

CRITICAL EXPONENTS OF SAW AT THE θ POINT IN TWO DIMENSIONS

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Duplantier and Saleur¹ have proposed the "exact" values of tricritical exponents: γ_t (entropy exponent) = $8/7$, ν_t (shape exponent) = $4/7$ and ϕ (crossover exponent) = $3/7$ for the self-avoiding walk (SAW) at the θ point in two dimensions. These values are derived, however, for a special model of SAW on the hexagonal lattice with randomly forbidden hexagons at the percolation threshold using Coulomb-gas techniques and conformal invariance. Here we examine these results from exact enumeration of SAW on the square lattice on the basis of the Orr model of a polymer chain in dilute solutions.

The partition function of the chain with nearest-neighbor interaction energy ε is written as

$$C_n(\omega) = \sum_{m=0} C_{n,m} e^{m\omega} \quad (1)$$

where $\omega = -\varepsilon/kT$ and $C_{n,m}$ is the number of n -step SAW with m non-bonded nearest-neighbors. The reduced free energy is given by

$$F^*(\omega) = \lim_{n \rightarrow \infty} n^{-1} \log C_n(\omega) \quad (2)$$

The reduced internal energy and the specific heat are obtained from $E^*(\omega) = \partial F^*(\omega) / \partial \omega$ and $C^*(\omega) = \partial^2 F^*(\omega) / \partial \omega^2$, respectively. The mean-square distance is also written as

$$R_n^2(\omega) = \left(\sum_{m=0} r_{n,m}^2 e^{m\omega} \right) / C_n(\omega) \quad (3)$$

where $r_{n,m}^2$ is the unnormalized square end-to-end distance.

Exact values of $C_{n,m}$ and $r_{n,m}^2$ were enumerated for $n \leq 24$. The locations $\omega_n^{(0)}$ of maximum points on $C_n^*(\omega)$ curves (see Fig.1) for different n are determined by exploiting the series $\{C_{n,m}\}$ obtained. The Neville tables for linear and quadratic extrapolants

$$\omega_n^{(r)} = \left(n\omega_n^{(r-1)} - (n-2r)\omega_{n-2}^{(r-1)} \right) / 2r \quad (4)$$

are formed for $r = 1$ and 2 . The tricritical point $\omega_t (= \lim_{n \rightarrow \infty} \omega_n^{(0)})$ is

determined by plotting these extrapolants against n^{-1} (Fig.2) after taking the average $(\omega_n^{(r)} + \omega_{n+1}^{(r)})/2$ in order to lessen the odd-even effect.

Following scaling assumptions are introduced:

$$C_n(\omega_t) \sim \mu_t^n n^{\gamma_t-1}; R_n^2(\omega_t) \sim n^{2\nu_t} \quad (5)$$

The growth parameter μ_t and the exponents γ_t and ν_t are determined from a conventional technique using Neville tables combined with the ratio method. The crossover exponent ϕ is also estimated from three kinds of approaches based on

$$|\Delta\omega_n| = |\omega_n^{(0)} - \omega_t| \sim n^{-\phi}; R_n^2(\omega) \sim n^{2\nu_t} f(|\Delta\omega_n|n^\phi) \quad (6a)$$

$$C_n(\omega) \sim \mu_t^n n^{\gamma_t-1} F(|\Delta\omega_n|n^\phi) \quad (6b)$$

The results thus estimated are listed in Table I together with those of Chang and Meirovich² from the scanning Monte Carlo method.

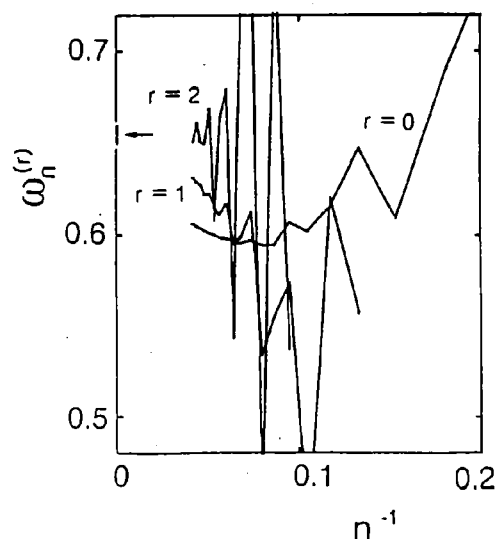
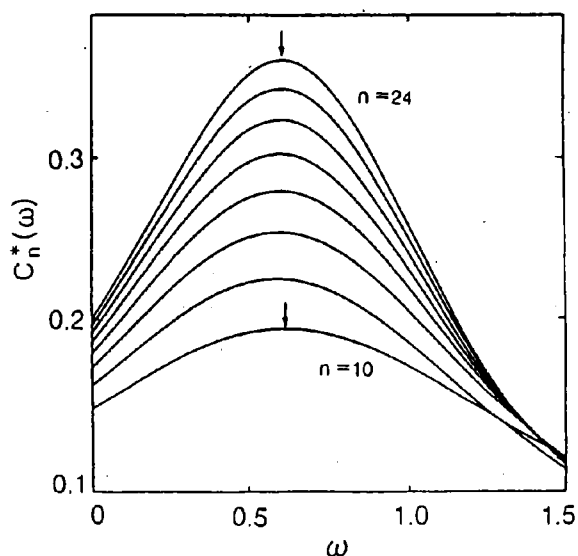


Fig.1 $C_n^*(\omega)$ curves for different n . Fig.2 Determination of ω_t .

Table I. Exponents for SAW at ω_t in two dimensions.

Method	ω_t	μ_t	ν_t	γ_t	ϕ
Series analysis	0.655	3.209	0.575	1.095	1.09
Monte Carlo [1]	0.658	3.213	0.574	1.11	0.594
Exact values[2]			0.571...	1.142...	0.428...

1. D.Duplantier and H.Saleur, Phys.Rev.Lett. 59, 539(1987).
2. I.Chang and H.Meirovich, Phys.Rev.E 48, 3656(1993).