcal{O}(1)$ where sizable chiral gravitational waves are produced (see the previous subsection). For simplicity, we take the background gauge field function $Q(t)$ to be fixed in the following calculations. For the time window $m_Q \ll x \lesssim \Lambda$, the equations of motions for perturbations at the leading order are given by

$$\frac{d^2\Delta_\varphi}{dx^2} + \left(1 + \frac{\Lambda^2 m_Q^2}{x^2}\right)\Delta_\varphi + \frac{\sqrt{2}\Lambda m_Q}{x}\frac{d\Delta_Q}{dx} + \frac{\sqrt{2}\Lambda}{x}\Delta_U \simeq 0\,, \tag{4.86}$$

$$\frac{d^2\Delta_Q}{dx^2} + \Delta_Q - \frac{\sqrt{2}\Lambda m_Q}{x}\frac{d\Delta_\varphi}{dx} \simeq 0\,, \tag{4.87}$$

$$\frac{d^2\Delta_U}{dx^2} + \Delta_U + \frac{\sqrt{2}\Lambda}{x}\Delta_\varphi \simeq 0\,. \tag{4.88}$$

In order to analyze above system, we seek for the solution in the WKB form

$$\Delta_\varphi = C_\varphi(x)e^{iS(x)}\,, \quad \Delta_Q = C_Q(x)e^{iS(x)}\,, \quad \Delta_U = C_U(x)e^{iS(x)}\,. \tag{4.89}$$

Then one can obtain

$$\left(1 + \frac{\Lambda^2 m_Q^2}{x^2} - S_x^2\right)C_\varphi + \frac{\sqrt{2}\Lambda m_Q}{x}iS_x C_Q + \frac{\sqrt{2}\Lambda}{x}C_U \simeq 0\,, \tag{4.90}$$

$$\left(1 - S_x^2\right)C_Q - \frac{\sqrt{2}\Lambda m_Q}{x}iS_x C_\varphi \simeq 0\,, \tag{4.91}$$

$$\left(1 - S_x^2\right)C_U + \frac{\sqrt{2}\Lambda}{x}C_\varphi \simeq 0\,, \tag{4.92}$$

where $S_x \equiv dS/dx$ is an angular frequency and we neglected the time dependence of $C_i(x)$. From (4.90) - (4.92), we find the following relation

$$\left(1 + \frac{\Lambda^2 m_Q^2}{x^2} - \omega^2\right)\left(1 - \omega^2\right) - \frac{2\Lambda^2 m_Q^2}{x^2}\omega^2 - \frac{2\Lambda^2}{x^2} = 0\,, \qquad \omega \equiv S_x \tag{4.93}$$

and hence we have two modes

$$\omega_\pm^2 = \frac{1}{2x^2}\left(2x^2 + 3\Lambda^2 m_Q^2\left(1 \pm \sqrt{1 + \frac{8x^2}{9\Lambda^2 m_Q^4}\left(m_Q^2 + 1\right)}\right)\right)\,. \tag{4.94}$$

Recalling that we are considering the following time interval $x \lesssim \Lambda m_Q$, the plus mode ω_+^2 is approximately given by

$$\omega_+^2 \simeq \frac{3\Lambda^2 m_Q^2}{x^2}\,. \tag{4.95}$$

Apparently this mode is positive and stable. On the other hand for a minus mode ω_-^2 , by expanding the square root in (4.94) with respect to $x^2/(\Lambda^2 m_Q^2)$, we get

$$\omega_-^2 \simeq \frac{m_Q^2 - 2}{3m_Q^2} \,. \tag{4.96}$$

Therefore this mode has the instability in the region $m_Q \lesssim \sqrt{2}$, which is the same result as in the previous works [60, 61]. In order to confirm this instability, we numerically solved the full scalar equations of motions (4.78) and plotted the time evolution of the coefficient $D_{\varphi\varphi}$ with several m_Q values are plotted in Fig.4.4. We can see that in the region $m_Q < \sqrt{2}$ the instability actually occurs. Based on this fact, from now on we consider the parameter region $m_Q \geq \sqrt{2}$ for the evaluation of curvature perturbation.

At super horizon regime, gauge fluctuations have no instability and therefore dilute away. Hence the curvature perturbation in flat slicing is given by

$$\zeta = -\frac{H}{\dot{\varphi}}\delta\varphi \tag{4.97}$$

and we find its dimensionless two-point function as follows

$$\Delta_\zeta = \frac{k^3}{2\pi^2}\frac{H^2}{\dot{\varphi}^2}\left(\left|\frac{D_{\varphi\varphi}}{a}\right|^2 + \left|\frac{D_{\varphi Q}}{a}\right|^2 + \left|\frac{D_{\varphi U}}{a}\right|^2\right) . \tag{4.98}$$

We numerically solved the amplitude of power-spectrum at super-horizon regime and depicted it in Fig.4.5 as a function of $m_Q - \sqrt{2}$. Note that in this figure we decided the Hubble parameter value so as to satisfy the COBE normalization at $m_Q \lesssim 4.5$. In order to estimate the spectral index, we approximate Δ_ζ by the following fitting functions

$$\log_{10}\Delta_\zeta(m_Q) \simeq \begin{cases} -2.95\log_{10}\left[m_Q - \sqrt{2}\right] - 7.85 & (m_Q \lesssim 2.5) \\ 0.95\left(\log_{10}\left[m_Q - \sqrt{2}\right] - 1\right)^2 - 8.80 & (m_Q \gtrsim 2.5) \end{cases} . \tag{4.99}$$

The spectral index is therefore given by

$$ns - 1 = \frac{d\ln\Delta_\zeta}{dN}\frac{dN}{d\ln k}\bigg|_{k=k_*} \simeq \frac{d\ln\Delta_\zeta}{dm_Q}\frac{dm_Q}{dN}\bigg|_{k=k_*} \tag{4.100}$$

with

$$\frac{dm_Q}{dN} = \frac{dm_Q}{d\tilde{\varphi}}\frac{d\tilde{\varphi}}{dN} = -\frac{2}{3}\frac{f}{\lambda}(1+m_Q^2)\left(\tan\tilde{\varphi} - \frac{2\sin\tilde{\varphi}}{1-\cos\tilde{\varphi}}\right) . \tag{4.101}$$

In terms of background values used in the previous subsection, we describe the spectral index as a function of m_Q in Fig.4.6. We can see that the allowed parameter window by CMB data is around $m_Q \gtrsim 4$, where chiral gravitational waves are however too much produced to satisfy the current upper bound on the tensor-to-scalar ratio (see Fig.4.3). This result is consistent with the previous works [61], where they reveal the observational inconsistency of chromo-natural inflation with more samples about parameter value.

This is the negative result of the model of axionic inflation with non-Abelian gauge field. However, it is not necessarily needed for the gauge field to couple with the inflaton.

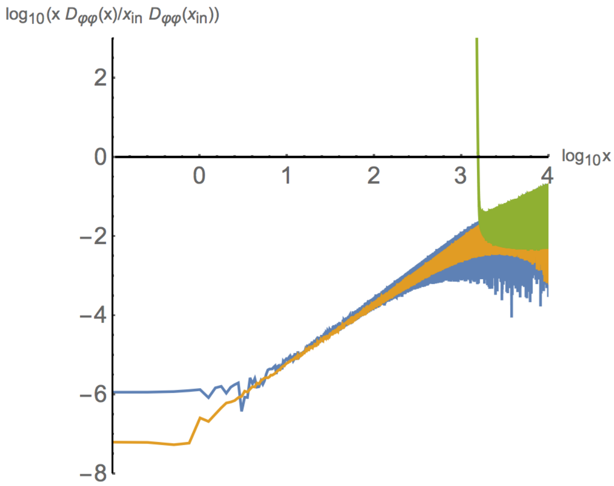

Figure 4.4: In this plot, we show the time evolution of $xD_{\varphi\varphi}$ normalized by initial amplitude with $m_Q = 3$ (blue line), $m_Q = 2$ (orange line) and $m_Q = 1$ (green line). The oscillating feature derives from the mixing effect between inflaton and gauge field. The blue and orange lines are stable, while the green line experiences a rapid enhancement due to the instability. Note that we set $\Lambda = 1.6 \times 10^3$, $x_{\rm in} = 5 \times 10^4$ in this plot.

For example, if the coupled scalar field is a spectator during inflation, we have a chance to circumvent the inconsistency with CMB observations [70]. Also, we can consider the existence of another scalar field which is coupled with the gauge field through its kinetic form. In the next chapter, we discuss the possibility of simultaneously satisfying the CMB constraint and generating observable chiral primordial gravitational waves at smaller scales.

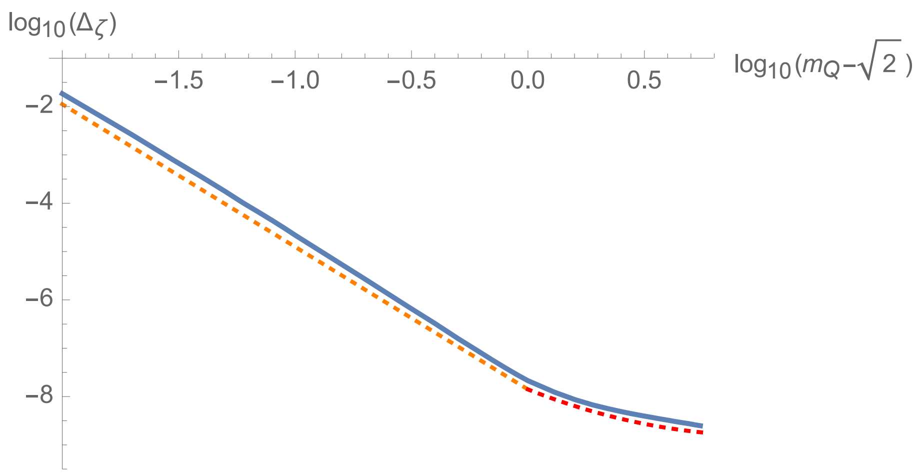

Figure 4.5: The m_Q dependence on the amplitude of scalar power spectrum. The dashed lines are fitting functions: $y \simeq -2.95x - 7.85$ (orange line) and $y \simeq 0.95(x-1)^2 - 8.80$ (red line). Note that we set $\Lambda = 1.6 \times 10^3$, $H = 5 \times 10^{-5} M_p$, $x_{\rm in} = 5 \times 10^4$ in this plot.

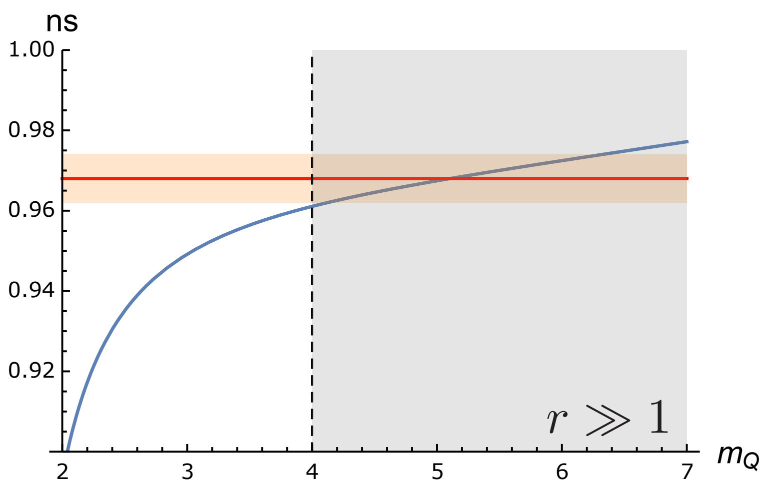

Figure 4.6: The m_Q dependence on the spectral index of scalar power spectrum with $\Lambda = 1.6 \times 10^3$, $H = 5 \times 10^{-5} M_p$. The red and orange contour shows the allowed region by current CMB observations. On the gray band, however, chiral gravitational wave is exponentially produced and it easily excesses current upper bound on the tensor-to-scalar ratio..

Chapter 5

Delayed Chromo-Natural Inflation

As we have mentioned in the introduction, recent works have revealed roles of two types of the coupling between the inflaton and the gauge field separately. From the point of view of supergravity or superstring theory, however, we should incorporate both the dilaton and the axion at the same time. In this chapter, we focus on this possibility and find a novel effect induced by the presence of both fields, which can be probed by primordial gravitational waves [65]. The point is that the gauge kinetic function of the dilaton makes the gauge field increase and controlls the strength of the axion-gauge field interaction. As initial conditions, we choose the gauge field small enough so that we can ignore the effective coupling of the axion to the gauge field in the early stage of inflation. Therefore, the tachyonic instability plays no role when primordial fluctuations on CMB scales are generated. However, the gauge field grows due to the dilaton coupling through the gauge kinetic function until the non-linear effect of gauge field appears in the background dynamics. Eventually, the gauge field settles in an attractor and realizes delayed chromo-natural inflation [36] in the late stage of inflation, Remarkably, sizable chiral gravitational waves are generated only on small scales, whose amplitude can be detectable by future space interferometers and pulsar timing arrays such as DECIGO, LISA and SKA missions.

This chapter is organized as follows. In Sec.5.1, we present an inflation model with a dilaton and an axion, both of which are coupled with a SU(2) gauge field. We then derive equations of motions for the homogeneous fields and study the background dynamics. We show two different inflationary stages are realized in Sec.5.1.1. In Sec.5.2, first we decompose perturbations into scalar, vector, and tensor perturbations and give the gauge conditions. Next, we analyze tensor dynamics and show that chiral gravitational waves are produced on scales smaller than CMB scales. In Sec.5.3, we also discuss the dynamics of scalar perturbations. We estimate the amplitude of curvature perturbations on CMB scales and check the stability of scalar dynamics on small scales. In Sec.5.4, we discuss phenomenological predictions in this model. In the Appendix B, we list equations used in the numerical calculations.

In this chapter, we use the units $\hbar = c = 1$ and $M_{\rm pl} = (8\pi G)^{-1/2} = 1$.

5.1 Inflation model with dilaton and axion coupled to SU(2) gauge field

In this section, we present an inflationary model and derive equations of background motions. Specifically, we consider a dilaton field φ and an axion field σ, both of which are coupled with a SU(2) gauge field A^a_μ. The field strength of the gauge field $F^a_{\mu\nu}$ is defined by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu \,, \tag{5.1}$$

where g is the gauge coupling constant and ϵ^{abc} is the Levi-Civita symbol whose components are structure constants of SU(2) gauge field. The dual field strength tensor $\tilde{F}^{a\mu\nu}$ is defined by (4.3). The action reads

$$\begin{aligned} S &= S_{\rm EH} + S_{\rm dilaton} + S_{\rm axion} + S_{\rm gauge} + S_{\rm CS} \\ &= \int dx^4 \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi) - \frac{1}{2}(\partial_\mu\sigma)^2 - W(\sigma) - \frac{1}{4}I(\varphi)^2 F^{a\mu\nu}F^a_{\mu\nu} - \frac{1}{4}\lambda\frac{\sigma}{f}\tilde{F}^{a\mu\nu}F^a_{\mu\nu}\right] . \end{aligned} \tag{5.2}$$

Here, g is the determinant of the metric $g_{\mu\nu}$ (note that it is not related with the gauge coupling constant), R is the Ricci scalar, and f is the decay constant of axion. Moreover, we introduced a coupling constant of axion to the gauge field λ. We have also introduced the potential functions $V(\varphi)$ and $W(\sigma)$.

As to the metric, we adapt the ADM parametrization

$$ds^2 = -N^2 dt^2 + q_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \,, \tag{5.3}$$

where N is the lapse function and N^i is the shift function, which are Lagrange multipliers of the system. The induced metric q_{ij} on the three dimensional spatial hypersurface is used to raise or lower the index as

$$q^{ik}q_{kj} = \delta^i_j \,, \qquad N^i = q^{ij}N_j \,. \tag{5.4}$$

Thus, the metric can be expressed by

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N_i N^i & N_j \\ N_i & q_{ij} \end{pmatrix} , \qquad g^{\mu\nu} = \begin{pmatrix} -N^{-2} & N^{-2}N^j \\ N^{-2}N^i & q^{ij} - N^{-2}N^i N^j \end{pmatrix} . \tag{5.5}$$

Using these variables, we can rewrite the action (5.2) as

$$S_{\rm EH} = \int dx^4 N\sqrt{q}\left[\frac{1}{2}\left({}^{(3)}R + K_{ij}K^{ij} - K^2\right)\right] , \tag{5.6}$$

$$S_{\rm dilaton} = \int dx^4 N\sqrt{q}\left[\frac{1}{N^2}\left(\frac{1}{2}\dot\varphi^2 - N^i\dot\varphi\varphi_{,i} + \frac{1}{2}(N^i\varphi_{,i})^2\right) - \left(\frac{1}{2}q^{ij}\varphi_{,i}\varphi_{,j} + V(\varphi)\right)\right] , \tag{5.7}$$

$$S_{\rm axion} = \int dx^4 N\sqrt{q}\left[\frac{1}{N^2}\left(\frac{1}{2}\dot\sigma^2 - N^i\dot\sigma\sigma_{,i} + \frac{1}{2}(N^i\sigma_{,i})^2\right) - \left(\frac{1}{2}q^{ij}\sigma_{,i}\sigma_{,j} + W(\sigma)\right)\right] , \tag{5.8}$$

$$S_{\rm gauge} = \int dx^4 N\sqrt{q}\left[\frac{1}{2N^2}I(\varphi)^2 q^{ik}(F^a_{0i} + F^a_{ij}N^j)(F^a_{0k} + F^a_{kl}N^l) - \frac{1}{4}I(\varphi)^2 q^{ik}q^{jl}F^a_{ij}F^a_{kl}\right] , \tag{5.9}$$

$$S_{\rm CS} = \int dx^4 \left[-\frac{1}{2}\lambda\frac{\sigma}{f}\epsilon^{ijk}F^a_{0i}F^a_{jk}\right] . \tag{5.10}$$

Note that Chern-Simons interaction does not include metric variables. Here, q is the determinant of the metric q_{ij}. We defined the extrinsic curvature of a hypersurface

$$K_{ij} \equiv \frac{1}{2N}(\dot{q}_{ij} - 2N_{(i|j)}) \,, \tag{5.11}$$

and the Ricci scalar of the hypersurface

$${}^{(3)}R = (q_{ij,kl} + q_{mn}{}^{(3)}\Gamma^{m(3)}_{ij}\Gamma^{n}_{kl})(q^{ik}q^{jl} - q^{ij}q^{kl}) \,, \tag{5.12}$$

where

$${}^{(3)}\Gamma^{i}_{jk} = \frac{1}{2}q^{il}(q_{lj,k} + q_{lk,j} - q_{jk,l}) \,. \tag{5.13}$$

Let us consider the homogeneous background dynamics in this set-up. For the metric, we use a spatially flat metric

$$N = N(t) \,, \quad N_i = 0 \,, \quad q_{ij} = a(t)^2\delta_{ij} \,. \tag{5.14}$$

After taking the variation of the action, we set $N(t) = 1$. Thus, the time function t becomes the cosmic time. For the dilaton and the axion we take homogeneous configurations $\varphi = \varphi(t)$, $\sigma = \sigma(t)$. As a gauge condition, we choose the temporal gauge

$$A^a_0 = 0 \,. \tag{5.15}$$

We also take an ansatz

$$A^a_i = A(t)\delta^a_i = a(t)Q(t)\delta^a_i \tag{5.16}$$

in the model of Chromo-Natural inflation. Note that $Q(t)$ is a scalar under the rotational transformation and we use this variable later. The field strength $F^a_{\mu\nu}$ can be deduced as

$$F^a_{0i} = \frac{dA}{dt}\delta^a_i \equiv aE(t)\delta^a_i \,, \qquad F^a_{ij} = g\epsilon^{abc}A^b_iA^c_j = gA^2\epsilon^a_{ij} \equiv a^2B(t)\epsilon^a_{ij} \,, \tag{5.17}$$

where we defined electric and magnetic components $E(t)$, $B(t)$. Substituting these configurations into the action, we obtain the following background action

$$S = \int d^4x\frac{a^3}{N}\left[-3\frac{\dot{a}^2}{a^2} + \frac{1}{2}\dot{\varphi}^2 - N^2V + \frac{1}{2}\dot{\sigma}^2 - N^2W + \frac{3}{2}I^2\left(E^2 - N^2B^2\right) - 3N\frac{\lambda}{f}\sigma EB\right] \,, \tag{5.18}$$

where a dot denotes a derivative with respect to the time variable t . Taking the variation with respect to N and setting $N = 1$ after the variation, we obtain the Hamiltonian constraint

$$3H^2 = \frac{1}{2}\dot{\varphi}^2 + V + \frac{1}{2}\dot{\sigma}^2 + W + \rho_E + \rho_B \,, \tag{5.19}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter. Here, we defined the following energy densities of electric and magnetic fields

$$\rho_E \equiv \frac{3}{2}I^2E^2 = \frac{3}{2}I^2\frac{\dot{A}^2}{a^2} \,, \qquad \rho_B \equiv \frac{3}{2}I^2B^2 = \frac{3}{2}I^2\frac{g^2A^4}{a^4} \,. \tag{5.20}$$

The equations for the dilaton, the axion and the gauge field read

$$\ddot{\varphi} + 3H\dot{\varphi} + V_\varphi = 2\frac{I_\varphi}{I}\left(\rho_E - \rho_B\right) \ , \tag{5.21}$$

$$\ddot{\sigma} + 3H\dot{\sigma} + W_\sigma = -3\frac{\lambda}{f}EB \ , \tag{5.22}$$

$$\ddot{A} + \left(H + 2\frac{\dot{I}}{I}\right)\dot{A} + 2g^2\frac{A^3}{a^2} = \frac{\lambda}{f}\dot{\sigma}g\frac{A^2}{aI^2} \ , \tag{5.23}$$

where

$$V_\varphi \equiv \frac{dV}{d\varphi} \ , \quad W_\sigma \equiv \frac{dW}{d\sigma} \ , \quad I_\varphi \equiv \frac{dI}{d\varphi} \ . \tag{5.24}$$

The equation for the scale factor $a(t)$ is given by

$$\dot{H} = -\left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}\dot{\sigma}^2 + \frac{2}{3}(\rho_E + \rho_B)\right) \ . \tag{5.25}$$

Now that we have obtained the basic equations, we can discuss the inflationary dynamics.

5.1.1 Inflationary dynamics

In this subsection, we analyze the background dynamics which eventually produces chiral gravitational waves on small scales.

First, we discuss initial conditions. We assume that the dilaton is energetically dominant and plays the role of an inflaton in the early stage of inflation. The dynamics during inflation can be described by slow-roll equations

$$3H^2 \simeq V \ , \qquad 3H\dot{\varphi} + V_\varphi \simeq 0 \ . \tag{5.26}$$

In addition to those, we assume that the axion also satisfies a slow-roll equation

$$3H\dot{\sigma} + W_\sigma \simeq 0 \ . \tag{5.27}$$

Now, we introduce the slow-roll parameters in this system

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \epsilon_\varphi + \epsilon_\sigma + \epsilon_E + \epsilon_B \ , \tag{5.28}$$

$$\eta_H \equiv \frac{\dot{\epsilon}_H}{H\epsilon_H} = 2\left(\epsilon_H + \frac{\epsilon_\varphi}{\epsilon_H}\frac{\ddot{\varphi}}{H\dot{\varphi}} + \frac{\epsilon_\sigma}{\epsilon_H}\frac{\ddot{\sigma}}{H\dot{\sigma}} + \frac{\epsilon_E}{\epsilon_H}\left(\frac{\dot{I}}{HI} + \frac{\dot{E}}{HE}\right) + \frac{\epsilon_B}{\epsilon_H}\left(\frac{\dot{I}}{HI} + \frac{\dot{B}}{HB}\right)\right) \ , \tag{5.29}$$

with

$$\epsilon_\varphi \equiv \frac{1}{2}\frac{\dot{\varphi}^2}{H^2} \ , \quad \epsilon_\sigma \equiv \frac{1}{2}\frac{\dot{\sigma}^2}{H^2} \ , \quad \epsilon_E \equiv \frac{2}{3}\frac{\rho_E}{H^2} \ , \quad \epsilon_B \equiv \frac{2}{3}\frac{\rho_B}{H^2} \ . \tag{5.30}$$

Here, we assume the relation $\epsilon_\sigma \ll \epsilon_H$ which can be realized easily. We can express the scale factor as a function of φ :

$$a \simeq a_0 \exp\left[-\int_{\varphi_0}^{\varphi}\frac{V}{V_\varphi}d\varphi\right] = a(\varphi) \ , \tag{5.31}$$

where the index "0" represents an initial value. Inspired by this result, to obtain an approximate relation $I(\varphi) \propto a^n$ during inflation we put the gauge kinetic function as[1]

$$I(\varphi) = I_0 \exp\left[-n \int_{\varphi_0}^{\varphi} \frac{V}{V_\varphi} d\varphi\right] , \tag{5.32}$$

where I_0 is a constant and n is a parameter which controls the strength of the gauge coupling.

In this paper, we consider a gauge field with a weak coupling constant. Namely, $I(\varphi)$ is a decreasing function $(n < 0)$ and we choose its initial value I_0 very large in order to get $g^{-1}I \sim O(1)$ at the end of inflation. This assumption is necessary to avoid the strong coupling problem. Hence the electric and magnetic components must be very small in order to satisfy slow-roll equations (5.26). The naturalness of this requirement depends on the effective potential of the gauge field, as we discuss later. Owing this assumption, we can neglect their non-linear effects as the leading order approximation. Then, Eq.(5.23) can be reduced to

$$\ddot{A} + \left(H + 2\frac{\dot{I}}{I}\right)\dot{A} \simeq 0 , \tag{5.33}$$

which can be integrated as

$$aI^2\dot{A} = \text{const.} \equiv C . \tag{5.34}$$

Thus, the energy density of electric field evolves as

$$\rho_E = \frac{3C^2}{2a^4I^2} \propto a^{-2(n+2)} . \tag{5.35}$$

Since we are interested in the solution where the energy density of the gauge field does not decay, we consider $n \leq -2$ region. On the other hand, the growing mode of magnetic energy density is proportional to $a^{-2(3n+4)}$ so that ρ_B grows more rapidly than ρ_E . Therefore, we can separate the inflationary history of this model into two stages. In the early stage of inflation, ρ_B is negligibly small compared to ρ_E, and hence its effect on the dynamics can be ignored in the background motion. We assume that primordial fluctuations on CMB scales are generated during this stage. In the late stage of inflation, however, ρ_B grows sufficiently fast and catches up with ρ_E . As we explicitly show later, during this stage, the interaction between the gauge field and the axion becomes important and an inflationary dynamics goes into an attractor solution, which is quite similar to the dynamics of chromo-natural inflation [36] . From now on, we explain these two inflationary stages separately.

The early stage of inflation

In an initial period, the energy density of the gauge field is so small that only a dilaton contributes an inflationary dynamics. However, when ρ_E grows sufficiently, the slow-roll condition for φ will be no more valid and modified as

$$3H\dot{\varphi} + V_\varphi \simeq 2\frac{I_\varphi}{I}\rho_E . \tag{5.36}$$

[1] Note that this specific choice is due to the analytical convenience for our discussion below. In principle, we can consider more general functional form for the gauge kinetic function. We expect that the results do not change even for other choices as long as the functional shape is similar in the relevant range of the dilaton field.

Substituting (5.35) into (5.36) , we can solve this equation [20] and find the gauge kinetic function as

$$a^4 I^2 \simeq -\frac{n^2}{n+2}\frac{3C^2}{2\epsilon_V V}(1 + D_1 a^{2(n+2)}) \,, \tag{5.37}$$

where D_1 is a constant of integration and $\epsilon_V \equiv \frac{1}{2}(\frac{V_\varphi}{V})^2$ is the slow-roll parameter in terms of the potential. Here, we note the relation

$$\epsilon_E \simeq -\frac{2(n+2)}{n^2}\epsilon_V \,. \tag{5.38}$$

We neglected the time dependence of $\epsilon_V V$ in the above. We can see that the second term in (5.37) becomes soon negligible when $n < -2$. Thus, ρ_E settles in a nearly constant value during this period

$$\rho_E \simeq -\frac{n+2}{n^2}\epsilon_V V \,. \tag{5.39}$$

That implies the effective n goes to -2 [14]. At this time the relation between ϵ_V and ϵ_φ reads

$$\sqrt{\epsilon_\varphi} \simeq \sqrt{\epsilon_V} + \frac{n\epsilon_E}{\sqrt{2\epsilon_V}} \,. \tag{5.40}$$

Remarkably, an electric component of gauge field finally supports the slow-roll inflation in this period. Then Hubble slow-roll parameters read

$$\epsilon_H \simeq \epsilon_\varphi + \epsilon_E \,, \tag{5.41}$$

$$\eta_H \simeq 2\left(\epsilon_H + \frac{\epsilon_\varphi}{\epsilon_H}\frac{\ddot{\varphi}}{H\dot{\varphi}} + \frac{\epsilon_E}{\epsilon_H}\left(\frac{\dot{I}}{HI} + \frac{\dot{E}}{HE}\right)\right) \,. \tag{5.42}$$

The intermediate stage of inflation

Up to here, we have neglected the non-linear effect of the gauge field. However, the energy density of magnetic field ρ_B grows and eventually becomes greater than ρ_E for $n \leq -2$. As the non-linear effect becomes important, we cannot use Eq.(5.34). Instead, we have to consider the full equation of motion for the gauge field (5.23). Here, we rewrite Eq.(5.23) into the equation for Q defined by (5.16)

$$\ddot{Q} + \left(1 + \frac{2}{3}\frac{\dot{I}}{HI}\right)3H\dot{Q} = -\left(\left(2 - \epsilon_H + 2\frac{\dot{I}}{HI}\right)H^2 Q + 2g^2 Q^3 - \frac{\lambda}{f}\dot{\sigma}\frac{gQ^2}{I^2}\right) \,, \tag{5.43}$$

which looks like an equation for a scalar field. Hence we can interpret the right hand side of Eq.(5.43) as an effective potential force of the gauge field. It is easy to read off the effective potential from Eq.(5.43) as

$$U_{\text{eff}}(Q) = \left(1 + \frac{\dot{I}}{HI}\right)H^2 Q^2 - \frac{1}{3}\frac{\lambda}{f}\dot{\sigma}\frac{gQ^3}{I^2} + \frac{1}{2}g^2 Q^4 \,. \tag{5.44}$$

Note that we neglected the slow-roll corrections.

We can understand the background dynamics of the gauge field from this effective potential. Let us focus on the coefficient of Q^2 term. Before inflation occurs, we assume that this coefficient was positive and Q is near the origin of its potential. However, we assume that at a certain time $\frac{\dot{I}}{HI} \simeq n \leq -2$ is realized and Q starts to roll down on the potential. Even if ρ_E modifies slow-roll dynamics, $\frac{\dot{I}}{HI} = -2 + O(\epsilon_H)$ holds because of (5.37) . Therefore, in an early period Q grows up. However, Q stops its growth when non-linear magnetic Q^4 term becomes important. Moreover, due to axion-gauge interaction Q^3 term, this effective potential gets two different minimum values. There is a trajectory where the gauge field finally settles in the deeper minimum of this potential. At this time, σ and Q obey slow-roll equations

$$3H\dot{\sigma} + W_\sigma = -3\frac{\lambda}{f}(\dot{Q} + HQ)gQ^2 , \tag{5.45}$$

$$\left(1 + \frac{2}{3}\frac{\dot{I}}{HI}\right)3H\dot{Q} + 2\left(1 + \frac{\dot{I}}{HI}\right)H^2Q + 2g^2Q^3 = \frac{\lambda}{f}\dot{\sigma}\frac{gQ^2}{I^2} . \tag{5.46}$$

Diagonalizing this system, we have

$$\left(1 + \frac{2}{3}\frac{\dot{I}}{HI} + \frac{1}{3}\Lambda^2 m_Q^2\right)3H\dot{\sigma} + \left(1 + \frac{2}{3}\frac{\dot{I}}{HI}\right)\left(W_\sigma + 3\frac{\lambda}{f}HgQ^3\right) - 2\left(1 + \frac{\dot{I}}{HI} + m_Q^2\right)\frac{\lambda}{f}HgQ^3 = 0 , \tag{5.47}$$

$$\left(1 + \frac{2}{3}\frac{\dot{I}}{HI} + \frac{1}{3}\Lambda^2 m_Q^2\right)3H\dot{Q} + \left(1 + \frac{\dot{I}}{HI} + \left(\frac{1}{2}\Lambda^2 + 1\right)m_Q^2\right)2H^2Q + \frac{1}{3I}\Lambda m_Q W_\sigma = 0 , \tag{5.48}$$

where we defined two model parameters

$$\Lambda \equiv \frac{\lambda}{If}Q , \qquad m_Q \equiv \frac{gQ}{H} . \tag{5.49}$$

If $\Lambda^2 \gg 1$ and $m_Q^2 \gg \Lambda^{-2}$ hold, we get the following results

$$Q \simeq Q_{\min} = -\left(\frac{fW_\sigma}{3\lambda gH}\right)^{1/3} , \tag{5.50}$$

$$\frac{1}{2}\frac{\lambda}{I^2}\frac{\dot{\sigma}}{fH} \simeq m_Q + \frac{1}{m_Q}\left(1 + \frac{\dot{I}}{HI}\right) . \tag{5.51}$$

Remarkably, the background gauge field settles in the minimum value of its effective potential. This is essentially an attractor solution in chromo-natural inflation [36]. In our model, however, the condition $\Lambda \gg 1$ and the slow-roll equation (5.51) can be controlled by the gauge kinetic function. So we can realize this slow-roll trajectory for a broad parameter region compared to the original chromo-natural inflation. Interestingly, this inflationary dynamics generates parity-violating chiral gravitational waves. We discuss the detail of this mechanism later.

Finally, let us check the evolution of the gauge kinetic function during this period. In this period, the energy densities of the electric and the magnetic fields are given by

$$\rho_E \simeq \frac{3}{2}I^2H^2Q_{\min}^2 , \quad \rho_B \simeq \frac{3}{2}I^2g^2Q_{\min}^4 = m_Q^2\,\rho_E . \tag{5.52}$$

Hence if $m_Q \gtrsim 1$ holds, the slow-roll equation for φ reads

$$3H\dot{\varphi} + V_\varphi \simeq 2\frac{I_\varphi}{I}(\rho_E - \rho_B) . \tag{5.53}$$

Substituting (5.52) into the above equation, we obtaine the time evolution of $I(\varphi)$ as

$$I \simeq \left[\frac{3H^2 Q_{\rm min}^2}{4\epsilon_V V}(m_Q^2 - 1) + D_2\, a^{-2nV/3H^2} \right]^{-1/2} , \tag{5.54}$$

where D_2 is a constant of integration. Note that we neglected the terms which are suppressed in the slow-roll approximation. The first term in the square brackets is almost constant. Moreover, when the energy density of the axion becomes dominant in the intermediate stage of inflation $3H^2 \simeq W \gg V$, the second term in the square brackets is also almost constant. Hence, I decreases very slowly.

5.1.2 Numerical analysis

As a numerical example, let us show the background dynamics for the following potentials

$$V(\varphi) = \Lambda_\varphi^4 \exp\left[r\varphi\right] + V_{\rm st}(\varphi) , \tag{5.55}$$

$$W(\sigma) = \Lambda_\sigma^4 \left(1 - \cos(\sigma/f)\right) + W_{\rm st}(\sigma) , \tag{5.56}$$

where Λ_φ and Λ_σ characterize energy densities of the dilaton and the axion. Here, r and f are constant parameters. In this model, we assume the potentials $V_{\rm st}$, $W_{\rm st}$ such that each scalar field gets stabilized in order to end the inflation at 60 number of e-foldings. However, in this thesis we do not discuss how to end this inflationary mechanism, leaving it for future work.

In Fig.5.1, we plotted the time evolution of the velocity of axion normalized by its energy density, and the time evolution of the gauge field $Q(t)$. As you can see, ρ_B catches up with ρ_E after 15 e-folds and the transition of the dynamics of the gauge field happens due to the non-linear effect. We can also see that $Q(t)$ stops its growth and oscillates when the magnetic energy density grows sufficiently, and finally settles in a constant value. Both variable finally settle in the attractor solution of Chromo-Natural inflation. Note that we used a super-Planckian decay constant $f > M_{\rm pl}$ in the plot by assuming it can be effectively produced by the combination of sub-Planckian decay constants derived from aligned multiple axions [30].

In the next section, we show that, in the late stage of inflation, one of circular polarization states of gauge field fluctuations are enhanced by the tachyonic instability and the rapid growth of the gauge field produces sizable circularly polarized gravitational waves.

5.2 Chiral gravitational waves

In this section, we consider a perturbed universe. To analyze the dynamics, we used the same notations in Sec.4.2. As to the two (pseudo) scalar fields, we decompose them into their background and perturbation variables

$$\varphi = \bar{\varphi}(t) + \delta\varphi , \qquad \sigma = \bar{\sigma}(t) + \delta\sigma . \tag{5.57}$$

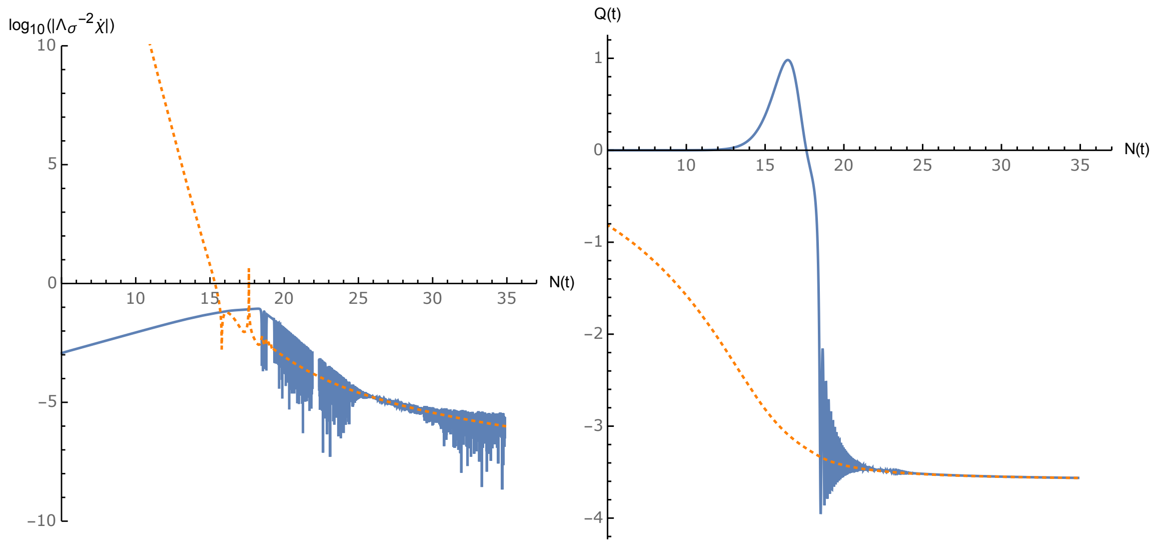

Figure 5.1: We plotted time evolutions of the amplitude of $\dot{\sigma}$ (solid blue line of left figure) and Q (solid blue line of right figure). Around at 15 e-folds, ρ_B catches up with ρ_E and the transition of the dynamics of the gauge field happens. By the non-linear effect, they settle in the attractor solution of delayed chromo-natural inflation (5.50) and (5.51) (dotted orange lines) after 20 e-folds. Note that the period when primordial CMB fluctuations are generated is assumed to be about from 5 to 10 e-folds in this plot. In both figures, we used initial conditions $(\varphi_0\ ,\sigma_0/f) = (4\ ,\pi/3)$ and the parameters $(\ \Lambda_\varphi,\ \Lambda_\sigma,\ r,\ n,\ g,\ \lambda,\ f\ ) = (\ 10^{-2},\ 2\times10^{-3}, 1,\ -2.01,\ 10^{-6},\ 10^{-1},\ 10\ )$.

5.2.1 Tensor perturbation dynamics

Let us analyze the dynamics of tensor perturbations in this model. Using a new variable $\psi_{ij} \equiv a(\tau)h_{ij}$, the quadratic action $S_{\rm EH}$ for tensor perturbations is given by

$$\delta S_{\rm EH} = \int dx d\tau \frac{1}{2}\left[\frac{1}{4}\psi'^{ij}\psi'_{ij} - \frac{1}{4}\psi^{ij,k}\psi_{ij,k} - \frac{1}{2}\left(\frac{a''}{a} - \left(\frac{a'}{a}\right)^2\right)\psi^{ij}\psi_{ij}\right], \tag{5.58}$$

where a prime represents a derivative with respect to a conformal time τ . We also have contributions to the quadratic action for tensor perturbations from the (pseudo) scalar actions as

$$\delta S_{\rm dilaton} + \delta S_{\rm axion} = \int dx d\tau \left(-\frac{a^2}{4}\psi^{ij}\psi_{ij}\right)\left[\frac{1}{2a^2}(\bar{\varphi}'^2 + \bar{\sigma}'^2) - V(\bar{\varphi}) - W(\bar{\sigma})\right]. \tag{5.59}$$

Moreover, using a new variable $t_{ai} \equiv a(\tau)I(\bar{\varphi})T_{ai}$, we obtain the quadratic actions for the tensor perturbations from the gauge sector

$$\begin{aligned}\delta S_{\rm gauge} \;=\; & \int dx d\tau \left(-\frac{1}{4}\right)\Bigg[-I^2\frac{(aQ)'^2}{2a^2}\psi^{ij}\psi_{ij} + \frac{3}{2}a^2I^2g^2Q^4\psi^{ij}\psi_{ij} - 2\left(t_i'^a t_i'^a + \frac{I''}{I}t_i^a t_i^a\right) + 2t^a_{i,j}t^a_{i,j} \\ & -4agQ\epsilon^{abi}t^b_j t^a_{j,i} + \frac{4I}{a}(aQ)'\psi^{ij}\left(t'_{ij} - \frac{I'}{I}t_{ij}\right) \\ & -4aIgQ^2\psi^{jm}\epsilon^a_{ij}t^a_{m,i} - 4a^2Ig^2Q^3\psi^{ij}t_{ij}\Bigg]\end{aligned} \tag{5.60}$$

and that from the Chern-Simons sector

$$\delta S_{\rm CS} = \int d\boldsymbol{x} d\tau \frac{1}{2I^2}\frac{\lambda}{f}\bar{\sigma}' \left(\epsilon^{ijk} t_i^a t_{k,j}^a - agQ t^{ij} t_{ij}\right) . \tag{5.61}$$

Using the slow-roll parameters and the following approximate scale factor

$$a(\tau) \simeq -\frac{1}{H\tau} , \tag{5.62}$$

we can write the total quadaratic action for tensor perturbations at the second order as

$$\begin{aligned}
\delta S_{\rm tensor} &\equiv \delta S_{\rm EH} + \delta S_{\rm scalar} + \delta S_{\rm gauge} + \delta S_{\rm CS} \\
&= \int d^3\boldsymbol{x} d\tau \frac{1}{2}\left[\frac{1}{4}\psi'^{ij}\psi'_{ij} - \frac{1}{4}\psi^{ij,k}\psi_{ij,k} + \frac{1}{2\tau^2}\psi^{ij}\psi_{ij}\right] \\
&+ \int d^3\boldsymbol{x} d\tau \left[\frac{1}{2}t_i'^a t_i'^a - \frac{1}{2}t_{i,j}^a t_{i,j}^a + \frac{1}{2}\left(\frac{I''}{I} - \frac{2m_Q\xi}{\tau^2}\right) t_i^a t_i^a - \frac{1}{\tau}\left(m_Q + \xi\right)\epsilon^{ijk} t_i^a t_{k,j}^a\right] \\
&+ \int d^3\boldsymbol{x} d\tau \left[\frac{\sqrt{\epsilon_E}}{\tau}\psi^{ij}\left(t'_{ij} - \frac{I'}{I}t_{ij}\right) - \frac{\sqrt{\epsilon_B}}{\tau}\psi^{jm}\epsilon^a_{ij} t^a_{m,i} + \frac{\sqrt{\epsilon_B} m_Q}{\tau^2}\psi^{ij} t_{ij} + \frac{\epsilon_E - \epsilon_B}{4\tau^2}\psi^{ij}\psi_{ij}\right] ,
\end{aligned} \tag{5.63}$$

where we defined a new variable

$$\xi \equiv \frac{\lambda}{2I^2}\frac{\dot{\bar{\sigma}}}{fH} . \tag{5.64}$$

In order to calculate the power spectrum of gravitational waves, we quantize these variables. The canonical quantization gives rise to the following expansion

$$\psi_{ij}(\boldsymbol{x},\tau) = 2\sum_{A=\pm}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\psi^A_{\boldsymbol{k}}(\tau) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} e^A_{ij}(\hat{\boldsymbol{k}}) , \tag{5.65}$$

$$t_{ij}(\boldsymbol{x},\tau) = \sum_{A=\pm}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3} t^A_{\boldsymbol{k}}(\tau) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} e^A_{ij}(\hat{\boldsymbol{k}}) . \tag{5.66}$$

where $e^A_{ij}(\hat{\boldsymbol{k}})$ are the polarization tensors which satisfy the following normalization relation, $e^{Aij}(\hat{\boldsymbol{k}}) e^B_{ij}(-\hat{\boldsymbol{k}}) = \delta^{AB}$, and the index "$A = \{+,-\}$" represents a circular polarization state defined by $ik^i\epsilon^a_{ij} e^{\pm}_{jm}(\hat{\boldsymbol{k}}) = \pm k e^{\pm}_{am}(\hat{\boldsymbol{k}})$.

Let us analyze the dynamics of tensor fluctuations of gauge field. Ignoring the backreaction of gravitational waves to gauge field, $t^{\pm}_{\boldsymbol{k}}$ obeys the following equation of motion

$$\frac{d^2 t^{\pm}_k}{dx^2} + \left(1 - \frac{d^2 I/dx^2}{I} + \frac{2m_Q\xi}{x^2} \mp \frac{2(m_Q + \xi)}{x}\right) t^{\pm}_k \simeq 0 , \qquad x \equiv -k\tau \tag{5.67}$$

On the other hand, the equation of motion for $\psi^{\pm}_{\boldsymbol{k}}$ with the backreaction of gauge field is approximately given by

$$\begin{aligned}
&\frac{d^2\psi^{\pm}_{\boldsymbol{k}}}{dx^2} + \left(1 - \frac{2}{x^2}\right)\psi^{\pm}_{\boldsymbol{k}} \simeq J^{\pm}_{\boldsymbol{k}} , \\
&J^{\pm}_{\boldsymbol{k}} \equiv 2\sqrt{\epsilon_E}\left(\frac{1}{x}v^{\pm}_{\boldsymbol{k}} + \frac{m_Q(m_Q \mp x)}{x^2} t^{\pm}_{\boldsymbol{k}}\right) , \quad v^{\pm}_{\boldsymbol{k}} \equiv \frac{dt^{\pm}_{\boldsymbol{k}}}{dx} - \frac{d(\ln I)}{dx} t^{\pm}_{\boldsymbol{k}} .
\end{aligned} \tag{5.68}$$

As to the homogeneous solution for graviton $\psi^{\pm}_{V\boldsymbol{k}}$, we can take the usual Bunch-Davis (BD) mode functions

$$\psi^{\pm}_{V\boldsymbol{k}} = \psi_{V\boldsymbol{k}} = \frac{1}{\sqrt{2k}}\left(1 + \frac{i}{x}\right)e^{ix} . \tag{5.69}$$

Here, we are interested in the peculiar solution $\psi^{\pm}_{S\boldsymbol{k}}$ sourced by gauge field

$$\psi^{\pm}_{S\boldsymbol{k}} = \int_0^{\infty} dx' G(x,x') J^{\pm}_k(x') , \tag{5.70}$$

$$G(x,x') = 2i\Theta(x'-x)\mathrm{Im}\left[\psi_{V\boldsymbol{k}}(x)\psi_{V\boldsymbol{k}}(x')\right] . \tag{5.71}$$

For the behaviour of $t^A_{\boldsymbol{k}}$, however, it is not easy to give its analytic solution because $I(\bar{\varphi})$, m_Q and ξ depend on time. Therefore, we approximately solve t^A_k in each inflationary stage as discussed in Sec.5.1.1 , and estimate the GW spectrum analytically in each stage.

5.2.2 Tensor spectrum in the early stage of inflation

Let us estimate the GW spectrum in the early stage of inflation. During this stage, m_Q and ξ are negligible so that the equation of motion for $t^{\pm}_k$ (5.67) is approximately given by

$$\frac{d^2 t^{\pm}_k}{dx^2} + \left(1 - \frac{d^2 I/dx^2}{I}\right) t^{\pm}_k \simeq 0 . \tag{5.72}$$

The time evolution of $I(\bar{\varphi})$ depends on n and the initial value of the gauge field. If the gauge field grows sufficiently, however, we obtain the transition to the attractor $I \propto a^{-2}$ as we showed in (5.37) . Therefore, we can approximately calculate as

$$\frac{dI/dx}{I} \sim \frac{2}{x} , \quad \frac{d^2 I/dx^2}{I} \sim \frac{2}{x^2} . \tag{5.73}$$

Thus, the mode fluctuations for the gauge field in this stage are

$$t^{\pm}_k \simeq \frac{1}{\sqrt{2k}}\left(1 + \frac{i}{x}\right)e^{ix} . \tag{5.74}$$

Therefore, in the super-horizon regime we obtain the following two-point function of gravitational waves

$$\Delta^{\pm}_h(k) \simeq \frac{H^2}{\pi^2 M_p^2}\left[1 + 4\left(\epsilon_E - m_Q\sqrt{\epsilon_E\epsilon_B} + \frac{2}{9}m_Q^2\epsilon_B\right)(\ln x)^2\right] . \tag{5.75}$$

The amplitude is not constant in the super-horizon due to the interaction with the gauge field. However, its modification will not be so serious unless the duration of this period is too large. This tensor spectrum in this stage is generated around CMB scales and we can see no parity-violation in the spectrum.

5.2.3 Chiral GWs in the intermediate stage of inflation

Now, we calculate the spectrum in the late stage of inflation. In this stage, ξ and m_Q become significant in the background dynamics. From the slow-roll equation (5.51), we have

$$\xi \simeq m_Q + \frac{1}{m_Q}\left(1 + \frac{\dot{I}}{HI}\right) . \tag{5.76}$$

The gauge kinetic function evolves as $x^{-nV/3H^2}$ ($\because$ (5.54)). Hence, we have

$$\frac{dI/dx}{I} \sim -\frac{\boldsymbol{n}}{x} , \quad \frac{d^2I/dx^2}{I} \sim \frac{\boldsymbol{n}(\boldsymbol{n}+1)}{x^2} , \quad \boldsymbol{n} \equiv nV/3H^2 \lesssim 1 . \tag{5.77}$$

Since the energy density of the axion is greater than that of the dilaton in this stage, this term becomes subdominant in the equation. Hence, (5.67) is approximated by

$$\frac{d^2 t_k^\pm}{dx^2} + \left(1 + \frac{A}{x^2} \mp \frac{2B}{x}\right) t_k^\pm \simeq 0 , \tag{5.78}$$

where

$$A \simeq 2\left(m_Q^2 + 1\right) , \qquad B \simeq 2m_Q + \frac{1}{m_Q} . \tag{5.79}$$

Moreover, we can neglect time dependence of A and B because m_Q is nearly constant during this time interval. Therefore, we get the same equation for $t_{\boldsymbol{k}}^\pm$ as in the cosmological perturbation of chromo-natural inflation. Here, we consider the parameter region where both A and B have positive values. We can see that the mass term of t_k^- is always positive, so this mode is stable. On the other hand, t_k^+ has the tachyonic instability during the time interval

$$\frac{1}{2}(B - \sqrt{B^2 - A}) < x < \frac{1}{2}(B + \sqrt{B^2 - A}) \tag{5.80}$$

and this instability can enhance one of helicity modes of gravitational waves.

Based on the discussion in the previous chapter, we reveal the chiral GW power spectrum given by (4.66) and (4.67) on small scales. We conclude that this model can avoid the overproduction of chiral gravitational waves on CMB scales discussed in previous works [60, 61] and produce a sizable circulary polarized gravitational waves on small scales. In the next section, we also study the dynamics of scalar perturbations and show that this model is stable under the scalar fluctuations and consistent with the CMB data.

5.3 Viability of the model

In this section, we check the dynamics of scalar perturbations. Specifically, we compute the curvature perturbation and its spectrum in the early stage of inflation where fluctuations on CMB scales are created[2]. Moreover, we discuss the stability of scalar perturbations in the late stage of inflation.

Let us derive the quadratic action for scalar perturbations. The Einstein-Hilbert action gives

$$\delta S_{\rm EH} = \int d\tau d\boldsymbol{x} \left[-12a^4H^2\phi^2 - 4a^2H\phi\partial^2\beta\right] , \tag{5.81}$$

[2]Throughout this model, we assume that curvature perturbation is provided by dilaton field.

where ϕ and β are perturbed lapse and shift functions defined in (4.27) . The (pseudo) scalar quadratic actions are given by

$$\begin{aligned}\delta S_{\text{dilaton}} =& \int d\tau d\boldsymbol{x} \Big[2a^4\dot{\bar{\varphi}}^2\phi^2 - 2\phi(a^3\dot{\bar{\varphi}}\delta\varphi' + a^4 V_{\bar{\varphi}}\delta\varphi) + a^2\dot{\bar{\varphi}}\beta\partial^2\delta\varphi \\ &+\frac{1}{2}a^2\delta\varphi'^2 + \frac{1}{2}a^2\delta\varphi\partial^2\delta\varphi - \frac{1}{2}a^4 V_{\bar{\varphi}\bar{\varphi}}\delta\varphi^2 \Big] , \\ \delta S_{\text{axion}} =& \int d\tau d\boldsymbol{x} \Big[2a^4\dot{\bar{\sigma}}^2\phi^2 - 2\phi(a^3\dot{\bar{\sigma}}\delta\sigma' + a^4 W_{\bar{\sigma}}\delta\sigma) + a^2\dot{\bar{\sigma}}\beta\partial^2\delta\sigma \\ &+\frac{1}{2}a^2\delta\sigma'^2 + \frac{1}{2}a^2\delta\sigma\partial^2\delta\sigma - \frac{1}{2}a^4 W_{\bar{\sigma}\bar{\sigma}}\delta\sigma^2 \Big] . \end{aligned} \tag{5.82}$$

For the quadratic action δS_{gauge} , we split it into two parts $\delta S^{\mathrm{I}}_{\text{gauge}} + \delta S^{\mathrm{II}}_{\text{gauge}}$: the former one includes only gauge field fluctuations

$$\begin{aligned}\delta S^{\mathrm{I}}_{\text{gauge}} =& \int d\tau d\boldsymbol{x} I^2 \Big[\frac{3}{2}(a\delta Q)'^2 - (aU)'\partial^2(aU)' + \frac{1}{2}a^4 Y\partial^4 Y - a^6 g^2 Q^2 Y\partial^2 Y \\ &-a^2 Y\left(\partial^2(a\delta Q)' + 2agQ\partial^2(aU)' - 2a^3\frac{\dot{(aQ)}}{a}g\partial^2 U\right) \\ &+a^2\delta Q\partial^2\delta Q - a^2\delta U\partial^4\delta U - 6a^3 gQ\delta Q\partial^2 U - a^4 g^2 Q^2\left(9\delta Q^2 - 2U\partial^2 U\right)\Big] , \end{aligned} \tag{5.83}$$

and the latter one has the interaction with other fields

$$\begin{aligned}\delta S^{\mathrm{II}}_{\text{gauge}} =& \int d\tau d\boldsymbol{x} \Big[4a^4\rho_E\phi^2 - \frac{2}{3}a^2\rho_B\beta\partial^2\beta - 4a^4\phi\frac{I_{\bar{\varphi}}}{I}(\rho_E + \rho_B)\delta\varphi \\ &+2\beta a I^2\left(\frac{\dot{(aQ)}}{a}a\partial^2\delta Q + gQ^2(a\partial^2 U)' + a^3 g^2 Q^3\partial^2 Y - \frac{\dot{(aQ)}}{a}a^2 gQ\partial^2 U\right) \\ &-2a^2 I^2(\phi - \frac{I_{\bar{\varphi}}}{I}\delta\varphi)\frac{\dot{(aQ)}}{a}\left(3(a\delta Q)' - a^2\partial^2 Y\right) \\ &-4a^3 I^2(\phi + \frac{I_{\bar{\varphi}}}{I}\delta\varphi)gQ^2\left(3agQ\delta Q + \partial^2 U\right) \\ &+\left(\frac{I_{\bar{\varphi}\bar{\varphi}}}{I} + \frac{I^2_{\bar{\varphi}}}{I^2}\right)(\rho_E - \rho_B)a^4\delta\varphi^2\Big] . \end{aligned} \tag{5.84}$$

Finally, the quadratic action δS_{CS} reads

$$\begin{aligned}\delta S_{\text{CS}} =& \int a d\tau d\boldsymbol{x} \Big[\frac{\lambda\dot{\sigma}}{f}\left(2\delta Q\partial^2 U + agQ(3\delta Q^2 - U\partial^2 U)\right) \\ &+\frac{\lambda}{f}\left(Ya^2 gQ^2\partial^2\delta\sigma + 6agQ^2\delta\sigma'\delta Q - 2\dot{(aQ)}\delta\sigma\partial^2 U\right)\Big] . \end{aligned} \tag{5.85}$$

5.3.1 Curvature perturbation spectrum in the early stage of inflation

The curvature perturbations in the uniform density gauge is defined by

$$-\zeta \equiv \psi + \frac{H}{\dot{\bar{\rho}}}\delta\rho , \tag{5.86}$$

where we defined the energy density of matter fields as $\rho = \bar{\rho} + \delta\rho$. The energy momentum tensor of matter is given by

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\text{matter}}}{\delta g^{\mu\nu}} \tag{5.87}$$

$$\equiv (\rho + P)u_\mu u_\nu + P g_{\mu\nu} \ , \tag{5.88}$$

where P is the pressure of matter and $u^\mu = \frac{dx^\mu}{d\tau}$ is the four-velocity which satisfies

$$u^\mu u_\mu = -1 \ . \tag{5.89}$$

Therefore, the background energy density reads

$$\begin{aligned}\bar{\rho} &= -\bar{T}^0_0 \\ &= \frac{1}{2}\dot{\bar{\varphi}}^2 + V(\bar{\varphi}) + \frac{1}{2}\dot{\bar{\sigma}}^2 + W(\bar{\sigma}) + \rho_E + \rho_B \ .\end{aligned} \tag{5.90}$$

and its fluctuation at the first order is given by

$$\delta\rho = -\delta T^0_0 \tag{5.91}$$

$$\equiv \delta\rho_{\text{dilaton}} + \delta\rho_{\text{axion}} + \delta\rho_{\text{gauge}} \ , \tag{5.92}$$

where we split $\delta\rho$ into three variables in the second line, which are derived from each action S_{dilaton} , S_{axion} and S_{gauge} . More concretely, we have

$$\delta\rho_{\text{dilaton}} = \frac{1}{a^2}\bar{\varphi}'\delta\varphi' + V_{\bar{\varphi}}\delta\varphi - \frac{2}{a^2}\bar{\varphi}'^2\phi \ , \tag{5.93}$$

$$\delta\rho_{\text{axion}} = \frac{1}{a^2}\bar{\sigma}'\delta\sigma' + W_{\bar{\sigma}}\delta\sigma - \frac{2}{a^2}\bar{\sigma}'^2\phi \tag{5.94}$$

and

$$\begin{aligned}\delta\rho_{\text{gauge}} &= 6I^2\frac{(aQ)'^2}{a^4}\phi + 3\left(\frac{(aQ)'^2}{a^4} + g^2Q^4\right)II_{\bar{\varphi}}\delta\varphi \\ &+ I^2\frac{(aQ)'}{a^2}\left(3\frac{(a\delta Q)'}{a^2} - \partial^2 Y\right) + \frac{2}{a}I^2gQ^2\partial^2U + 6I^2g^2Q^3\delta Q \ .\end{aligned} \tag{5.95}$$

In the early stage of inflation, the background effect of the magnetic field is negligibly small. So we can set $g = 0$ and estimate the power spectrum of the curvature perturbation. We can easily eliminate β and solve for ϕ as

$$\phi = \frac{1}{2}\left(\frac{\dot{\bar{\varphi}}}{H}\delta\varphi + \frac{\dot{\bar{\sigma}}}{H}\delta\sigma + \frac{2}{aH}\frac{d(aQ)}{dt}I^2\delta Q\right) \ . \tag{5.96}$$

Eliminating Y and defining the following variables

$$\begin{aligned}\delta\varphi &= \frac{\Delta_\varphi}{a} \ , \quad \delta\sigma = \frac{\Delta_\sigma}{a} \ , \\ \delta Q &= \frac{\Delta_Q}{\sqrt{2}aI} \ , \quad U = \frac{\Delta_U}{\sqrt{2}aI} \ ,\end{aligned} \tag{5.97}$$

we get the following quadratic action for scalar perturbations

$$
\begin{aligned}
\delta S_{\text{scalar}} \equiv\ & \delta S_{\text{EH}} + \delta S_{\text{scalar}} + \delta S_{\text{gauge}} + \delta S_{\text{CS}} \\
=\ & \int d^3\boldsymbol{x} d\tau \frac{1}{2}\left[\Delta_\varphi'^2 + \Delta_\varphi \partial_i^2 \Delta_\varphi + \Delta_\sigma'^2 + \Delta_\sigma \partial_i^2 \Delta_\sigma + \Delta_Q'^2 + \Delta_Q \partial_i^2 \Delta_Q - \Delta_U' \partial_i^2 \Delta_U' - \Delta_U \partial_i^4 \Delta_U\right] \\
& - \int d^3\boldsymbol{x} d\tau \frac{1}{2}\left[M_\varphi^2 \Delta_\varphi^2 + M_\sigma^2 \Delta_\sigma^2 + M_Q^2 \Delta_Q^2 - M_U^2 \Delta_U \partial_i^2 \Delta_U\right] \\
& + \int d^3\boldsymbol{x} d\tau \left[-2\frac{\sqrt{2\epsilon_E}}{\tau}\frac{I_{\bar\varphi}}{I}\Delta_Q' \Delta_\varphi\right] \\
& + \int d^3\boldsymbol{x} d\tau \left[M_{\varphi\sigma}\Delta_\varphi \Delta_\sigma + M_{\varphi Q}\Delta_\varphi \Delta_Q + M_{\sigma Q}\Delta_\sigma \Delta_Q - M_{\sigma U}\Delta_\sigma \partial_i^2 \Delta_U - M_{QU}\Delta_Q \partial_i^2 \Delta_U\right] ,
\end{aligned}
\tag{5.98}
$$

where we defined effective mass parameters

$$M_Q^2 \equiv -\frac{2}{\tau^2} + \frac{\epsilon_E}{\tau^2}(6 - \epsilon_\varphi - \epsilon_\sigma - \epsilon_E) , \tag{5.99}$$

$$M_U^2 \equiv -\frac{2}{\tau^2} , \tag{5.100}$$

$$M_\varphi^2 \equiv -\frac{2}{\tau^2} + \frac{V_{\bar\varphi\bar\varphi} + \sqrt{2}V_{\bar\varphi}\sqrt{\epsilon_\varphi} - H^2\left(\epsilon_\varphi + \epsilon_\sigma + \epsilon_E\right)\epsilon_\varphi}{H^2\tau^2} - \frac{\epsilon_E}{\tau^2}\left(3\frac{I_{\bar\varphi\bar\varphi}}{I} - \left(\frac{I_{\bar\varphi}}{I}\right)^2 - \sqrt{2}\frac{I_{\bar\varphi}}{I}\sqrt{\epsilon_\varphi}\right) , \tag{5.101}$$

$$M_\sigma^2 \equiv -\frac{2}{\tau^2} + \frac{W_{\bar\sigma\bar\sigma} + \sqrt{2}W_{\bar\sigma}\sqrt{\epsilon_\sigma} - H^2\left(\epsilon_\varphi + \epsilon_\sigma + \epsilon_E\right)\epsilon_\sigma}{H^2\tau^2} \tag{5.102}$$

and

$$M_{\varphi\sigma} \equiv -\sqrt{\frac{\epsilon_\sigma}{2}}\frac{\epsilon_E I_{\bar\varphi}}{\tau^2 I} - \sqrt{\frac{\epsilon_\sigma}{2}}\frac{V_{\bar\varphi}}{H^2\tau^2} - \sqrt{\frac{\epsilon_\varphi}{2}}\frac{W_{\bar\sigma}}{H^2\tau^2} + \frac{\sqrt{\epsilon_\varphi}\sqrt{\epsilon_\sigma}\left(\epsilon_\sigma + \epsilon_\varphi + \epsilon_E\right)}{\tau^2} , \tag{5.103}$$

$$M_{\varphi Q} \equiv \sqrt{\frac{\epsilon_E}{2}}\frac{I_{\bar\varphi}}{I}\frac{(8 - \epsilon_E)}{\tau^2} + \frac{\sqrt{\epsilon_E \epsilon_\varphi}\left(-3 + \epsilon_\sigma + \epsilon_\varphi + \epsilon_E\right)}{\tau^2} - \sqrt{\frac{\epsilon_E}{2}}\frac{V_{\bar\varphi}}{H^2\tau^2} , \tag{5.104}$$

$$M_{\sigma Q} \equiv \frac{\sqrt{\epsilon_E \epsilon_\sigma}\left(-3 + \epsilon_\sigma + \epsilon_\varphi + \epsilon_E\right)}{\tau^2} - \sqrt{\frac{\epsilon_E}{2}}\frac{W_{\bar\sigma}}{H^2\tau^2} , \tag{5.105}$$

$$M_{\sigma U} \equiv -\frac{\lambda}{I^2 f}\frac{\sqrt{2\epsilon_E}}{\tau} , \tag{5.106}$$

$$M_{QU} \equiv \frac{\xi}{\tau} . \tag{5.107}$$

Note that in the above calculations we used the following

$$\frac{I'}{I} \sim \frac{2}{\tau} , \quad \frac{I''}{I} \sim \frac{2}{\tau^2} \tag{5.108}$$

and assumed $\dot{\bar\varphi}$, $\dot{\bar\sigma} > 0$ so that we can get $\frac{\dot{\bar\varphi}}{H} = \sqrt{2\epsilon_\varphi}$ and $\frac{\dot{\bar\sigma}}{H} = \sqrt{2\epsilon_\sigma}$. The fluctuation of the energy density is given by

$$\begin{aligned}\delta\rho = -\frac{1}{a^3}\Bigg[&\frac{\sqrt{2\epsilon_\varphi}}{\tau}\Delta'_\varphi + \frac{\sqrt{2\epsilon_\sigma}}{\tau}\Delta'_\sigma + \frac{\sqrt{2\epsilon_E}}{\tau}\left(\Delta'_Q - \frac{2}{\tau}\Delta_Q\right)\\ &+\left(\frac{\sqrt{2\epsilon_\varphi}}{\tau^2} - a^2 V_{\bar\varphi} - \frac{I_{\bar\varphi}}{I}\frac{\epsilon_E}{\tau^2}\right)\Delta_\varphi + \left(\frac{\sqrt{2\epsilon_\sigma}}{\tau^2} - a^2 W_{\bar\sigma}\right)\Delta_\sigma\\ &+\frac{\epsilon_\varphi+\epsilon_\sigma-2\epsilon_E}{\tau^2}\left(\sqrt{2\epsilon_\varphi}\Delta_\varphi + \sqrt{2\epsilon_\sigma}\Delta_\sigma + \sqrt{2\epsilon_E}\Delta_Q\right)\Bigg] . \end{aligned} \tag{5.109}$$

In order to estimate the curvature perturbation at the leading order, let us compare the magnitude of each coefficient (slow-roll parameters) in $\delta\rho$. From the discussion in Sec.5.1.1, ϵ_φ and ϵ_E are dominant in the early stage of inflation while ϵ_σ is negligible. Hence ζ in the flat slicing gauge is approximately given by

$$\zeta \simeq \frac{H\tau^2}{3\sqrt{2\epsilon_\varphi}}\left(1+\mathcal{E}_E^2\right)^{-1}\left[\Delta'_\varphi + \mathcal{E}_E\left(\Delta'_Q - \frac{2}{\tau}\Delta_Q\right) + 4\frac{1+\mathcal{E}_E^2}{\tau}\Delta_\varphi\right] , \tag{5.110}$$

where we defined a new variable

$$\mathcal{E}_E \equiv \sqrt{\frac{\epsilon_E}{\epsilon_\varphi}} . \tag{5.111}$$

We can see that $\mathcal{E}_E$ can be of order unity.

Let us calculate the power spectrum of ζ by using interaction picture. We canonically quantize the fields as

$$\begin{aligned}\Delta_\varphi(\boldsymbol{x},\tau) &= \int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\hat\Delta_{\varphi\boldsymbol{k}}(\tau)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\\ &= \int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\Delta_{\varphi k}(\tau)c_{\boldsymbol{k}} + \Delta^*_{\varphi k}(\tau)c^\dagger_{-\boldsymbol{k}}\right]e^{i\boldsymbol{k}\cdot\boldsymbol{x}} ,\end{aligned} \tag{5.112}$$

$$\begin{aligned}\Delta_Q(\boldsymbol{x},\tau) &= \int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\hat\Delta_{Q\boldsymbol{k}}(\tau)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\\ &= \int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\Delta_{Qk}(\tau)d_{\boldsymbol{k}} + \Delta^*_{Qk}(\tau)d^\dagger_{-\boldsymbol{k}}\right]e^{i\boldsymbol{k}\cdot\boldsymbol{x}} ,\end{aligned} \tag{5.113}$$

where $\Delta_{\varphi k}$ and Δ_{Qk} are mode functions in de Sitter spacetime. From (5.98), we can choose mode functions corresponding the BD vacuum as

$$\Delta_{\varphi k} \simeq \frac{1}{\sqrt{2k}}\left(1+\frac{i}{x}\right)e^{ix} , \tag{5.114}$$

$$\Delta_{Qk} \simeq \frac{1}{\sqrt{2k}}\left(1+\frac{i}{x}\right)e^{ix} . \tag{5.115}$$

The creation and annihilation operators $c_{\boldsymbol{k}}$, $d_{\boldsymbol{k}}$ satisfy

$$[c_{\boldsymbol{k}}\, , c^\dagger_{-\boldsymbol{k}'}] = [d_{\boldsymbol{k}}\, , d^\dagger_{-\boldsymbol{k}'}] = (2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k}') , \qquad [c_{\boldsymbol{k}}\, , d_{\boldsymbol{k}'}] = [c_{\boldsymbol{k}}\, , d^\dagger_{-\boldsymbol{k}'}] = 0 \tag{5.116}$$

and the vacuum state is defined by

$$c_{\boldsymbol{k}}\,|0\rangle = d_{\boldsymbol{k}}\,|0\rangle = 0 , \qquad \langle 0|0\rangle = 1 . \tag{5.117}$$

The interaction Hamiltonian at the leading order reads

$$H_I(\tau) = 4\int d\boldsymbol{x}\left[\frac{\mathcal{E}_E^2}{\tau^2}\Delta_\varphi^2 - \frac{\mathcal{E}_E}{\tau}\left(\Delta_Q' - \frac{2}{\tau}\Delta_Q\right)\Delta_\varphi\right] . \tag{5.118}$$

Thus, we expect that in the super-horizon limit $x \to 0$, the power spectrum of ζ takes the following form

$$\mathcal{P}_\zeta(k) = \langle in|\,\zeta_k(\tau)^2\,|in\rangle \simeq \frac{H^2}{4\epsilon_\varphi k^3}\left(1 + \mathcal{A}\,\mathcal{E}_E^2\,(\ln x)^2\right) , \tag{5.119}$$

where $\mathcal{A} \sim O(10)$ is a numerical factor. Here, we neglected the time-derivative of slow-roll parameters in the above estimation. This result is consistent with that for dilatonic inflation coupled to a triad of gauge fields [103]. This power spectrum is also not constant in the super-horizon due to the interaction with the gauge field.

5.3.2 Stability of scalar modes in the intermediate stage of inflation

We are interested in the stability conditions of scalar perturbations in the intermediate stage of inflation when the background gauge field settles in the attractor value and chiral gravitational waves are generated. For simplicity, we take the background gauge field function $Q(t)$ to be fixed in the following calculations. Moreover, we neglect the perturbations of the metric and set $\phi = \beta = 0$ because these contributions are suppressed by slow-roll parameters [60, 61].

After elimination of Y, we can define the canonical fields Δ_i by

$$\delta\varphi = \frac{\Delta_\varphi}{a} , \quad \delta\sigma = \frac{\Delta_\sigma}{a} ,$$
$$\delta Q = \frac{\Delta_Q}{\sqrt{2}aI} , \quad U = \frac{1}{\sqrt{2}kaI}\left(\frac{agQ}{k}\Delta_Q + \sqrt{\frac{k^2 + 2a^2g^2Q^2}{k^2}}\Delta_U\right) . \tag{5.120}$$

We derived the equations of motions for these scalar perturbations in the Appendix B. We are interested in the stability condition of scalar perturbations in the region $m_Q \gtrsim O(1)$ where sizable chiral gravitational waves are produced (see the previous section). For the time window $m_Q \ll x \lesssim \Lambda$, the equations of motions for perturbations at the leading order are given by

$$\frac{d^2\Delta_\varphi}{dx^2} + \left(1 + \frac{4I_{\bar\varphi}^2Q^2}{x^2}\right)\Delta_\varphi \simeq 0 , \tag{5.121}$$

$$\frac{d^2\Delta_\sigma}{dx^2} + \left(1 + \frac{\Lambda^2 m_Q^2}{x^2}\right)\Delta_\sigma - \frac{\sqrt{2}\Lambda m_Q}{x}\frac{d\Delta_Q}{dx} - \frac{\sqrt{2}\Lambda}{x}\Delta_U \simeq 0 , \tag{5.122}$$

$$\frac{d^2\Delta_Q}{dx^2} + \Delta_Q + \frac{\sqrt{2}\Lambda m_Q}{x}\frac{d\Delta_\sigma}{dx} \simeq 0 , \tag{5.123}$$

$$\frac{d^2\Delta_U}{dx^2} + \Delta_U - \frac{\sqrt{2}\Lambda}{x}\Delta_\sigma \simeq 0 . \tag{5.124}$$

Note that, for the coefficient of gauge-dilaton interactions $I_{\bar\varphi}Q$, we assumed that the background parameter region satisfies $|I_{\bar\varphi}Q| \ll \Lambda$, and neglected them in the above derivation. This assumption can be justified since I needs to become small enough to get a large Λ value, which is proportional to I^{-1}. Hence dilaton perturbations do not affect the stability on sub-horizon scales. In order to check the stability condition of the scalar dynamics, we focus on one peculiar solution. We use the WKB method for analyzing these equations. Substituting

$$\Delta_\varphi = C_1(x)e^{iS(x)}\,, \quad \Delta_\sigma = C_2(x)e^{iS(x)}\,, \quad \Delta_Q = C_3(x)e^{iS(x)}\,, \quad \Delta_U = C_4(x)e^{iS(x)}\,, \tag{5.125}$$

into Eqs.(5.121) - (5.124), we get

$$\left(1 + \frac{4I_\varphi^2 Q^2}{x^2} - S_x^2\right) C_1 \simeq 0\,, \tag{5.126}$$

$$\left(1 + \frac{\Lambda^2 m_Q^2}{x^2} - S_x^2\right) C_2 - \frac{\sqrt{2}\Lambda m_Q}{x} iS_x C_3 - \frac{\sqrt{2}\Lambda}{x} C_4 \simeq 0\,, \tag{5.127}$$

$$\left(1 - S_x^2\right) C_3 + \frac{\sqrt{2}\Lambda m_Q}{x} iS_x C_2 \simeq 0\,, \tag{5.128}$$

$$\left(1 - S_x^2\right) C_4 - \frac{\sqrt{2}\Lambda}{x} C_2 \simeq 0\,, \tag{5.129}$$

where $S_x \equiv dS/dx = \omega$ is the angular frequency and we neglected the time dependence of $C_i(x)$. Let us focus on the solution with $C_1 = 0$. Then, from (5.127) - (5.129), we get the relation

$$\left(1 + \frac{\Lambda^2 m_Q^2}{x^2} - \omega^2\right)\left(1 - \omega^2\right) - \frac{2\Lambda^2 m_Q^2}{x^2}\omega^2 - \frac{2\Lambda^2}{x^2} = 0\,. \tag{5.130}$$

Thus, we have two modes

$$\omega_\pm^2 = \frac{1}{2x^2}\left(2x^2 + 3\Lambda^2 m_Q^2\left(1 \pm \sqrt{1 + \frac{8x^2}{9\Lambda^2 m_Q^4}\left(m_Q^2 + 1\right)}\right)\right)\,. \tag{5.131}$$

Recalling that we are considering the region $x \lesssim \Lambda m_Q$, the plus mode ω_+^2 is approximately given by

$$\omega_+^2 \approx \frac{3\Lambda^2 m_Q^2}{x^2}\,. \tag{5.132}$$

Apparently this mode is positive and stable. On the other hand, for a minus mode ω_-^2 by expanding the square root in (5.131) with respect to $x^2/\Lambda^2 m_Q^2$, we get

$$\omega_-^2 \approx \frac{m_Q^2 - 2}{3m_Q^2}\,. \tag{5.133}$$

Thus, this mode has the instability in the region $m_Q < \sqrt{2}$ and we get the same result as in the previous works [60, 61], because the interaction of dilaton perturbations is not dominant in this stage. The scalar dynamics in the region $x \lesssim m_Q$ have the similar feature as that of Chromo-Natural inflation [61]. In this region, gauge fluctuations have no instability and Δ_φ and Δ_σ show the conventional power law behavior.

5.4 Discussions

In this section, we discuss constraints from the CMB observations and the possibility of generating chiral gravitational waves in this model.

In Sec.5.3.1 , we estimated the curvature perturbation spectrum on CMB scales. Its dimensionless power spectrum is given by

$$\frac{k^3}{2\pi^2}\mathcal{P}_\zeta(k) \simeq \frac{H^2}{8\pi^2\epsilon_\varphi}\left(1 + \mathcal{A}\,\mathcal{E}_E^2\,(\ln x_f)^2\right) \tag{5.134}$$

at the end of inflation $x = x_f$. Primordial fluctuations of CMB temperature are produced between $-60 \leq \ln x_f \leq -50$, so the second term is dominant unless $\mathcal{E}_E^2$ is negligibly small. The spectral index n_s at the end of inflation is given by

$$n_s - 1 = \frac{d\ln k^3\mathcal{P}_\zeta}{d\ln k} \simeq \frac{2\mathcal{A}\,\mathcal{E}_E^2\,(\ln x_f)}{1 + \mathcal{A}\,\mathcal{E}_E^2\,(\ln x_f)^2}\,. \tag{5.135}$$

This takes the value $2/(\ln x_f) \sim -0.04$ if the second term in the denominator in (5.135) is larger than the unity. Thus the spectral index will be nicely consistent with Planck data, even with $\mathcal{E}_E^2 \sim 1$ [6]. Moreover, from the power spectrum of gravitational waves given in (5.75) , we can estimate the tensor-to-scalar ratio as

$$r = \frac{\mathcal{P}_h^+ + \mathcal{P}_h^-}{\mathcal{P}_\zeta} \simeq 16\epsilon_\varphi^2\frac{\epsilon_\varphi^{-1} + 4\mathcal{E}_E^2(\ln x_f)^2}{1 + \mathcal{A}\,\mathcal{E}_E^2\,(\ln x_f)^2} \tag{5.136}$$

and the spectral tilt of tensor modes at the end of inflation

$$n_t = \frac{d\ln k^3\mathcal{P}_h^\pm}{d\ln k} \simeq \frac{8\epsilon_E\,(\ln x_f)}{1 + 4\epsilon_E\,(\ln x_f)^2}\,. \tag{5.137}$$

Note that we neglected the contribution of ϵ_B in (5.136) and (5.137) since its coefficient m_Q is sufficiently small in this epoch. Remarkably, r can be suppressed by a factor $\epsilon_\varphi/\mathcal{A}$ compared with the conventional single field slow-roll inflation. Moreover, n_t is also red but much closer to the scale invariance than that of scalar perturbations if $\mathcal{E}_E^2$ is sufficiently small. Therefore, without any difficulty, we can choose appropriate $\mathcal{E}_E^2$ and make the model consistent with the CMB constraints. Though we have not calculated the bispectrum of curvature perturbations and check the qualitative features of non-gaussianity, we expect that its constraint is not so strong unless the axion-gauge interactions are dominant on CMB scales [104].

The most remarkable prediction in this model is that primordial chiral gravitational waves are generated on small scales, which happens after 25 e-folds in our numerical example (see Fig.5.1). The frequencies of gravitational waves corresponding to CMB scales are around 10^{-18} Hz today [85], so chiral gravitational waves in our model can be detectable by gravitational wave interferometers whose sensitivity peaks are located above nHz frequency regions . From the discussion in Sec.5.2, the ratio of the amplitude of gravitational wave spectrum is approximately given by

$$\frac{\mathcal{P}_h^+(k)}{\mathcal{P}_h^-(k)} \simeq \begin{cases} 1 & (f \lesssim \text{nHz}) \\ 1 + 8|C_2|^2 Q_{\min}^2 \left|\mathcal{I}_0(m_Q) - m_Q\mathcal{I}_1(m_Q) + m_Q^2\mathcal{I}_2(m_Q)\right|^2 & (f \gtrsim \text{nHz})\,, \end{cases} \tag{5.138}$$

where $f = \frac{k}{2\pi}$ is the frequency of gravitational waves today. We expect that this two branches are smoothly connected at the transition epoch of two inflationary stages ($f \sim$ nHz).

Finally, let us estimate the intensity of gravitational waves in this model. The intensity derived for vacuum fluctuations is given by [85]

$$h_0^2\Omega_{\rm vac}(f) \simeq 10^{-15}\left(\frac{H}{10^{-5}}\right)^2 . \tag{5.139}$$

Considering CMB normalization in (5.134), Hubble parameter can be estimated as $10^{-7} \lesssim H \lesssim 10^{-5}$ for certain $\mathcal{E}_E^2$ region, then $h_0^2\Omega_{\rm vac}$ satisfies $10^{-19} \lesssim h_0^2\Omega_{\rm vac}(f) \lesssim 10^{-15}$. In the late stage of inflation, the enhancement of chirality from the vacuum case is like Fig.4.3, getting large value as m_Q increases, so there are some sets of parameter region $(Q_{\rm min},\ m_Q)$ where chiral gravitational waves can be observed by various detectors such as DECIGO ($h_0^2\Omega_{\rm GW} \gtrsim 10^{-15}$), LISA ($h_0^2\Omega_{\rm GW} \gtrsim 10^{-12}$) and SKA ($h_0^2\Omega_{\rm GW} \gtrsim 10^{-14}$).

Chapter 6

Summary & Discussions

In this thesis, we developed cosmologically new features through the physics of primordial gravitational waves sourced by the particle production of gauge field coupled to an axion during inflation. More precisely, we discussed models of axionic inflation with Abelian gauge field and non-Abelian gauge field, respectively, and explored their observable predictions of chiral gravitational waves with gravitational wave detectors.

In chapter 3, we introduced the phenomenology of axion-U(1) interactions during inflation and discussed the amplification of chiral gravitational waves sourced by the quantum loop effect of gauge field. Simultaneously, we also calculated the generation of curvature perturbation and clarified the correlation between the production of GWs and that of PBHs. In the conventional single gauge field model, the prediction of an observable primordial gravitational waves also leads to the detection of PBHs at small scales. At the end of this chapter, however, we introduced a model which overcomes above problems by including many U(1) gauge sectors coupled with axion. Moreover, we studied a new feature of generating chiral primordial gravitational waves from axion-gauge couplings with combinations of different signs, whose coupling forms are inspired by dimensional reductions of string theory. The main prediction is that several types of blue tenor spectra are provided due to the amplifications of each gauge group. If the couplings of axion to more than two gauge fields with different signs are included, both polarization modes of gauge fields experiences a tachyonic instability and enhances both helicity of metric tensor modes during inflation. As a result, we developed the possibility of generating chiral gravitational waves whose chirality is characterized by the number of U(1) gauge sectors from each gauge group with opposite signs. This prediction is completely different from the previous results since a sizable amplitude of tensor power spectrum is totally parity-violating in small scales. As an example, we described the dynamics of generating tensor power spectrum in the case of a Starobinsky-type potential. We found that both amplified helicity modes of graviton have the possibility to be detected by the future gravitational wave experiments such as LISA, DECIGO/BBO and SKA project. As to the dynamics of scalar curvature perturbation, we checked the time evolution of its two-point function and showed that it safely avoids the overproduction of PBHs in small scales by the frictional power of many Abelian gauge components.

In this work, however, we have not estimated the dynamics of the inflaton taking into account the back-reaction of gauge fields self-consistently so that the careful treatment of the evolution of curvature perturbation will be actually needed near the end of inflation where the back-reaction is strong [105, 106]. For this reason, we did not discuss the mechanism responsible for ending inflation. The back-reaction of gauge fields prolongs the total number of e-foldings compared with that without back-reaction. Hence, our present work might not give a precise prediction about the amount of gravitational waves since their whole signal should be effectively shifted towards lower frequencies due to the back-reaction effect. We expect, however, that the effect of back-reaction can be avoided if the velocity of axion has a peak value at interferometer scales and ξ is suppressed near the end of inflation, by considering an effect of a local potential form or a non-minimal coupling of axions to gravity for an example [40, 41].

It would be instructive to consider how robust our observational prediction is. It seems that similar results could appear when two axions couple with the gauge fields with the same signs and roll down on their potentials in an opposite direction each other. In this case, however, we will need fine-tuning for each parameter of axion's velocity or the number of gauge fields to realize our result because in the equation of motion for each gauge field the ξ parameter is not in common. In addition to that, there is another mechanism of generating chiral primordial gravitational waves [107–112], so it is worth considering whether we can discriminate between these theories through the observation of parity-violating tensor spectrum. Moreover, it is intriguing to study the case where axions play a role of spectator fields during inflation. We expect that in this case the similar result of this work would be also realized. Finally, in this work we investigated the dynamics of Abelian gauge fields for simplicity and neglected the effect of non-linear interaction terms of non-Abelian gauge fields. Hence, it will be important to explore the dynamics of axions coupled to non-Abelian gauge sectors during inflation. We leave these issues for our future work.

In chapter 4, we reviewed the physics of axionic inflation with SU(2) gauge field dubbed Chromo-Natural inflation and discussed its theoretical and observational consistency by computing perturbation dynamics. In this chapter, we show that this model is in conflict with the CMB measurement due to the overproduction of chiral gravitational waves sourced by the axion-gauge interactions. In chapter 5 we studied an inflationary dynamics with a dilaton and an axion coupled to a SU(2) gauge field. In particular, inspired by supergravity, we introduced a non-trivial gauge kinetic function to Chromo-Natural inflation. Consequently, the effective coupling constant of the axion to the gauge field is dynamically controlled due to the time variation of gauge kinetic function. For the initial conditions, we required that the gauge field is near the origin of its effective potential and the dilaton playing the role of inflaton initially is energetically dominant. Since the interaction of the axion to the gauge field is sufficiently small, we could neglect the axion in the background dynamics on CMB scales. We derived the power-spectrum of scalar and tensor perturbations in this early stage of inflation, and discussed its observational implication by looking at the spectral index and the tensor-to-scalar ratio. We found that they are characterized by the kinetic energy density of the gauge field and can be adjusted so that the model becomes consistent with the constraints from the current CMB observations. In particular, since the gauge field is inert in the early stage of inflation, the model does not show any conflict with the CMB constraints. The main prediction of this chapter is the generation of chiral gravitational waves on small scales. We found an inflationary solution where the gauge field grows due to the gauge-kinetic function and finally settles in a finite attractor value $Q_{\rm min}$, which realizes the delayed Chromo-Natural inflation. Then we analyzed tensor dynamics in the stage of Chromo-Natural inflation and computed gravitational wave spectrum by employing Green function method, and got the qualitatively similar result as Chromo-Natural inflation [61]. Namely, the general feature of tensor perturbations is the enhancement of chiral gravitational waves which is controlled by an attractor value $Q_{\rm min}$ and the mass parameter of the gauge field m_Q. Moreover, we checked the stability condition of scalar dynamics and showed that there is a stable parameter region $m_Q > \sqrt{2}$. Note that during this period we have chosen the gauge-kinetic function so that the interaction of the dilaton to the gauge field does not contribute to perturbation dynamics. We found that there is a parameter region where chiral gravitational waves are produced in an interesting frequency range, higher than nHz, which might be detectable in future gravitational wave interferometers and pulsar timing arrays, such as DECIGO, LISA and SKA.

In this work we did not clarify how to stabilize axion field on the way of inflation and how to end the inflationary stage. As to the next step we need to investigate the reheating stage in this model. We expect from our previous work [63] that at this epoch the effective mass parameter m_Q grows and then chiral gravitational waves are more enhanced, which might reach the current observational bounds on ground-based interferometers. In this paper, we studied our model with an isotropic initial condition. While in the case of Chromo-Natural inflation the initial anisotropy decays and the isotropic configuration is stable [101], the anisotropy generally remains in models with a gauge kinetic function for a non-Abelian gauge field [113]. Thus, it is intriguing to investigate the fate of the initial anisotropy in our model.

We leave these issues for future work.

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Appendix A

The Axion-Gauge Coupling From the String Theory

In this appendix, we briefly derive two types of the axion-gauge interaction from the string theory, especially the weakly coupled heterotic $E_8 \times E_8$ superstring as an example [114–116][1]. The relevant bosonic part of its low-energy effective action in the string frame reads [119]

$$S = \frac{1}{2\kappa_{10}^2} \int_{M_{10}} e^{-2\phi_{10}} \left[d^{10}x\sqrt{-g_{10}}R - \frac{1}{2}H_3 \wedge *_{10}H_3 - \frac{\kappa_{10}^2}{g_s^2}\mathrm{tr}(F_2 \wedge *_{10}F_2) \right] , \tag{A.1}$$

where R is Ricci scalar (g_n is a determinant of a n-dimensional metric), H_3 is the field strength of the Neveu-Schwarz 2-form B_2 , and F_2 is the field strength of $E_8 \times E_8$ gauge group living in the ten-dimensional space-time M_{10}. This action contains three coupling constants: the dilaton coupling $e^{\phi_{10}}$, the Yang-Mills coupling g_s and the gravitational coupling κ_{10} with dimensions of (length)^0, (length)^3 and (length)^4 respectively. We write the "tr" for the trace in the fundamental representation of E_8 . The symbol "$*$" is the Hodge dual operator so that $X \wedge *_n X = d^n x\sqrt{|g_n|}X_{M_1 M_2 \ldots M_p}X^{M_1 M_2 \ldots M_p}/p!$ is satisfied if X is a p-form. Hereafter, we usually write the products and powers of forms without the wedge symbol "$\wedge$" to keep notations compact, so that $X^2 \equiv X \wedge X$ and $|X|_n^2 \equiv X \wedge *_n X$. Note that we assume that the dilaton has been stabilized and integrated out in the action.

The gauge-invariant field strength H is given by $H_3 = dB_2 - \frac{\alpha'}{4}(\omega_{3Y} - \omega_{3L})$, where α' is the string tension and ω_{3Y} , ω_{3L} are the Yang-Mills and Lorentz Chern-Simons 3-forms ensuring absence of anomalies, so that the following Bianchi identity

$$0 = \int_\Sigma dH_3 = -\frac{\alpha'}{4} \int_\Sigma \left[\mathrm{tr}(F_2^2) - \boldsymbol{tr}(R_2^2) \right] \tag{A.2}$$

holds for the integration on any four-dimensional submanifold Σ embedded in the internal manifold. Note that the Riemann tensor is regarded as a 2-form $R_2 = (R_2)^p{}_q = d\omega_1{}^p{}_q + \omega_1{}^p{}_r \wedge \omega_1{}^r{}_q$ which is a tangent space matrix represented by the connection 1-form $\omega_1{}^p{}_q$ and the "$\boldsymbol{tr}$" is over the trace on the tangent space indices. As to the trace of the gauge field, we explicitly take care of two E_8 gauge factors $F_2^{(1)}$ and $F_2^{(2)}$ and separate it into $\mathrm{tr}(F_2^2) = \mathrm{tr}(F_2^{(1)2}) + \mathrm{tr}(F_2^{(2)2})$.

[1]The result of this section is also derived from the strong coupling limit of the $E_8 \times E_8$ heterotic superstring (M-theory) [117, 118].

Let us derive the axion-gauge couplings from the above action. As to the metric, we assume that it is separated into the four-dimensional space-time X_4 and the conpactified six-dimensional space Y_6 : $ds^2(\mathrm{M}_{10}) = ds^2(\mathrm{X}_4) + ds^2(\mathrm{Y}_6)$. Pseudo-scalar fields with axion-like properties generically arise from the component of B_2 field. Focusing on the zero-mode, we expand it as

$$B_2(\mathrm{M}_{10}) = b_2(\mathrm{X}_4) + l_s^2 \sum_{k=1}^{n} b_{0(k)}(\mathrm{X}_4)\, \omega_k(\mathrm{Y}_6) , \tag{A.3}$$

where b_2 and $b_{0(k)}$ are the 2-form and the 0-form. As we show later, b_2 is reduced to the model-independent axion; namely it does not depend the choice of Y_6 very much. On the other hand $b_{0(k)}$ depends on the compactification of Y_6 , so it is called the model-dependent axion. Here, $n = \dim H^2(\mathrm{Y}_6)$ is the dimension of the basis of closed non-exact 2-form ω_k dual to the 2-cycles C_l on Y_6 representing a basis of $H_2(\mathrm{Y}_6)$ modulo torsion: $\int_{C_l} \omega_k = \delta_{lk}$. For dimensional reasons, we introduced the string length scale $l_s = 2\pi\sqrt{\alpha'}$. By the dimensional reduction from the kinetic term of B_2 field, we can simply get the following four-dimensional action

$$\frac{e^{-2\phi_{10}}}{4\kappa_{10}^2} L^6 \int_{\mathrm{M}_4} \left[-\frac{1}{2}|h_3|_4^2 - \left(\frac{l_s}{L}\right)^4 \sum_{k=1}^{n} |db_{0(i)}|_4^2 + \frac{\theta}{\pi}\left\{ dh_3 + \frac{\alpha'}{4}(\mathrm{tr}(F_2^2) - \boldsymbol{tr}(R_2^2)) \right\} \right] , \tag{A.4}$$

where L is the unit length of Y_6 , h_3 is the four-dimensional component of H_3 and we chose the basis ω_i to satisfy $\int_{\mathrm{Y}_6} \omega_i \wedge *_6\omega_j = L^2\delta_{ij}$. Here, we introduced a Lagurange multiplier θ in order to satisfy the Bianchi identity in our four-dimensional space-time. Integrating h_3 out and canonically normalizing each variable, we get the kinetic terms of axions and Chern-Simons terms coupled to the model-independent axion

$$\int d^4x \sqrt{-g_4} \left[-\frac{1}{2}\partial_\mu a \partial^\mu a - \sum_{k=1} \frac{1}{2}\partial_\mu a_k \partial^\mu a_k + \frac{\lambda}{4}\frac{a}{f_\theta}\left(\mathrm{tr}(F^{(1)}_{\mu\nu}\tilde{F}^{\mu\nu}_{(1)}) + \mathrm{tr}(F^{(2)}_{\mu\nu}\tilde{F}^{\mu\nu}_{(2)}) \right) \right] , \tag{A.5}$$

where

$$f_\theta \equiv \frac{e^{-\phi_{10}}L^3}{2\sqrt{2}\pi\kappa_{10}} = \frac{M_p}{2\sqrt{2}\pi} , \qquad \lambda \equiv 2\pi\alpha' f_\theta^2 = \frac{1}{16\pi^3}\left(\frac{M_p}{M_s}\right)^2 \tag{A.6}$$

is a decay constant of model-independent axion and a dimensionless coupling constant of its Chern-Simons gauge interaction characterized by the string mass scale $M_s \equiv l_s^{-1}$.

On the other hand, as to the model-dependent axions, they acquire the gauge couplings from the Green-Schwarz anomaly cancellation action [120]

$$S_{\mathrm{GS}} = \frac{1}{288(2\pi)^5\alpha'} \int_{\mathrm{M}_{10}} B \left[\mathrm{Tr}(F_2^4) - \frac{1}{300}\left[\mathrm{Tr}(F_2^2)\right]^2 - \frac{1}{10}\mathrm{Tr}(F_2^2)\boldsymbol{tr}(R_2^2) + ... \right] , \tag{A.7}$$

where the trace "Tr" is over the adjoint representation of E_8, which is related to the tr as Tr = 30 tr. Note that we have omitted the purely gravitational anomaly cancellation terms "..." in the above action in order to focus on the coupling of the gauge field. In terms of the following relationships $\mathrm{Tr}(F_2^4) = [\mathrm{Tr}(F_2^2)]^2/100$, Tr = 30 tr, we can get the following model-dependent axion-gauge coupling

$$\frac{1}{48(2\pi)^3}\sum_{k=1}^{n}\int_{\mathrm{X}_4} b_{0(k)}\left[\ \mathrm{tr}(F_2^{(1)2})\int_{\mathrm{Y}_6}\omega_k\left[\mathrm{tr}(F_2^{(1)2})-\frac{1}{2}\boldsymbol{tr}(R_2^2)\right]\right.$$
$$\left.+\mathrm{tr}(F_2^{(2)2})\int_{\mathrm{Y}_6}\omega_k\left[\mathrm{tr}(F_2^{(2)2})-\frac{1}{2}\boldsymbol{tr}(R_2^2)\right]\ \right]\ . \tag{A.8}$$

Note that we separated the trace of the gauge field $\mathrm{tr}(F_2^2)$ into $\mathrm{tr}(F_2^{(1)2})+\mathrm{tr}(F_2^{(2)2})$ in above equation. Then, using the Bianchi identity (A.2), we have

$$(\mathrm{A.8})=\frac{1}{48(2\pi)^3}\sum_{k=1}^{n} I_k\int_{\mathrm{X}_4} b_{0(k)}\ \left(\mathrm{tr}(F_2^{(1)2})-\mathrm{tr}(F_2^{(2)2})\right)\ , \tag{A.9}$$

where

$$I_k\equiv\int_{\mathrm{Y}_6}\omega_k\left[\mathrm{tr}(F_2^{(1)2})-\frac{1}{2}\boldsymbol{tr}(R_2^2)\right]=-\int_{\mathrm{Y}_6}\omega_k\left[\mathrm{tr}(F_2^{(2)2})-\frac{1}{2}\boldsymbol{tr}(R_2^2)\right]\ . \tag{A.10}$$

Therefore, the coupling of model-dependent axions to gauge fields in the four dimension has the following form

$$\sum_{k=1}^{n}\int d^4x\,\sqrt{-g_4}\frac{\lambda_k}{4}\frac{a_k}{f_k}\left(\mathrm{tr}(F^{(1)}_{\mu\nu}\tilde{F}^{\mu\nu}_{(1)})-\mathrm{tr}(F^{(2)}_{\mu\nu}\tilde{F}^{\mu\nu}_{(2)})\right)\ , \tag{A.11}$$

where

$$f_k\equiv\left(\frac{l_s}{L}\right)^2 f_\theta\ ,\qquad \lambda_k\equiv\frac{I_k}{12(2\pi)^3}\ . \tag{A.12}$$

Remarkably, the sign of each Chern-Simons interaction is opposite.

Thus, we get the two types of axion-gauge couplings in (A.5) and (A.11) from the dimensional reduction of $\mathrm{E}_8\times\mathrm{E}_8$ superstring. Other than the heterotic string theory, these sign combination of axion-gauge couplings could also be derived from the compactifications with D-branes through the generalized Green-Shwarz mechanism [121]. In Sec.3.5, we show that this interaction forms could provide the blue tensor spectra whose both polarization modes are largely enhanced in small scales.

Appendix B

EoMs for Scalar Perturbations

In this appendix, we list up the full equations of motion used for numerical calculations in Chapter 5. We set $\phi = \beta = 0$. Then, we have the following equations of motions for scalar perturbations

$$\frac{d^2\Delta_Q}{dx^2} + Q_1\Delta_Q + Q_2\Delta_U + Q_3\frac{d\Delta_\varphi}{dx} + Q_4\Delta_\varphi + Q_5\frac{d\Delta_\sigma}{dx} + Q_6\Delta_\sigma = 0\,, \tag{B.1}$$

$$\frac{d^2\Delta_U}{dx^2} + \mathcal{U}_1\Delta_U + \mathcal{U}_2\Delta_Q + \mathcal{U}_3\frac{d\Delta_\varphi}{dx} + \mathcal{U}_4\Delta_\varphi + \mathcal{U}_5\frac{d\Delta_\sigma}{dx} + \mathcal{U}_6\Delta_\sigma = 0\,, \tag{B.2}$$

$$\frac{d^2\Delta_\varphi}{dx^2} + \mathcal{P}_1\Delta_\varphi + \mathcal{P}_2\Delta_\sigma + \mathcal{P}_3\frac{d\Delta_Q}{dx} + \mathcal{P}_4\Delta_Q + \mathcal{P}_5\frac{d\Delta_U}{dx} + \mathcal{P}_6\Delta_U = 0\,, \tag{B.3}$$

$$\frac{d^2\Delta_\sigma}{dx^2} + \mathcal{S}_1\Delta_\sigma + \mathcal{S}_2\Delta_\varphi + \mathcal{S}_3\frac{d\Delta_Q}{dx} + \mathcal{S}_4\Delta_Q + \mathcal{S}_5\frac{d\Delta_U}{dx} + \mathcal{S}_6\Delta_U = 0\,. \tag{B.4}$$

As for the coefficients Q_i $(i = 1, ..., 6)$, we get

$$\begin{aligned}
Q_1 = &\frac{1}{x^4(x^2+2m_Q^2)}\Big(4m_Q^6 + 2m_Q^4\left(2 - 2d\ln I/dx\, x + 5x^2\right) + \left(1 - (d\ln I/dx)^2\right)x^6 \\
&+2m_Q^2x^2\left(1 - d\ln I/dx\, x + \left(3 - (d\ln I/dx)^2\right)x^2\right) \\
&-2\xi\left(2m_Q^5 + 5m_Q^3x^2 + 2m_Qx^4\right)\Big) - \frac{d^2I/dx^2}{I} + \frac{(dI/dx)^2}{I^2}\,,
\end{aligned} \tag{B.5}$$

$$\begin{aligned}
Q_2 = &\frac{1}{x^4(x^2+2m_Q^2)^{3/2}}\Big(8m_Q^7 + 8m_Q^5(1 - d\ln I/dx\, x) + 2m_Q^3(4 - 4d\ln I/dx\, x - 3x^2)x^2 \\
&+2m_Q(1 - d\ln I/dx\, x - x^2)x^4 - \xi(8m_Q^6 + 4m_Q^4x^2 - 2m_Q^2x^4 - x^6)\Big)\,,
\end{aligned} \tag{B.6}$$

$$Q_3 = -\frac{2\sqrt{2}I_{\bar\varphi}Q}{x}\,, \tag{B.7}$$

$$Q_4 = \frac{2\sqrt{2}I_{\bar\varphi}Q}{x^2}\left(1 - d\ln I/dx\, x + 2m_Q^2\right) - \frac{2\sqrt{2}Q dI_{\bar\varphi}/dx}{x}\,, \tag{B.8}$$

$$Q_5 = \frac{5}{\sqrt{2}}\frac{\lambda}{If}\frac{Qm_Q}{x}\,, \tag{B.9}$$

$$Q_6 = \frac{5}{\sqrt{2}}\frac{\lambda}{If}\frac{Qm_Q}{x^2}\,. \tag{B.10}$$

As for $\mathcal{U}_i$ $(i = 1, ..., 6)$, we have

$$\begin{aligned}
\mathcal{U}_1 =& \frac{1}{x^4(x^2+2m_Q^2)^2}\Big(16m_Q^8 + 16m_Q^6\left(1 - d\ln I/dx\, x + 2x^2\right) + \left(1-(d\ln I/dx)^2\right)x^8 \\
&+4m_Q^4x^2\left(6(1-d\ln I/dx\, x) + \left(6-(d\ln I/dx)^2\right)x^2\right) \\
&+2m_Q^2x^4\left(7-4d\ln I/dx\, x + 2\left(2-(d\ln I/dx)^2\right)x^2\right) \\
&-2\xi(8m_Q^7 + 12m_Q^5x^2 + 6m_Q^3x^4 + m_Qx^6)\Big) - \frac{d^2I/dx^2}{I} + \frac{(dI/dx)^2}{I^2}\,, \qquad \text{(B.11)}\\
\mathcal{U}_2 =& \mathcal{Q}_2\,, \qquad \text{(B.12)}\\
\mathcal{U}_3 =& \frac{2\sqrt{2}I_{\bar\varphi}Qm_Q}{x(x^2+2m_Q^2)^{1/2}}\,, \qquad \text{(B.13)}\\
\mathcal{U}_4 =& -\frac{2\sqrt{2}I_{\bar\varphi}Qm_Q}{x^2(x^2+2m_Q^2)^{3/2}}\Big(4m_Q^4 + 2m_Q^2(1-d\ln I/dx\, x - d\ln I_{\bar\varphi}/dx\, x + 2x^2) \\
&+(3-d\ln I/dx\, x - d\ln I_{\bar\varphi}/dx\, x + x^2)x^2\Big)\,, \qquad \text{(B.14)}\\
\mathcal{U}_5 =& -\frac{\lambda}{If}\frac{\sqrt{2}Qm_Q^2}{x(x^2+2m_Q^2)^{1/2}}\,, \qquad \text{(B.15)}\\
\mathcal{U}_6 =& -\frac{\lambda}{If}\frac{\sqrt{2}Q}{x^2(x^2+m_Q^2)^{3/2}}\left(2m_Q^4 + m_Q^2x^2 + x^4\right)\,. \qquad \text{(B.16)}
\end{aligned}$$

Then, as for $\mathcal{P}_i$ $(i = 1, ..., 6)$, we have

$$\begin{aligned}
\mathcal{P}_1 =& 1 + \frac{1}{x^2}\left(3\left(\frac{I_{\bar\varphi}^2}{I^2} + \frac{I_{\bar\varphi\bar\varphi}}{I}\right)(\epsilon_B - \epsilon_E) - 2 + \frac{V_{\bar\varphi\bar\varphi}}{H^2} + \frac{4I_{\bar\varphi}^2Q^2}{x^2+2m_Q^2}x^2\right)\,, \qquad \text{(B.17)}\\
\mathcal{P}_2 =& -\frac{\lambda}{If}\frac{2I_{\bar\varphi}Q^2m_Q}{x^2+2m_Q^2}\,, \qquad \text{(B.18)}\\
\mathcal{P}_3 =& -\mathcal{Q}_3\,, \qquad \text{(B.19)}\\
\mathcal{P}_4 =& \frac{2\sqrt{2}I_{\bar\varphi}Q}{x^2}\left(2m_Q^2 - d\ln I/dx\, x\right)\,, \qquad \text{(B.20)}\\
\mathcal{P}_5 =& -\mathcal{U}_3\,, \qquad \text{(B.21)}\\
\mathcal{P}_6 =& -\frac{2\sqrt{2}I_{\bar\varphi}Qm_Q}{x^2(x^2+2m_Q^2)^{3/2}}\left(4m_Q^4 + 2m_Q^2(-d\ln I/dx\, x + 2x^2) + (1-d\ln I/dx\, x + x^2)x^2\right)\,. \\
& \qquad \text{(B.22)}
\end{aligned}$$

Finally, as for $\mathcal{S}_i$ $(i = 1, ..., 6)$, we have

$$\mathcal{S}_1 = 1 + \frac{1}{x^2}\left(-2 + \frac{W_{\bar{\sigma}\bar{\sigma}}}{H^2} + \frac{\alpha^2}{I^2 f^2}\frac{Q^2 m_Q^2}{x^2 + 2m_Q^2}x^2\right) , \tag{B.23}$$

$$\mathcal{S}_2 = \mathcal{P}_2 , \tag{B.24}$$

$$\mathcal{S}_3 = -\mathcal{Q}_5 , \tag{B.25}$$

$$\mathcal{S}_4 = \frac{10}{\sqrt{2}}\frac{\alpha}{If}\frac{Qm_Q}{x^2}\left(1 + \frac{1}{2}d\ln I/dx\, x\right) , \tag{B.26}$$

$$\mathcal{S}_5 = -\mathcal{U}_5 , \tag{B.27}$$

$$\mathcal{S}_6 = -\frac{\lambda}{If}\frac{\sqrt{2}Q}{x^2(x^2 + 2m_Q^2)^{3/2}}\left(2m_Q^4(2 + d\ln I/dx\, x) + m_Q^2(3 + d\ln I/dx\, x)x^2 + x^4\right) . \tag{B.28}$$

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